DECENTRALIZED CONTROL OF WINDING SYSTEMS: A HYBRID EVOLUTIONARY-ALGEBRAIC APPROACH

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Abstract: This paper considers the problem of controlling a web transport system, using a 3-input 3-output nonlinear model that has been extensively validated against the real plant. There are two difficulties that arise when designing a controller for this industrial plant. The first one is the wide variation in the system dynamics due to the variation of radius and inertia of the rollers. Secondly, the multivariable controller should be decentralized to increase the reliability of the control loop. An approach for designing decentralized controllers that satisfy a H_2-performance measure and place the closed loop poles in a prescribed region is proposed. The approach is based on the combined use of Genetic Algorithms and Lyapunov equation solvers. A decentralized controller that meets all design specifications is presented, and the simplicity of the design procedure is demonstrated. Among the attractive features of the proposed technique is its computational efficiency and the fact that it does not need to be initialized.
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Keywords: Decentralized control, Genetic algorithms, Lyapunov Equation.

1. INTRODUCTION

This paper considers the problem of robust control of an elastic web transmission system, which appears often in industrial applications. The objective here is to transmit the web with the maximum speed without breaking or folding it. The high degree of coupling between tension and velocity of the web makes it difficult to suppress the cross-coupling influences (Koc *et al.*, 2002). Another important issue here is the reliability and integrity of the control loop, which can be better achieved with a decentralized controller structure, such that if one control loop fails the others continue to function safely.

It is well known that model based modern techniques such as LQG and H_∞ design lead to controllers of the same order as the plant. Moreover, imposing structural constraints like decentralized control becomes a difficult task, see e.g. (Iwasaki, 1999) . This fact has prevented such techniques from gaining widespread acceptance in industrial applications. The problem with fixed-structure design is the non-convexity of the problem (Geromel *et al.*, 1999). There have been many attempts to solve this problem, but common drawbacks of the approaches proposed so far are the excessive computation time required for the design, the sensitivity on initial values and the conservatism of the resulting controllers. For these reasons it turns out that in many practical situations, modern control techniques are not considered an attractive alternative to the use of manually-tuned PID controllers.

Control of winding systems for elastic webs has been considered in (Koc *et al.*, 2002), where it was shown that applying modern control design techniques like H_∞ synthesis using Linear Matrix Inequalities can improve the performance compared to classical PID controllers. The problem of fixed-structure controller design - which is the main concern here - was however not considered in this work. The method presented in this paper is related to the approach proposed in (Farag and Werner, 2004*b*), (Farag and Werner, 2004*a*); the main difference is the formulation of the problem in terms of a Lyaponuv equation rather than a Ricatti equation, and the use of a simpler performance measure (H_2 rather than H_∞). This change helped to reduce the computation time and increased the reliability of the method. On the other hand, if a more complex control objective is desired such as robust H_2 with stability multiplier, then the approach in (Farag and Werner, 2004*b*) will be more suitable.

2. PLANT MODEL

This section presents a nonlinear model of the web transmission system. The derivation of this model is based on a set of physical laws and omitted here for briefness, the interested reader can refer to (Koc *et al.*, 2002) for full details. The plant - shown in Figure 1 - is governed by the differential equations

$$\frac{d}{dt}\left(\frac{L}{1+\varepsilon_2}\right) = \frac{V_1}{1+\varepsilon_1} - \frac{V_2}{1+\varepsilon_2} \quad (1)$$

$$\frac{d(J_k\Omega_k)}{dt} = R_k(T_{k+1} - T_k) + K_kU_k + C_f \quad (2)$$

with the physical parameters

L: web length under stress,

T: tension of the elastic web,

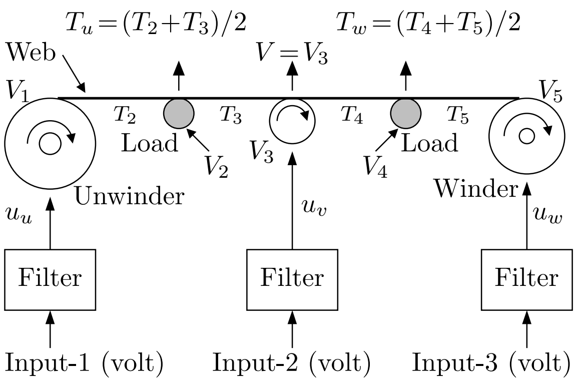

Fig. 1. Winding system

ε: web strain,

R_k: radius of the kth roll,

V_k: velocity of the kth roll,

Ω_k: rotational speed $\frac{V_k}{R_k}$ of the kth roll,

K_kU_k: motor torque of the kth roll,

J_k: inertia of the kth roll,

f_k: viscous friction of the kth roll,

C_f: sum of friction torques,

T_k, T_{k+1}: web tension before and after the roll k,

T_w, T_u: winding and unwinding tension respectively

u_u, u_v, u_w: the voltage inputs of the unwinding motor, traction motor, and winding motor respectively.

K_u, K_t, K_w: the torque constants of each motor.

E_0 : a constant depending on the elasticity modulus.

The control inputs are u_u, u_v and u_w, and the controlled outputs are the unwinding web tension T_u, the traction motor linear velocity V, and winding web tension T_w. Note that a low pass filter is added at the input of each motor to suppress measurement noise; the use of these filters is compulsory so they are considered as part of the model.

The controller design technique proposed in the next section requires a linear model of the above nonlinear system. This model is obtained by linearizing a simplified version of equations (1) and (2), around a nominal web tension and velocity T_0, V_0 respectively. A linear, time-varying state space model of the above system can then be written as

$$E_m\dot{x} = A(t)x + Bu$$
$$y = Cx$$

where

$$x^T = \left[\, J_1\Omega_1 \;\; T_2 \;\; V_2 \;\; T_3 \;\; V_3 \;\; T_4 \;\; V_4 \;\; T_5 \;\; J_5\Omega_5 \,\right]$$

$$u^T = \left[\, u_u \;\; u_v \;\; u_w \,\right]$$

$$y^T = \left[\frac{T_2+T_3}{2} \;\; V_3 \;\; \frac{T_4+T_5}{2}\right] = \left[\, T_u \;\; V \;\; T_w \,\right]$$

$$E_m = diag(1, L_1, J_2, L_3, J_3, L_3, J_4, L_4, 1)$$

$$A_1(t) = \begin{bmatrix} -\frac{f_1(t)}{J_1(t)} & R_1(t) & 0 & 0 \\ -E_0\frac{R_1(t)}{J_1(t)} & -V_0 & E_0 & 0 \\ 0 & -R_2^2 & -f2 & R_2^2 \\ 0 & V_0 & -E_0 & -V_0 \\ 0 & 0 & 0 & -R_3^2 \\ 0 & 0 & 0 & V_0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ E_0 & 0 & 0 & 0 & 0 \\ -f_3 & R_3^2 & 0 & 0 & 0 \\ -E_0 & -V_0 & E_0 & 0 & 0 \\ 0 & -R_4^2 & -f_4 & R_4^2 & 0 \\ 0 & V_0 & -E_0 & -V_0 & E_0\frac{R_5(t)}{J_5(t)} \\ 0 & 0 & 0 & -R_5(t) & -\frac{f_5(t)}{J_5(t)} \end{bmatrix}$$

$$A(t) = \left[\, A_1(t) \;\; A_2(t) \,\right]$$

$$B = \begin{bmatrix} -K_u & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & K_tR_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_w \end{bmatrix},\; C^T = \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Note that $A(t)$ is divided into two parts just to fit the column.

It is clear that the system matrix $A(t)$ varies with time due to the variation in radius and inertia $R_k(t)$ and $J_k(t)$, respectively, of the rollers. Obviously, R_k and J_k increases on the winding roller and decreases on the unwinding roller, their variation can be as much as 300%.

Controller Structure

To apply the design method proposed in this paper a nominal model must be selected. The starting phase is very important, to guarantee good overall performance, consequently, the nominal operating point is chosen with the radius and inertia corresponding to the starting phase, (full roller at unwinder and empty roller at winder). A useful observation given in (Koc *et al.*, 2002), is that the dc gain of the transfer functions from the inputs u_u and u_w to the web tensions T_u and T_w, are inversely proportional to the radii R_u and R_w, respectively. This observation can be used to construct a simple gain-scheduling strategy that reduces the influence of the variation in the roller radius, and improves robust performance of any designed controller. Following the gain scheduling methodology proposed in (Koc *et al.*, 2002), the motor inputs u_u and u_w are multiplied by the measured radii R_u and R_w respectively.

The standard H_2-synthesis used in this paper does not provide any direct tuning parameters that can be used to improve the reference tracking properties of the closed loop system. A standard technique to improve the tracking capabilities of H_2 controllers is to augment the nominal model with integral action as shown in Figure 2.

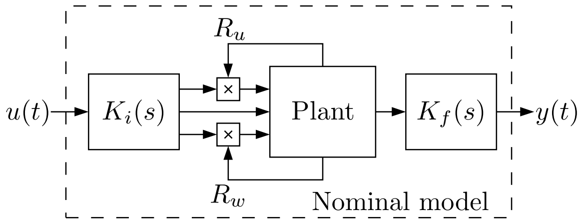

Fig. 2. Augmented nominal control

The transfer function of the integral controller $K_i(s)$ and the noise rejection filter $K_f(s)$ shown in Figure 1 are

$$K_i(s) = \begin{bmatrix} \frac{1}{s} & 0 & 0 \\ 0 & \frac{1}{s} & 0 \\ 0 & 0 & \frac{1}{s} \end{bmatrix} \tag{3}$$

$$K_f(s) = \begin{bmatrix} \frac{1}{\tau_1 s+1} & 0 & 0 \\ 0 & \frac{1}{\tau_2 s+1} & 0 \\ 0 & 0 & \frac{1}{\tau_3 s+1} \end{bmatrix} \tag{4}$$

The state space model of the augmented nominal model can be derived in straight forward manner as

$$\dot{x} = A_a x + B_a u$$
$$y = C_a x$$

3. CONTROLLER DESIGN

The physical model (5) is first embedded in a generalized plant model as shown in Figure 3, with state-space realization

$$\begin{aligned} \dot{x} &= A_a x + B_w w + B_a u \\ z &= C_z x + D_z u \\ y &= C_a x + D_w w \end{aligned} \tag{5}$$

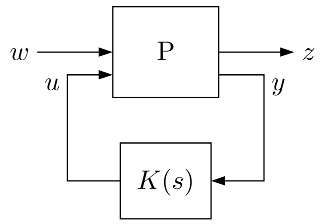

Fig. 3. Generalized plant

To represent an LQG cost function, the weighting matrices C_z, D_z, B_w and D_w are selected as

$$C_z = \begin{bmatrix} Q^{1/2} \\ 0 \end{bmatrix}, \quad D_z = \begin{bmatrix} 0 \\ R^{1/2} \end{bmatrix},$$
$$B_w = [Q_e^{1/2} \;\; 0], \quad D_w = [0 \;\; R_e^{1/2}] \tag{6}$$

where Q, R, Q_e and R_e are the usual LQG tuning parameters. The problem considered in this section is to determine a controller$K(s)$ with state space realization

$$\begin{aligned} \dot{\zeta} &= A_K\zeta + B_K y \\ u &= C_K\zeta \end{aligned} \tag{7}$$

such that the H_2 norm of the closed loop system is minimized. Substituting the controller (7) in (5) gives a closed-loop system $T(s)$ with state space realization

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}_w w \\ z &= \bar{C}_z\bar{x} \end{aligned} \tag{8}$$

where

$$\bar{x} = \begin{bmatrix} x \\ \zeta \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A_a & B_aC_K \\ B_KC_a & A_K \end{bmatrix}$$
$$\bar{B} = \begin{bmatrix} B_w \\ B_KD_w \end{bmatrix}, \quad \bar{C} = [C_z \;\; D_zC_K] \tag{9}$$

It is well known that the norm $\|T(s)\|_2$ of the above closed loop system can be computed by first solving

$$\bar{A}P + P\bar{A}^T + \bar{B}\bar{B}^T = 0 \tag{10}$$

for P, and computing

$$\|T(s)\|_2^2 = \text{trace}\,\bar{C}P\bar{C}^T$$

Note that the norm $\|T(s)\|_2$ will always be bounded for any stabilizing controller $K(s)$.

Hybrid Evolutionary-Algebraic (HEA) Design Approach

We now present an algorithm that uses a combination of GA and Lyaponuv equation solver to solve the above design problem. Note that designing a full order controller that minimizes $\|T(s)\|_2$ is a standard problem that, can be solved in a straightforward manner. However, with the controller order or structure fixed the problem becomes difficult to solve due to its non-convexity. In this paper both the controller order and structure are fixed by imposing the constraint that the controller belongs to the set

$$\mathcal{K} = \{K(s) = \text{diag}\,(K_1(s),\; K_2(s),\; K_3(s))\} \tag{11}$$

where

$$K_1(s) = \frac{b_{11}s + b_{01}}{a_{21}s^2 + a_{11}s + a_{01}}$$
$$K_2(s) = \frac{b_{12}s + b_{02}}{a_{22}s^2 + a_{12}s + a_{02}}$$
$$K_3(s) = \frac{b_{13}s + b_{03}}{a_{23}s^2 + a_{13}s + a_{03}}$$

Note that the number of parameters (decision variables) in each of the three transfer functions could be reduced to four by dividing all coefficients in $K_i(s)$ by a_{2i}, $i = 1, 2, 3$, respectively. However, the redundant variables a_{2i} are added deliberately to facilitate the initialization of the GA algorithm.

Let θ denote a parameter vector that contains all controller parameters, and let Θ denote the set of all admissible controller parameters such that $K(s) \in \mathcal{K}$. The design problem can be now stated as

$$\min_{\theta\in\Theta,P} \text{trace}\,\bar{C}(\theta)P\bar{C}^T(\theta)$$

subject to

$$\bar{A}(\theta)P + P\bar{A}^T(\theta) + \bar{B}(\theta)\bar{B}^T(\theta) = 0, \quad P > 0 \tag{12}$$

Note that the number of decision variables in θ is 15, while the number of decision variables in P is 210. With 225 decision variables an attempt use GA only to solve this problem is not advisable, because evolutionary techniques are unreliable for such large numbers of variables.

The idea now is to use GA to search for $K(s)$ (non-convex part); a standard Lyapunov solver is then used to calculate the unique solution P (convex part), to allow the computation of the objective function trace $\bar{C}P\bar{C}^T$. Note that a positive definite solution P will always exist as long as $K(s)$ is a stabilizing controller and $T(s)$ is strictly proper. These considerations motivate the following usage of a genetic algorithm.

Algorithm

- Generate an initial random population of controllers $\{\theta_1(s),\; \theta_2(s),\; \ldots,\; \theta_\mu(s)\}$
- Use the objective function

  $$f(\theta_i) = \begin{cases} \text{trace}\,\bar{C}(\theta)P\bar{C}^T(\theta), & \text{if } \bar{A} \text{ is stable} \\ \kappa(\bar{A}(\theta)) + \beta, & \text{if } \bar{A} \text{ is unstable} \end{cases}$$

  where $\kappa(\bar{A})$ stands for maximum real part of the eigenvalues of $\bar{A}$, and β is a penalty (e.g. 10^3) for destabilizing controllers
- Use ranking to determine fitness

It turns out that starting with a complete random population - which may contain controllers that do not stabilize the plant - creates no problem at all for the HEA algorithm. The above penalty based approach enables the HEA algorithm to search for stabilizing controllers in early generations, and to turn to the task of norm minimization later. This property gives the HEA approach an advantage over design techniques that cannot be started with random initialization.

The idea of breaking the problem up into a small non-convex part, solved by GA, and larger convex part solved by efficient algebraic solvers, has been presented before in (Farag and Werner, 2004*b*). The advantage of splitting the problem is the huge reduction in the size of the non-convex part (searching over 15 rather than 225 decision variables).

A straightforward application of the above idea to the linear model of the elastic web system leads initially to very slow controllers. An interesting observation is the fact that although the cost trace $\bar{C}P\bar{C}^T$ obtained using this approach is close to that obtained using the standard LQG approach, the time response is much slower. To overcome this problem, a pole region constraint is added to the problem formulation as follows:

$$\min_{\theta \in \Theta, P} \operatorname{trace} \bar{C}(\theta) P \bar{C}^T(\theta)$$

subject to $P > 0$ and

$$\bar{A}(\theta)P + P\bar{A}^T(\theta) + \bar{B}(\theta)\bar{B}^T(\theta) = 0, \quad \kappa(\bar{A}(\theta)) < -\kappa_0 \tag{13}$$

where $\kappa_0 > 0$ is a design parameter that controls the speed of response.

This additional constraint can be added by modifying the objective function according to

$$f(\theta_i) = \begin{cases} \operatorname{trace} \bar{C}(\theta)P\bar{C}^T(\theta), & \text{if } \kappa(\bar{A}(\theta)) < -\kappa_0 \\ \kappa(\bar{A}(\theta)) + \beta, & \text{if } \kappa(\bar{A}(\theta)) \geq -\kappa_0 \end{cases}$$

Thus the GA sees only a single objective function f, but this objective can be either a bound on the closed-loop pole locations or on the norm $\|T(s)\|_2^2$, depending on whether $\kappa(\bar{A}) < -\kappa_0$ is true or not. Experience has shown that the pole region constraint dominates the search only for a few early generations, while in the middle and final stage of the search the focus is on minimizing the norm $\|T(s)\|_2^2$.

4. SIMULATION AND RESULTS

The algorithm presented in the previous section is implemented on the 15th order linear model of the elastic web. The weighting matrices Q, Q_e, R and R_e in (6) are selected as:

$$Q = C^T C \ , \ R = \rho_r \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_e = BB^T \ , \ R_e = \rho_r \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This selection leaves the designer with two tuning parameters ρ_r and ρ_c, which simplifies the tuning strategy. All results given here were obtained using *Genetic Algorithm Direct Search Toolbox* developed by MathWorks (Goldberg, 1989). The population size and number of iterations by the GA are 40 and 500, respectively. The computer used is a Pentium4-2.2G with 512 MB Ram, the average computation time is 3-5 min. Once the controller $K(s)$ is designed using the HEA algorithm, a non-linear validation for its closed loop performance is performed using the control configuration shown on Figure 4. The pre-filter $K_r(s)$ is included to add a further degree of freedom, it can be tuned easily directly on the nonlinear simulation.

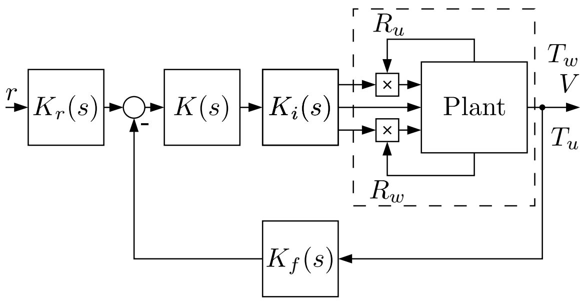

Fig. 4. Closed-loop system

Figures 5 and Figure 6 show nonlinear simulation results with a full-order LQG controller and a decentralized controller.The first observation is the high performance of the full-order LQG controller. On the other hand, for the decentralized controller it is more difficult to suppress the cross-coupling influences of both tension channels on the velocity channel. The large cross-perturbation in the velocity resulting from step changes in both tension channels is due to the restricted controller structure (the controller does not see what happens in the other channels). Nevertheless the decentralized controller achieves a better performance than the controllers presented in (Koc *et al.*, 2002), where it must be kept in mind however that the results shown in the latter work represent real-time experiments whereas the results presented here are only nonlinear simulations.

5. CONCLUSION

Modern robust design techniques like H_2 and H_∞ control can improve the achievable perfor-

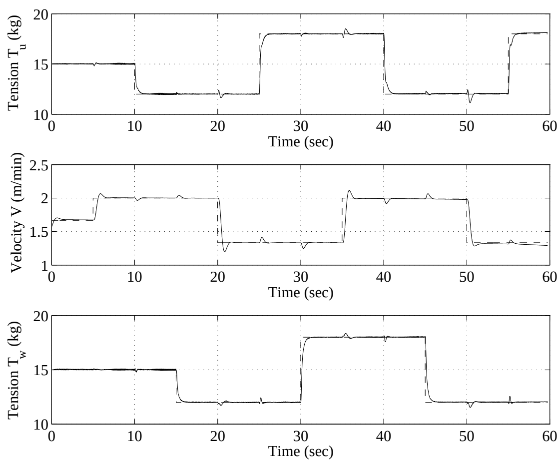

Fig. 5. Nonlinear simulation using full-order centralized controller, (dashed: reference signal, solid: system output)

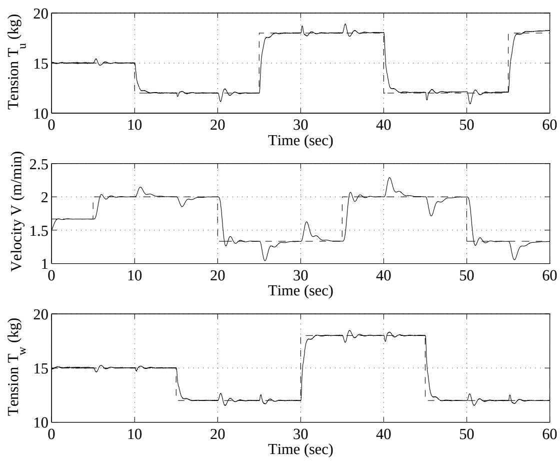

Fig. 6. Nonlinear simulation using decentralized controller, (Dashed: reference signal, solid: system output)

mance compared with classical PID controllers. But such techniques become very difficult to use when the controller structure is fixed. This paper illustrates a novel way of combining the power of global search algorithms such as GA with well established tools from linear control theory. Even though the idea is simple, it can lead to impressive results. Among the attractive features of the approach proposed here are its computational efficiency as well as the increased likelihood of converging to the global minimum. Moreover, the proposed technique does not require initialization with stabilizing controllers, which adds to its numerical reliability. Simulation results using a nonlinear model that has been extensively validated against experimental data confirms the high performance of the designed controllers compared with previously published results. The advantage claimed here is however not only the achieved performance but also the fact that only two tuning parameters were used to tune this controller. In contrast a large number of tuning parameters are involved in tuning the shaping filters of a mixed sensitivity loop-shaping approach. This advantage has an important impact on the reliability of the method presented here, since using a small number of tuning parameter leads to a reduction in the number of the design cycles required to tune a fixed-structure controller.

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