ROBUST STABILIZATION OF NONLINEAR PROCESSES USING HYBRID PREDICTIVE CONTROL [1]

Prashant Mhaskar, **Nael H. El-Farra** and **Panagiotis D. Christofides**

Department of Chemical Engineering, University of California, Los Angeles, CA 90095

Abstract: In this work, we consider nonlinear systems with input constraints and uncertain variables, and develop a robust hybrid predictive control structure that provides a safety net for the implementation of any model predictive control (MPC) formulation, designed with or without taking uncertainty into account. The key idea is to use a Lyapunov–based bounded robust controller, for which an explicit characterization of the region of robust closed–loop stability can be obtained, to provide a stability region within which any available MPC formulation can be implemented. This is achieved by devising switching laws that orchestrate switching between MPC and the bounded robust controller in a way that exploits the performance of MPC whenever possible, while using the bounded controller as a fall–back controller that can be switched in at any time to maintain robust closed–loop stability in the event that the predictive controller fails to yield a control move (due, for example, to computational difficulties in the optimization or infeasibility) or leads to instability (due, for example, to inappropriate penalties and/or horizon length in the objective function). The implementation and efficacy of the robust hybrid predictive control structure are demonstrated through simulations using a chemical process example.

Keywords: Input constraints, Lyapunov-based bounded control, Model predictive control, Controller switching, Hybrid processes and control, Stability region.

1. INTRODUCTION

Stabilization of nonlinear systems subject to uncertainty and manipulated input constraints is a fundamental control problem that has been the subject of significant research work. One of the control methods suited for handling constraints within an optimal control setting is model predictive control (MPC). Numerous research studies have investigated the stability properties of model predictive controllers for systems without uncertainty (see, for example, the review paper (Mayne *et al.*, 2000)).

The problem of analysis and design of predictive controllers for uncertain linear systems has been extensively investigated (see (Bemporad and Morari, 1999; Mayne *et al.*, 2000) for surveys of results in this area). For uncertain nonlinear systems, the problem of robust MPC design continues to be an area of ongoing research (see, for example, (Michalska and Mayne, 1993; Magni *et al.*, 2003)). While min-max formulations provide a natural setting within which to address this problem, computational problems with these

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approaches are well known, and stem in part from the nonlinearity of the model which typically makes the optimization problem non–convex and in part from performing the min–max optimization over the non-convex problem. Furthermore the robust stability guarantee in various MPC formulations (with or without stability conditions, and with or without robustness considerations) is contingent upon the assumption of initial feasibility, and the set of initial conditions starting from where feasibility and stability is guaranteed is not explicitly characterized.

Stabilizing control laws that provide explicitly–defined regions of attraction for the closed–loop system have been developed using Lyapunov techniques; the reader may refer to (Kokotovic and Arcak, 2001) for a survey of results in this area. In (El-Farra and Christofides, 2003*a*; El-Farra and Christofides, 2003*b*), a class of Lyapunov–based bounded robust nonlinear controllers, inspired by the results on bounded control originally presented in (Lin and Sontag, 1991), was developed. While these Lyapunov–based controllers have well–characterized stability and constraint–handling properties, they cannot, in general, be designed to be optimal with respect to a pre–specified, arbitrary cost function.

From the above discussion, it is clear that both MPC and Lyapunov–based analytic control approaches possess, by design, their own, distinct stability and optimality properties. Recently, we proposed a hybrid predictive control structure that employs switching between bounded control and MPC for the stabilization of linear systems under state (El-Farra *et al.*, 2004*b*) and output feedback (Mhaskar *et al.*, 2004), and in (El-Farra *et al.*, 2004*a*) for nonlinear systems, subject to input constraints while providing *a priori* (off–line) the set of initial conditions, for which closed–loop stability is guaranteed (through bounded control).

The presence of uncertainty, however, may alter the stability region of the nominal controllers (designed without taking the uncertainty into account) or even render the closed–loop system unstable. Furthermore, simply replacing the fall–back controller by an appropriate robust controller and implementing the same switching logics proposed in (El-Farra *et al.*, 2004*b*; El-Farra *et al.*, 2004*a*) may lead to switching that is too conservative, resulting in the implementation of the fall–back controller for almost all times. Motivated by these considerations, we consider in this work nonlinear systems with input constraints and uncertain variables, and develop a robust hybrid predictive control structure. The proposed method provides a safety net for the implementation of any available MPC formulation, designed with or without taking uncertainty into account, and allows for an explicit characterization of the set of initial conditions starting from where the closed–loop system is guaranteed to be stable. The key idea is to use a Lyapunov–based robust controller, for which an explicit characterization of the closed–loop stability region can be obtained, to provide a stability region within which MPC can be implemented. Switching laws are designed that exploit the performance of MPC whenever possible, while using the bounded controller to provide the stability guarantees. The implementation and efficacy of the robust hybrid predictive control structure are demonstrated through a chemical process example.

2. PRELIMINARIES

We consider nonlinear systems with uncertain variables and input constraints, described by:

$$\dot{x} = f(x) + G(x)u + W(x)\theta(t), \quad u \in \mathbf{U} \qquad (1)$$

where $x \in \mathbb{R}^n$ denotes the vector of state variables, $u \in \mathbb{R}^m$ denotes the vector of constrained manipulated inputs, taking values in a nonempty convex subset $\mathbf{U}$ of $\mathbb{R}^m$, where $\mathbf{U} = \{u \in \mathbb{R}^m : \|u\| \leq u_{max}\}$, $\|\cdot\|$ is the Euclidean norm of a vector, $u_{max} > 0$ is the magnitude of input constraints, and $\theta(t) = [\theta^1(t) \cdots \theta^q(t)]^T \in \Theta \subset \mathbb{R}^q$ denotes the vector of uncertain (possibly time–varying) but bounded variables taking values in a nonempty compact convex subset of $\mathbb{R}^q$ and $f(0) = 0$. The vector function $f(x)$, the matrices $G(x) = [g^1(x) \cdots g^m(x)]$ and $W(x) = [w^1(x) \cdots w^q(x)]$, where $g^i(x) \in \mathbb{R}^n$, $i = 1 \cdots m$, and $w^i(x) \in \mathbb{R}^n$, $i = 1 \cdots q$, are assumed to be sufficiently smooth on their domains of definition. The notation $L_f h$ denotes the standard Lie derivative of a scalar function $h(\cdot)$ with respect to the vector function $f(\cdot)$, the notation $x(T^-)$ denotes the limit of the trajectory $x(t)$ as T is approached from the left, i.e., $x(T^-) = \lim_{t \to T^-} x(t)$ and the notation $\partial\Omega$ is used to donate the boundary of a closed set, Ω. Throughout the manuscript, we assume that for any $u \in \mathbf{U}$ the solution of the system of Eq.1 exists and is continuous for all t, and we focus on the state feedback problem where measurements of the entire state, $x(t)$, are assumed to be available for all t.

2.1 Bounded robust Lyapunov–based control

Referring to the system of Eq.1, we assume that the uncertain variable term, $W(x)\theta$, is non–vanishing (in the sense that the origin is no longer the equilibrium point of the uncertain system) and that a robust control Lyapunov function (RCLF (Freeman and Kokotovic, 1996)), V exists. Consider also, the bounded state feedback control law (see (El-Farra and Christofides, 2003*a*;

El-Farra and Christofides, 2003*b*; El-Farra and Christofides, 2001) for details on controller design), $u = -k(x)(L_G V)^T$, where $k(x) =$

$$-\left(\frac{\alpha(x) + \sqrt{(\alpha_1(x))^2 + (u_{max}\beta(x))^4}}{(\beta(x))^2\left[1 + \sqrt{1 + (u_{max}\beta(x))^2}\right]}\right) \quad (2)$$

when $L_G V \neq 0$ and $u = 0$ when $L_G V = 0$, where $\alpha(x) = L_f V + (\rho\|x\| + \chi\theta_b\|L_W V\|)\left(\frac{\|x\|}{\|x\| + \phi}\right)$, $\alpha_1(x) = L_f V + \rho\|x\| + \chi\theta_b\|L_W V\|$, $\beta(x) = \|(L_G V)^T\|$, $L_G V = [L_{g^1}V \ \cdots \ L_{g^m}V]$ and $L_W V = [L_{w^1}V \ \cdots \ L_{w^q}V]$ are row vectors, θ_b is a positive real number such that $\|\theta(t)\| \leq \theta_b$, for all $t \geq 0$, and ρ, χ and ϕ are adjustable parameters that satisfy $\rho > 0$, $\chi > 1$ and $\phi > 0$. Let Π be the set defined by $\Pi(\theta_b, u_{max}) = \{x \in \mathbb{R}^n : \alpha_1(x) \leq u_{max}\beta(x)\}$ and assume that $\Omega := \{x \in \mathbb{R}^n : V(x) \leq c^{max}\} \subseteq \Pi(\theta_b, u_{max})$ for some $c^{max} > 0$. Then, given any positive real number, d, such that:

$$\mathbb{D} := \{x \in \mathbb{R}^n : \|x\| \leq d\} \subset \Omega \quad (3)$$

and for any initial condition $x_0 \in \Omega$, it can be shown that there exists a positive real number ϵ^* such that if $\phi/(\chi - 1) < \epsilon^*$, the states of the closed–loop system of Eqs.1–2 satisfy $x(t) \in \Omega \; \forall \; t \geq 0$ and $\limsup_{t\to\infty} \|x(t)\| \leq d$.

Remark 1: Referring to the above controller design, it is important to make the following remarks. First, a general procedure for the construction of RCLFs for nonlinear systems of the form of Eq.1 is currently not available. Yet, for several classes of nonlinear systems that arise commonly in the modeling of engineering applications, it is possible to exploit system structure to construct RCLFs. For example, for feedback linearizable systems, quadratic Lyapunov functions can be chosen as candidate RCLFs and can be made RCLFs with appropriate choice of the function parameters based on the process parameters (see, for example, (Freeman and Kokotovic, 1996)). Also, for nonlinear systems in strict feedback form, backstepping techniques can be employed for the construction of RCLFs (Freeman and Kokotovic, 1996). Second, given that an RCLF, V, has been obtained for the system of Eq.1, it is important to clarify the essence and scope of the additional assumption that there exists a level set, Ω, of V that is contained in Π. Specifically, the assumption that the set, Π, contains an invariant subset around the origin, is necessary to guarantee the existence of a set of initial conditions for which closed-loop stability is guaranteed (note that even though $\dot{V} < 0 \; \forall \; x \in \Pi\backslash\mathbb{D}$, there is no guarantee that trajectories starting within Π remain within Π for all times). Moreover, the assumption that Ω is a level set of V is made only to simplify the construction of Ω. This assumption restricts the applicability of the proposed control method because a direct method for the construction of an RCLF with level sets contained in Π is not available. However, the proposed control method remains applicable if the invariant set Ω is not a level set of V but can be constructed in some other way (which, in general, is a difficult task).

Remark 2: Regarding the choice of the above controller design, we note that the problem of designing control laws that guarantee stability in the presence of input constraints has been extensively studied (see, for example, (Lin and Sontag, 1991; Teel, 1992; Liberzon *et al.*, 2002; El-Farra and Christofides, 2003*a*; El-Farra and Christofides, 2003*b*)). The bounded robust controller design of Eq.2, proposed in (El-Farra and Christofides, 2003*a*; El-Farra and Christofides, 2003*b*) (inspired by the results on bounded control in (Lin and Sontag, 1991) for processes without uncertainty) is an example of a controller design that (1) guarantees robust stability in the presence of constraints, and (2) allows for an explicit characterization of the closed–loop stability region. The results of this paper are not limited to this particular choice of controllers and any other robust controller that satisfies (1) and (2) above, can be used.

2.2 Model Predictive Control

The MPC approach provides a framework with the ability to handle, among other issues, multivariable interactions, constraints on controls, and optimization requirements, all in a consistent, systematic manner. For the purpose of illustrating our results, we describe here a symbolic MPC formulation that incorporates most existing MPC formulations as special cases. This is not a new formulation of MPC; the general description is only intended for the purpose of highlighting the fact that the robust hybrid predictive control structure (to be proposed in the next section) can incorporate any available MPC formulation. In MPC, the control action at time t is conventionally obtained by solving, on–line, a finite horizon optimal control problem. The generic form of the optimization problem can be described as $u =$

$$\begin{aligned}&argmin\{max\{J_s(x,t,u(\cdot))|\theta(\cdot) \in \Theta\}|u(\cdot) \in S\}\\&s.t. \quad \dot{x}(t) = f(x(t)) + G(x)u + W(x)\theta(t) \quad (4)\\&x(0) = x_0, \;\; x(t+T) \in \Omega_{MPC}(x,t,\theta)\end{aligned}$$

where $J_s(x,t,u(\cdot)) =$

$$\int_t^{t+T} (x'(s)Qx(s) + u'(s)Ru(s))ds + F(x(t+T)) \quad (5)$$

and $S = S(t,T)$ is the family of piecewise continuous functions, with period Δ, mapping $[t, t+T]$ into the set of admissible controls, T is the horizon length and θ is the bounded uncertainty assumed to belong to a set Θ. A control $u(\cdot)$ in S is characterized by the sequence $\{u[k]\}$ where $u[k] := u(k\Delta)$ and satisfies $u(t) = u[k]$ for all $t \in [k\Delta, (k+1)\Delta)$. J_s is the performance index, R and Q are strictly positive definite, symmetric matrices and the function $F(x(t+T))$ represents a penalty on the states at the end of the horizon. The maximization over θ may not be carried out if the MPC version used is not a min-max type of formulation. The set $\Omega_{MPC}(x, t, \theta)$ could be a fixed, terminal set, or may represent inequality constraints (as in the case of MPC formulations that require some norm of the state, or a Lyapunov function for the process, to decrease at the end of the horizon). This stability constraint may or may not account for uncertainty. The stability guarantees in MPC formulations (with or without explicit stability conditions, and with or without robustness considerations, and whether or not it is a min-max type of formulation) are dependent on the assumption of initial feasibility. Obtaining an explicit characterization of the closed–loop stability region of the predictive controller under uncertainty and constraints remains a difficult task.

3. ROBUST HYBRID PREDICTIVE CONTROL STRUCTURE

The hybrid predictive controller is designed and implemented as follows (for a mathematical description of the controller and for more details, see (Mhaskar *et al.*, 2005)):

- Given the nonlinear process of Eq.1, θ_b and u_{max}, design the bounded robust controller of Eq.2, and calculate an estimate of its stability region Ω.
- Design/pick an MPC formulation (the MPC formulation could be min–max optimization based, linear or nonlinear, and with or without stability constraints). For convenience, we refer to the general MPC formulation of Eqs.4–5.
- Given any $x_0 \in \Omega$, check the feasibility of the optimization in Eqs.4–5 at $t = 0$, and if feasible, start implementing MPC (i.e., set $u(0) = M_s(x_0)$).
- If at any time, MPC becomes infeasible ($t = T_{inf}$), or the states of the closed–loop process approach the boundary of Ω ($t = T_s$) or the closed–loop states enter the set $\mathbb{D}$ ($t = T_D$) then switch to the bounded controller, else keep MPC active in the closed–loop process until a time T_{design}.
- Switch to the bounded robust controller at T_s, T_D, T_{design}, or T_{inf}, whichever comes earliest, to achieve practical closed–loop stability.

Remark 3: The purpose of switching to the bounded robust controller after the time T_{design} is to ensure convergence to $\mathbb{D}$ and avoid possible cases where the closed–loop states, under MPC, could wander inside Ω without actually converging to, and staying within, $\mathbb{D}$. Convergence to $\mathbb{D}$ could also be achieved (see, for example, (El-Farra *et al.*, 2004*b*; El-Farra *et al.*, 2004*a*)), by switching to the bounded controller when $\dot{V} \geq 0$ under MPC. However, in the presence of uncertainty, such a condition might be very restrictive in the sense that it may terminate MPC implementation too early. Note that if an MPC design is used that guarantees robust stability for the uncertain nonlinear process if initially feasible, it could be implemented for all time (T_{design} can be chosen to be practically infinity) to stabilize the closed–loop process. The stability safeguards, provided by the bounded controller, are still required because the stability of any MPC formulation, robust or otherwise, is based on the assumption of initial feasibility, which cannot be verified short of testing, via simulation, an initial condition for feasibility.

Remark 4: We note that while the MPC framework provides a transparent way of specifying a performance objective, the various MPC formulations, in general, may not be optimal, and only approximate the infinite horizon optimal cost to varying degrees of success. The choice of a particular MPC design can be made entirely on the basis of the desired tradeoff between performance and computational complexity because the stability guarantees of the robust hybrid predictive controller are independent of the specific MPC formulation being used.

Remark 5: Note that the switching scheme can be relaxed in order to take better advantage of the MPC performance. For example, instead of using a single backup controller, one may use a family of fall back controllers together with the MPC, to provide a larger region within which MPC can be safely implemented. Additionally, multiple switchings between MPC and the bounded controllers may be orchestrated, to allow for the possibility that the MPC, starting from a different initial condition, is able to stabilize the closed–loop system (for a demonstration, see the simulation example; for more details, see (Mhaskar *et al.*, 2005)). In particular, if the closed–loop state under the MPC starts to escape the stability region of the fall–back controller, the fall–back controller can be used to bring the closed–loop state trajectory further inwards, after which the supervisor may switch back to the MPC.

4. APPLICATION TO A CHEMICAL REACTOR

Consider the following model of an irreversible elementary exothermic reaction of the form $A \xrightarrow{k} B$ in a well–mixed continuous stirred tank reactor:

$$V_R \frac{dC_A}{dt} = F(C_{A0} - C_A) - k(T)C_A V \qquad (6)$$
$$V_R \frac{dT}{dt} = F(T_{A0} - T) - \frac{\Delta H}{\rho c_p} k(T)C_A V + \frac{Q}{\rho c_p}$$

where C_A denotes the concentration of species A, T and V_R denote the temperature and volume of the reactor, respectively, $k(T) = k_0 \exp\left(\frac{-E}{RT}\right)$ denotes the reaction rate, where k_0, E, ΔH denote the pre–exponential constant, the activation energy, and the enthalpy of the reaction, respectively, Q denotes the rate of heat input to the reactor, and c_p and ρ denote the heat capacity and density of the fluid in the reactor, respectively (the steady–state values and process parameters can be found in (Mhaskar *et al.*, 2005)). The control objective is to regulate both the reactor temperature and reactant concentration at the (open–loop) unstable equilibrium point by manipulating both the rate of heat input/removal and the inlet reactant concentration. Defining $x_1 = C_A - C_{As}$, $x_2 = T - T_s$, $u_1 = C_{A0} - C_{A0s}$, $u_2 = Q$, $\theta_1(t) = T_{A0} - T_{A0s}$, $\theta_2(t) = \Delta H - \Delta H_n$, where the subscript s denotes the steady–state value and ΔH_n denotes the nominal value of the heat of reaction, the process model of Eq.7 can be cast in the form of Eq.1. In all simulation runs, $\theta_1(t) = \theta_0 sin(3t)$, where $\theta_0 = 0.08T_{A0s}$ and $\theta_2(t) = 0.5\,(-\Delta H_{nom})$, and the manipulated input constraints were $|u_1| \leq 1.0\ Kmol/m^3$ and $|u_2| \leq 92\ KJ/s$.

A quadratic Lyapunov function was used to design the bounded robust controller of Eq.2, using $\chi = 1.01$, $\phi = 0.0001$, and $\rho = 0.01$. The term β^2 in the denominator of the control law of Eq.2 was replaced by the number $\nu = 0.001$ close to the origin to alleviate chattering of the control action (note that for this example, the denominator term $\beta^2 = 0$ if and only if $x = 0$). The set of nonlinear ODEs was integrated using the MATLAB solver, ODE45, and the optimization problem in MPC was solved using the MATLAB nonlinear constrained optimization solver, *fmincon*.

In the first scenario, a nominal nonlinear MPC formulation, without stability constraints, is used as part of the robust predictive control structure (setting $\Omega_{MPC} = \mathbb{R}^n$, $F(x(t+T)) = 0$, $T = 0.02$ minutes in Eqs.4–5) and with the design parameter $T_{design} = 10$ minutes. Shortly after the initial implementation of MPC, the supervisor detects, at $t = 0.6$ seconds, that the closed–loop states are close to the boundary of $\Omega(u_{max}, \theta_b)$ and therefore switches to the bounded robust controller to stabilize the closed–loop process (solid lines in Figures 1, 3). Note that the stability region information is completely contained in the value of the level set obtained at the time of the computation of the stability region (c^{max}) and the supervisor reaches this inference by simply evaluating the Lyapunov function, and comparing it to c^{max}. In the next scenario, a stabilizing formulation of MPC is used (requiring the states to go to a small invariant set at the end of the horizon), with a horizon length of $T = 0.02$ minutes and a $T_{design} = 20$ minutes. For the initial condition of the trajectory shown by the dashed lines in Figures 1 and 3, the MPC yields a feasible solution and drives the states close to the origin. For the initial condition depicted by the dotted lines in Figures 1, 3, however, the MPC does not yield a feasible solution, and therefore the supervisor initially implements the bounded robust controller, switching to the MPC at $t = 0.465$ minutes, when the MPC becomes feasible, and leads to closed–loop stability.

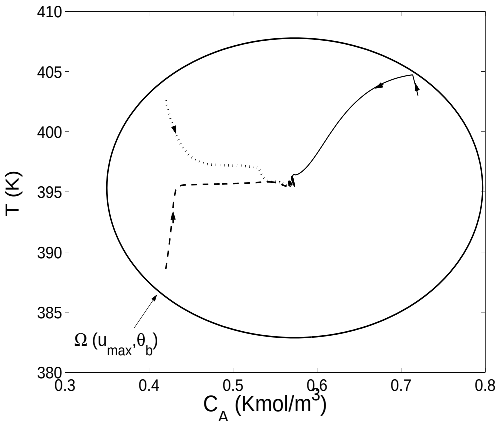

Fig. 1. Closed–loop state trajectory: implementation of the robust hybrid predictive controller using a nominal MPC formulation without stability constraints (solid line) and an MPC formulation with stability constraints, from two different initial conditions (dashed and dotted lines).

Finally, we demonstrate the relaxation of the switching scheme as discussed in Remark 5 using two Lyapunov functions. We use a nominal nonlinear MPC formulation, without stability constraints (setting $\Omega_{MPC} = \mathbb{R}^n$, $F(x(t+T)) = 0$, $T = 0.02$ minutes in Eqs.4–5) and with the design parameter $T_{design} = 3$ minutes. Starting from an initial condition within Ω_2 under MPC, the switching logic allows MPC to be implemented in the closed–loop even though the states escape out of Ω_2, since they are still within Ω_1 (see solid line in Figure 2 and dashed lines in Figure 3). Note also that in this case the nominal MPC does not enforce the desired degree of uncertainty attenuation (note the oscillations). After the time T_{design} has elapsed, the supervisor implements the bounded

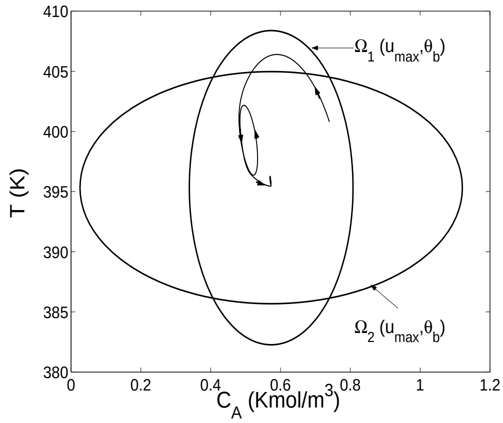

Fig. 2. Closed–loop state trajectory: implementation of the robust hybrid predictive controller discussed in Remark 5, using a nominal MPC formulation without stability constraints (solid line).

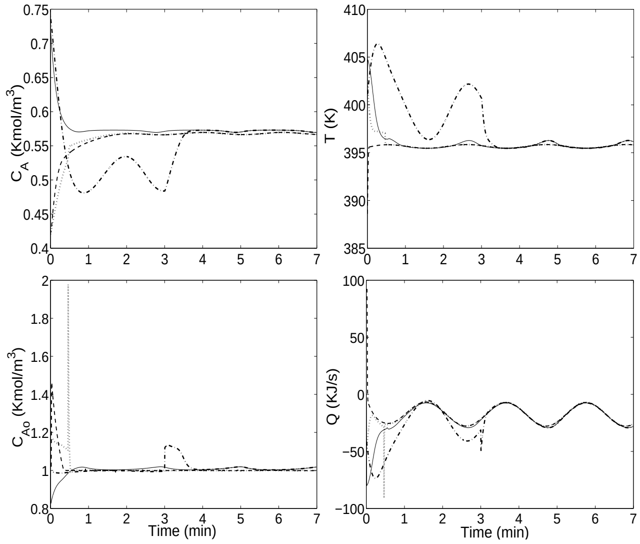

Fig. 3. Closed–loop state (top) and input (bottom) profiles: implementation of the robust hybrid predictive controller using a nominal MPC formulation without stability constraints (solid lines), an MPC formulation with stability constraints, from two different initial conditions (dashed and dotted lines) and the relaxed switching scheme discussed in Remark 5, using a nominal MPC formulation without stability constraints (dash–dotted lines).

controller (associated with Ω_2) in the closed–loop process to achieve practical stabilization.

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