NON-FRAGILE ADAPTIVE CONTROL OF A CLASS OF TIME-DELAY SYSTEMS

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Abstract: This paper focuses on the controller fragility and performance deterioration issues due to inaccuracies in controller implementation. It addresses the problem of non-fragile adaptive control problem for a class of continuous-time systems with state-delays and norm-bounded uncertainties against controller gain variations. Adaptive control schemes are constructed for the case of known gain pertubation bounds and then extended to accomodate unknown norm-bounded perturbations. All the developed results are conveniently expressed in LMI format. A detailed simulation results is presented to demonstrate the developed theory.

Keywords: Adaptive control, Time-delay systems, Stability, Non-fragile control.

1. INTRODUCTION

Considerable discussions on delays and their stabilization/destabilization effects in control systems have commanded the interests of numerous investigators in recent years Mahmoud (2000) and it become quite clear that there are various sources for delays including finite capabilities of information processing among different parts of the system, inherent phenomena like mass transport flow and recycling and/or by product of computational delays. On another research direction in the course of controller implementation based on different control design methods (including weighted $\mathcal{H}_\infty$, $\mathcal{H}_2$, μ and ℓ_1 synthesis techniques), it turns out that the controllers are very sensitive with respect to errors in the controller coefficients Keel and Bhattacharyya (1997). The sources for this include, but not limited to, imprecision in analogue-digital conversion, fixed word length, finite resolution instrumentation and numerical roundoff errors. By means of several examples, it is demonstrated Keel and Bhattacharyya (1997) that relatively small perturbations in controller parameters could even destabilize the closed loop system. Such controllers are often termed "fragile". Hence, it is considered beneficial that the designed (nominal) controllers should be capable of tolerating some level of controller gain variations. This illuminates the controller fragility problem for which some relevant results are available in Dorato (1998); Haddad and Corrado (1998) and further effort to alleviate this problem can also be found in Mahmoud (2004); Yang and Lin (2000); Yang and Wang (2001)

The objective of this paper is to contribute to the further development of non-fragile controllers for a class of uncertain systems. In the present work, we focus on the development of nonfragile adaptive controllers for a class of linear continuous-time systems with norm-bounded uncertainties and controller gain variations. We extend the results of Mahmoud (2004); Yang and Lin (2000); Yang and Wang (2001) to uncertain discrete-time

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systems with both both types of gain variations. Necessary and sufficient conditions are developed for both additive and multiplicative perturbations such that the resulting closed-loop feedback control system is quadratically stable for all admissible perturbations and uncertainties. These conditions are conveniently expressed in the form of linear matrix inequalities (LMIs). The feedback stabilization schemes are based on guaranteed cost control and $\mathcal{H}_\infty$ control approaches. System examples are provided to illustrate the theoretical developments.

Notations and Facts: In the sequel, the Euclidean norm is used for vectors. We use W^t, W^{-1}, $\lambda(W)$ and $||W||$ to denote, respectively, the transpose, the inverse, the eigenvalues and the induced norm of any square matrix W. We use $W > 0$ ($W < 0$) to denote a positive- (negative-) definite matrix W with $\sigma_M(W)$ being the maximum singular value of W. The Lebsegue space $L_2[0, \infty)$ consists of square-integrable functions on the interval $[0, \infty)$. The symbol $\bullet$ will be used in some matrix expressions to induce a symmetric structure, that is if given matrices $L = L^t$ and $R = R^t$ of appropriate dimensions, then

$$\begin{bmatrix} L + M + \bullet & \bullet \\ N & R \end{bmatrix} = \begin{bmatrix} L + M + M^t & N^t \\ N & R \end{bmatrix}$$

Fact 1: For any real matrices Σ_1 , Σ_2 and Σ_3 with appropriate dimensions and $\Sigma_3^t\ \Sigma_3 \ \leq\ I$, it follows that

$$\Sigma_1\Sigma_3\Sigma_2 + \Sigma_2^t\Sigma_3^t\Sigma_1^t \ \leq\ \alpha\ \Sigma_1\Sigma_1^t + \alpha^{-1}\ \Sigma_2^t\Sigma_2,$$
$$\forall\alpha > 0.$$

Fact 2: Let $\Sigma_1, \Sigma_2, \Sigma_3$ and $0 < R = R^t$ be real constant matrices of compatible dimensions and $H(t)$ be a real matrix function satisfying $H^t(t)H(t) \leq I$. Then for any $\rho > 0$ satisfying $\rho\Sigma_2^t\Sigma_2 < R$, the following matrix inequality holds:
$(\Sigma_3 + \Sigma_1 H(t)\Sigma_2)R^{-1}(\Sigma_3^t + \Sigma_2^t H^t(t)\Sigma_1^t) \leq \rho^{-1}\Sigma_1\Sigma_1^t + \Sigma_3\big(R - \rho\Sigma_2^t\Sigma_2\big)^{-1}\Sigma_3^t.$

2. PROBLEM STATEMENT

A schematic of the problem setup is displayed in Fig. 1 which shows a plant P subjected to uncertainties Δ_p and a controller K having gain perturbations Δ_c.

We consider the plant P to be represented by the following class of time-delay systems:

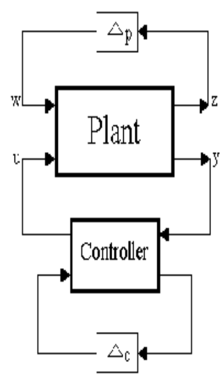
$$\dot{x}(t) = A_\Delta x(t) + B_o u(t) + A_{\Delta d}x(t-\tau)$$
$$y(t) = C_o x(t) \qquad (1)$$

where $x(t) \in \Re^n$ is the state vector; $u(t)\Re^p$ is the control input, $y(t) \in \Re^q$ is the controlled output, τ is a constant time-delay and the uncertain matrices $A_\Delta \in \Re^{n\times n}$, $B_\Delta \in \Re^{n\times p}$ and $A_{\Delta d} \in \Re^{n\times n}$, are represented by

$$[A_\Delta \quad A_{\Delta d}] = [A_o \quad A_d] + M\Delta_p(t)[N_a \quad N_d] \qquad (2)$$

whre $A_o \in \Re^{n\times n}$, $B_o \in \Re^{n\times p}$, $C_o \in \Re^{q\times n}$, $A_d \in \Re^{n\times n}$, $M \in \Re^{n\times\alpha}$, $N_a \in \Re^{\beta\times n}$ and $N_d \in \Re^{\beta\times n}$, are real and known constant matrices with $\Delta_p(t)$ is a matrix of uncertainties and bounded in the form $\Delta_p(t)\ \Delta_p^t(t) \ \leq\ I\ \forall\ t$. In the absence of uncertainties ($\Delta \equiv 0$), system (1) reduces to

$$\dot{x}(t) = A_o x(t) + B_o u(t) + A_d x(t-\tau)$$
$$y(t) = C_o x(t) \qquad (3)$$



It is a straightforward task to show that the nominal state-feedback controller

$$u(t) = 1/2\ B_o^t P\ x(t) \stackrel{\Delta}{=} K_o\ x(t) \qquad (4)$$

renders system (3) asymptotically stable for arbitrary constant delay $\tau \in [0 \rightarrow \tau^*]$ if given a matrix $0 < Q = Q^t \in \Re^{n\times n}$ there exists a matrix $0 < P = P^t \in \Re^{n\times n}$ such that the LMI

$$\begin{bmatrix} PA_o + A_o^t P & PA_d & PB_o \\ \bullet & -Q & 0 \\ \bullet & \bullet & -I \end{bmatrix} \ <\ 0 \qquad (5)$$

In practical situations, there are at least two sources of inaccuarcies when implementing controller (4). The first source is obviously due to the presence of uncertainties in the system matrices and the second source arises from gain perturbations due to various reasons Dorato (1998); Haddad and Corrado (1998). Therefore, it is naturally to consider, for a given nominal feedback controller $u(t) = K_o\ x(t)$, that the actual implemented controller is assumed to have two-terms:

$$u(t) = [\mu\ K_o + \Delta K(t)]\ u(t) \qquad (6)$$

where μ is an adjustable gain factor, K_o is the gain matrix to be determined and $\Delta K(t)$ represents the gain perturbation, which is assumed to be norm-bounded of the form:

$$||\Delta K(t)|| \leq \beta \tag{7}$$

where $\beta > 0$ is an upper bound to be dealt with in the subsequent analysis.

The problem of interest in this paper is to develop a feedback control scheme that ensures that the closed-loop system of (1)-(3) is asymptotically stable. Among the various possible approaches, we aim at constructing an adaptive scheme to achieve the cited design objective. Needless to stress the salient features of adaptive stabilization methods are well-established Narendra and Annaswamy (1989).

3. MAIN RESULTS

To achieve our goal, we will proceed in two stages. In the first stage, we attempt to construct an adaptive schemes for the uncertain time-delay system (1) assuming that the gain pertubation bound is known. Then in the second stage, we extend the results to accomodate bounded-but-known gain perturbations.

3.1 Known Perturbation Bound

When the gain perturbation bound is known, then the purpose of adaptation is to accomodate the uncertainties of system (1). The following adaptive scheme is proposed

$$\begin{aligned} u(t) &= [\tilde{\mu}\; K_o + \Delta K(t)]x(t) \\ \dot{\tilde{\mu}} &= -g\;\tilde{\mu} \;+\; x^t K_o^t \tilde{\mu}^{-1} K_o x \;\;, \\ \tilde{\mu}(0) &= \mu^+ \;\;,\;\; g > 0 \end{aligned} \tag{8}$$

where $K,\; g > 0$ represent, respectively, a control gain matrix and a growth factor, both will be determined in the sequel. A convnient Lyapunov functional $V(.)$ is given by

$$\begin{aligned} V_n(x,\mu) &= x^t(t)Px(t) + \int_{t-\tau}^{t} x^t(s)Qx(s)ds \\ &\quad + 1/2\;\tilde{\mu}^2 \end{aligned} \tag{9}$$

where $0 < P = P^t \in \mathbb{R}^{n\times n}$ and $0 < Q = Q^t \in \mathbb{R}^{n\times n}$. The following theorem summarizes the first main result:

Theorem 3.1. System (1) under the adaptive controller (8) is asymptotically stable if for a given matrix $0 < Q = Q^t$ and a scalar $\beta > 0$, there exist matrices $0 < X = X^t$, Y, Z and scalars $\varepsilon > 0$, $\varrho > 0$ such that the LMIs

$$\begin{bmatrix} \begin{array}{c} A_oX + XA_o^t \\ +Z + Z^t \\ +\beta B_oX \\ +\beta XB_o^t \end{array} & M & M & XQ & L & Y & A_d \\ \bullet & -\varepsilon I & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\varrho I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\varepsilon I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \begin{array}{c} -Q+ \\ \varrho N_d^t N_d \end{array} \end{bmatrix} < 0 \tag{10}$$

have a feasible solution. Moreover, the feedback gain is $K_o = YX^{-1}$.

Proof: Evaluation of the derivative of $V_n(x,\mu)$ along the solutions of system (1)-(2) using adaptive controller (8) with some algebraic manipulations yields:

$$\begin{aligned} \dot{V}_n(x,\tilde{\mu}) &= x^t(t)\Big[PA_{\Delta c} + A_{\Delta c}^t P + Q\Big]x(t) \\ &\quad + 2x^t PA_{\Delta d}x(t-\tau) \\ &\quad - x^t(t-\tau)\;Q\;x(t-\tau) + \tilde{\mu}\;\dot{\tilde{\mu}} \\ &= x^t(t)\;\big[PA_{\Delta c} + A_{\Delta c}^t P + Q \\ &\quad + PA_{\Delta d}Q^{-1}A_{\Delta d}^t P + K_o^t K_o\big]\;x(t) \\ &\quad - \Big[x^t(t-\tau) - x^t(t)PA_{\Delta d}\Big]Q^{-1} \\ &\quad \Big[x(t-\tau) - A_{\Delta d}^t Px(t)\Big] - g\tilde{\mu}^2 \\ &\leq x^t(t)\;\big[PA_{\Delta c} + A_{\Delta c}^t P + Q \\ &\quad + PA_{\Delta d}Q^{-1}A_{\Delta d}^t P + K_o^t K_o\big]\;x(t) \\ &\triangleq x^t(t)\;\Xi_n\;x(t) \end{aligned} \tag{11}$$

$$\begin{aligned} A_{\Delta c} &= A_\Delta + \tilde{\mu}\;B_oK_o + B_o\Delta K(t) \\ &= [A_o + \tilde{\mu}\;B_oK_o] + \Delta A + B_o\Delta K(t) \\ &= A_c + \Delta A + B_o\Delta K(t) \end{aligned} \tag{12}$$

From Lyapunov theory, it follows that $\dot{V}_n(x,\tilde{\mu}) < 0$ is guaranteed if $\Xi_n < 0$. By Mahmoud (2000) with **Facts 1-2** and the Schur complements, it is a straightforward task to show that the stability condition holds if there exist scalers $\varepsilon > 0,\; \varrho > 0$ such that :

$$\begin{aligned} &PA_{\Delta c} + A_{\Delta c}^t P + Q + \\ &PA_{\Delta d}Q^{-1}A_{\Delta d}^t P + K_o^t K_o < 0 \end{aligned} \tag{13}$$

or equivalently,

$$\begin{aligned} &PA_o + A_o^t P + Q + K_o^t K_o + \varepsilon N_a^t N_a + \beta PB_o + \\ &\beta B_o^t P + \tilde{\mu}\;PB_oK_o + \tilde{\mu}\;K_o^t B_o^t P + \varepsilon^{-1}PMM^tP \\ &+ \varrho^{-1}PMM^tP + PA_d[Q - \varrho N_d^t N_d]^{-1}A_d^t P < 0 \end{aligned}$$

or

$$\begin{bmatrix} \begin{matrix} PA_o + A_o^t P + \\ Q + K_o^t K_o + \varepsilon N_a^t N_a \\ + \beta P B_o + \beta B_o^t P + \\ \tilde{\mu}\ P B_o K_o + \tilde{\mu}\ K_o^t P \end{matrix} & PM & PM & PA_d \\ \bullet & -\varepsilon I & 0 & 0 \\ \bullet & \bullet & -\varrho I & 0 \\ \bullet & \bullet & \bullet & \begin{matrix} \varrho N_d^t N_d \\ -Q \end{matrix} \end{bmatrix} < 0 \quad (14)$$

Using the congruegce transformation $T = diag[X \quad I \quad I \quad I]$, $X = P^{-1}$ and defining $Y = K_o$, $Z = \tilde{\mu}\ B_o K_o P^{-1}$, $L = \varepsilon P^{-1}$, it follows that Schur complement operations convert (14) to (10) and thus the proof is completed. $\nabla\nabla\nabla$

Remark 3.1. The dynamical relation of $\tilde{\mu}$ consists of two-terms: one is growth factor and the other is a product of $\tilde{\mu}$ and x so as to preserve inter-coupling between the states and the gain factor. The selection of the growth factor $g > 0$ guarantees the asymptotic stability of system (8) and different values will only affect the speed of convergence. This is illustrated by the following example.

3.2 Example 1

This example is motivated by the dynamics of biostrata in water-quality studies on the river Nile. A pilot-scale model of the form (1) is described by:

$$A_o = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.9 & -0.3 \\ 0.8 & 0 & -1 \end{bmatrix}, \ A_d = \begin{bmatrix} 0 & 0 & 0 \\ -0.7 & 0 & 0 \\ 0 & -0.8 & 0 \end{bmatrix},$$

$$B_o = \begin{bmatrix} 0.8 & 0 \\ 0.2 & 0.3 \\ 0 & 0.4 \end{bmatrix}, \ C_o^t = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} -0.1 \\ 0.1 \\ 0.3 \end{bmatrix}, \ N_a^t = \begin{bmatrix} -0.1 \\ 0 \\ 0.1 \end{bmatrix}, \ N_d^t = \begin{bmatrix} -0.2 \\ 0 \\ 0.2 \end{bmatrix}$$

The feasible solution of LMIs (10) is given by

$$K = \begin{bmatrix} -2.146 & 0.01 \\ 0.157 & 0 \\ -3.804 & -1.626 \end{bmatrix}$$

In Figs. 2-3, the behavior of the output variables and the gain factor $\tilde{\mu}$ are displayed for different values of g, from which it is clear that relatively-high values of g tend to yield effective stabilization.

3.3 Unknown Gain Perturbation Bound

Now we consider the application of controller (6) subject to bound (7) where β is unknown. The following adaptive scheme is then proposed

$$\begin{aligned} u(t) &= \bar{\mu}\ Kx(t), \\ \dot{\bar{\mu}} &= -g\ \bar{\mu}\ +\ x^t K^t \bar{\mu}^{-1} Kx + \beta\ ||x||^2, \\ \dot{\beta} &= -h\ \beta - \bar{\mu}\ ||x||^2\ +\ \beta^{-1} x^t Rx, \\ \beta(0) &= \beta^* \ ,\ \ \bar{\mu}(0) = \mu^* \end{aligned} \quad (15)$$

where $h > 0$ represents a growth factor. Note that scheme (15) is constructed in the same way as scheme (8). A convenient Lyapunov functional $V(.)$ is given by

$$V_b(x, \bar{\mu}, \beta) = x^t(t)Px(t) + \int_{t-\tau}^{t} x^t(s)Qx(s)ds \\ +1/2\ \bar{\mu}^2 + 1/2\ \beta^2 \quad (16)$$

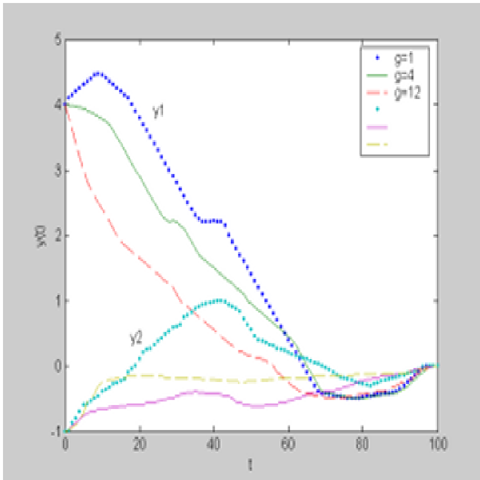


Fig. 2 Plot of Output Response for different values of g

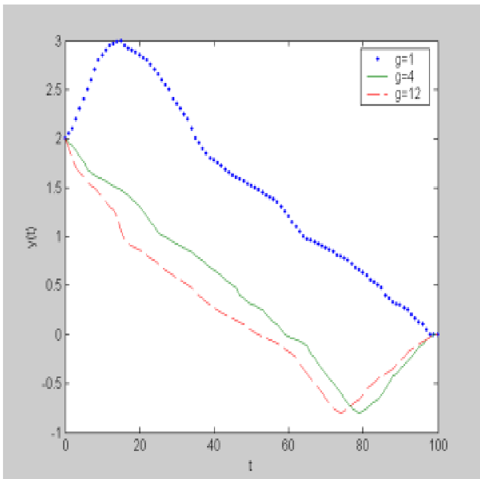


Fig. 3 Plot of Gain Factor $\tilde{\mu}$ for different values of g

The following theorem summarizes the second main result:

Theorem 3.2. System (1) under the adaptive controller (15) is asymptotically stable if for given matrices $0 < Q = Q^t$, $0 < R = R^t$ there exist *matrices* $0 < X = X^t,\ Y,\ L,\ Z$ *and scalars* $g > 0,\ \varepsilon > 0,\ \varrho > 0$ *such that the LMIs*

$$\begin{bmatrix} \begin{matrix} A_o X \\ +XA_o^t \\ +Z+Z^t \\ +L+L^t \end{matrix} & M & M & N_a^t & Y & XQ & XR & A_d \\ \bullet & -\varepsilon I & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\varrho I & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -\sigma I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \begin{matrix} -Q+ \\ \varrho N_d^t N_d \end{matrix} \end{bmatrix} < 0 \quad (17)$$

have a feasible solution. Moreover, the feedback gain is $K = YX^{-1}$ *and the adjustable factor* $\bar{\mu}^{-1} = B_o Y Z^{-1}$.

Proof: An evaluation of the derivative of $V_b(x, \bar{\mu}, \beta)$ along the solutions of system (1)-(2) using (12) and adaptive controller (8), yields:

$$\begin{aligned} \dot{V}_b(x, \bar{\mu}, \beta) &= x^t(t) \left[PA_{\Delta c} + A_{\Delta c}^t P + Q \right] x(t) \\ &\quad +2x^t PA_{\Delta d} x(t-\tau) \\ &\quad -x^t(t-\tau)\, Q\, x(t-\tau) \\ &\quad +\bar{\mu}\, \dot{\bar{\mu}} + \beta\, \dot{\beta} \\ &= x^t(t) \left[PA_{\Delta f} + A_{\Delta f}^t P + Q \right. \\ &\quad \left. +PA_{\Delta d} Q^{-1} A_{\Delta d}^t P + K^t K + R \right] x(t) \\ &\quad - \left[x^t(t-\tau) - x^t(t) PA_{\Delta d} \right] Q^{-1} \\ &\quad \left[x(t-\tau) - A_{\Delta d}^t P x(t) \right] - g\bar{\mu}^2 - h\beta^2 \\ &\le x^t(t) \left[PA_{\Delta f} + A_{\Delta f}^t P + Q \right. \\ &\quad \left. +PA_{\Delta d} Q^{-1} A_{\Delta f}^t P + K^t K + R \right] x(t) \\ &\stackrel{\Delta}{=} x^t(t)\, \Xi_f\, x(t) \qquad (18) \\ A_{\Delta f} &= [A_o + \bar{\mu}\, B_o K] + \Delta A = A_f + \Delta A \quad (19) \end{aligned}$$

Following parallel development to **Theorem (3.2)**, it is readily evident that the stability condition $\dot{V}_n(x, \tilde{\mu}) < 0$ holds if there exist scalers $\varepsilon > 0,\ \varrho > 0$

$$\begin{aligned} & PA_{\Delta f} + A_{\Delta f}^t P + Q + PA_{\Delta d} Q^{-1} A_{\Delta d}^t P + K^t K \\ & +R + S < 0 \Longrightarrow \\ & PA_o + A_o^t P + Q + K^t K + \varepsilon N_a^t N_a + \bar{\mu}\, PB_o K \\ & +\bar{\mu}\, K^t B_o^t P + \varepsilon^{-1} PMM^t P + \varrho^{-1} PMM^t P \\ & +PA_d [Q - \varrho N_d^t N_d]^{-1} A_d^t P < 0 \Longleftrightarrow \end{aligned} \qquad (20)$$

$$\begin{bmatrix} \begin{matrix} PA_o + A_o^t P \\ +R+Q \\ +K^t K + \varepsilon N_a^t N_a \\ +\bar{\mu}\, PB_o K \\ +\bar{\mu}\, K^t B_o^t P \end{matrix} & PM & PM & PA_d \\ \bullet & -\varepsilon I & 0 & 0 \\ \bullet & \bullet & -\varrho I & 0 \\ \bullet & \bullet & \bullet & \begin{matrix} -Q \\ +\varrho N_d^t N_d \end{matrix} \end{bmatrix} < 0 \quad (21)$$

Using the congruegce transformation $T = diag[X \quad I \quad I \quad I]$, $X = P^{-1}$ and defining $Y = KX$, $L = \bar{\mu}\ B_o K P^{-1}$, it follows that Schur operations converts (21) to (17) under the constraint $\varepsilon^{-1}\ \sigma = 1$ and hence the proof is completed. ∇∇∇

3.4 Example 2

The following example is motivated by the dynamics of machining chatter with the matrices of system (1) given by:

$$A_o = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -16 & 10 & 0 & 0 \\ -5 & -15 & 0.02 & -0.25 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_o = \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \\ 0.8 \end{bmatrix}, C_o^t = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$M = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.4 \\ -0.5 \end{bmatrix}, N_a^t = \begin{bmatrix} 0.1 \\ 0 \\ -0.1 \\ 0 \end{bmatrix}, N_d^t = \begin{bmatrix} 0 \\ 0.2 \\ 0 \\ -0.2 \end{bmatrix}$$

The feasible solution of LMI (10) is

$$K = [-0.387 \quad 1.245 \quad -0.336 \quad -0.804]\,,$$
$$\varepsilon = 1.345.$$

The system output y and the control input u are plotted in Fig. 4 for $g = 5,\ h = 4$ while the variation of $\bar{\mu}$ and β are displayed in Fig. 5. In all cases smooth behavior is recorded which supports the flexibility of the developed adaptive control scheme.

4. CONC1USIONS

The problem of non-fragile adaptive control for a class of continuous-time systems with state-delays and norm-bounded uncertainties against controller gain variations has been investigated. Adaptive control schemes have been constructed for the case of known gain pertubation bounds and then extended to accomodate unknown norm-bounded perturbations. All the developed results have been expressed in LMI format. A detailed simulation results has been presented. Extension of the present methodology to incorporate other adaptation laws and/or to deal with discrete-time systems requires additonal research efforts.

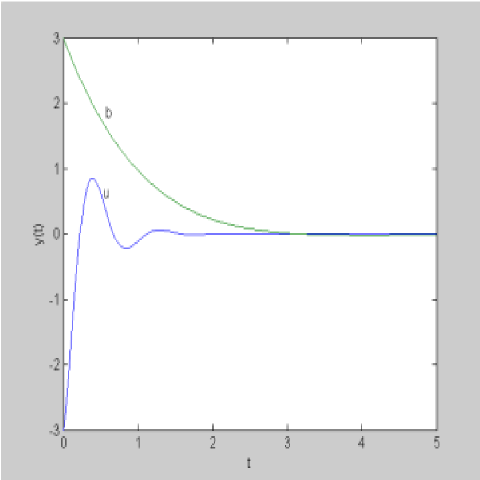


Fig. 4 Plot of Output Response and Control Input: Example 2

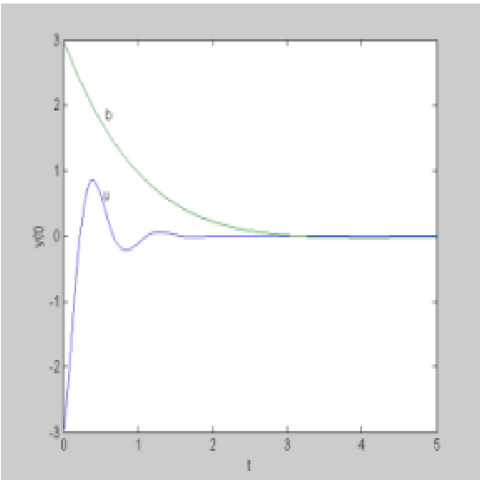


Fig. 5 Plot of $\bar{\mu}$ and β: Example 2

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