DENSITY AND VELOCITY ESTIMATION IN TRAFFIC FLOW [3]

Óscar Rosas-Jaimes [*,1] **Luis Alvarez-Icaza** [*,2]

[*] *Instituto de Ingeniería, Universidad Nacional Autónoma de México, 04510 Coyoacán DF, México*

Abstract: An estimation scheme that allows to recover vehicle density and velocity in a stretch of highway is presented. The design is based on a non-linear switching model, inspired by the cell transmission traffic flow model, that has a distinctive behavior for free flow or congested traffic. Lyapunov and linear matrix inequalities techniques are used to prove the stability of the estimation and observer schemes. The algorithm is applied to a set of data traffic collected in a stretch of highway in Pasadena, California, showing good performance.

Keywords: density estimation in traffic flow, velocity estimation in traffic flow, automated highway systems.

1. INTRODUCTION

Incorporating automated devices and advanced control techniques in surface transportation networks composed by highways, freeways or urban avenues and streets is becoming a very common solution to help alleviating daily impacts of congested traffic. Adaptive cruise control, advanced traction and braking schemes, driver alert systems, variable signal systems and real time on-ramp metering control are among the new technologies (Varaiya, 1993; Horowitz and Varaiya, 2000).

This paper focuses on the traffic information required to implement on-ramp metering schemes. In a normal highway, there are sensors already installed to provide some of this information. However, in many cases, measurements are not obtained in appropriate places for on-ramp metering control, or the available sensors are faulty. Therefore, it is necessary to use on-line estimation schemes to provide all the needed traffic information.

In this paper an estimation scheme to recover vehicle density and velocity is proposed. The design is based on the cell transmission traffic flow model introduced in (Daganzo, 1994; Daganzo, 1995). This is in appearance a simple traffic flow model and yet it is able to reproduce most of the phenomena observed in real traffic. There are many other traffic flow models in the literature, as those suggested, for example, in (Payne, 1971; Papageorgiou *et al.*, 1990; Broucke and Varaiya, 1996; Drew, 1968), that can also reproduce traffic flow, and in some cases with more accuracy than the cell transmission model. In this paper however, the cell transmission model is used as it characterizes traffic flow in a cell with only two parameters that represent the velocities of the traffic waves traveling downstream or upstream, for free flow and congested traffic, respectively. The value of these wave velocities can be recovered with real time identification schemes.

Based on the cell transmission model, a switching model was proposed in (Munoz *et al.*, 2003). This

[1] Graduated student, ORosasJ@iingen.unam.mx.
[2] Professor, alvar@pumas.iingen.unam.mx; corresponding author.
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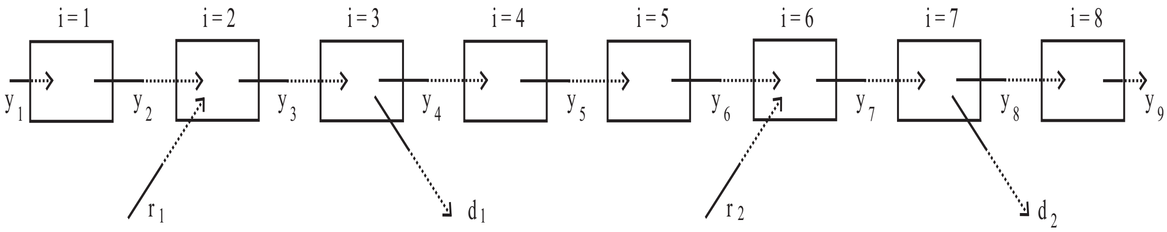

Fig. 1. Highway stretch (I-210 West in Southern California)

model switches between a set of linear models depending on the condition, free flow or congested traffic, in a set of cells. The switching model was applied to estimate vehicle density in a stretch of highway in Southern California, for which traffic data was available. It was assumed that wave velocities were known constants and equal for all cells.

This paper uses this switching model structure, although now assumes that vehicle densities and wave velocities are unknown and that can vary from cell to cell. This assumption implies that each one of the models of the switching structure becomes non-linear. To reduce the number of possible models involved in the switching structure, in this paper only two models are considered, corresponding to all the cells in the stretch of highway having free flow or congested traffic, respectively.

The first section of this paper describes the traffic flow model used. The following section contains the design of the adaptive observer, including the analytical proof of its convergence. The last two sections refer to simulation results and conclusions.

2. TRAFFIC FLOW MODEL

To explain the design of the velocity and density estimator, consider the stretch of highway depicted in Fig. 1, that is composed by eight cells, two on-ramps and two off-ramps. This stretch represents a portion of highway I-210W in Southern California.

The cell transmission model introduced in (Daganzo, 1994) defines the flow between cells according to

$$y_i = \min\{v_{i-1}k_{i-1}, w_i(k_{j_i} - k_i)\} \tag{1}$$

where k_i is the vehicle density, v_i the free-flow upstream wave velocity, w_i the congested downstream wave velocity and k_{j_i} the jam density, all of them referring to cell i. To better understand the meaning of this flow calculation, consider the density vs. flow "fundamental diagram" shown in Fig. 2. The region with positive slope, v, corresponds to free-flow traffic that can be calculated using the first argument in Eq. (1), while the region with negative slope, $-w$, to congested traffic that is calculated using the second argument in Eq. (1). When cell i is congested, the flow it can receive from cell $i-1$ is limited by the available space in cell i, otherwise when cell i is in free-flow, the limit of the incoming flow depends on the maximum velocity v_{i-1} at which vehicles coming from cell $i-1$ can travel.

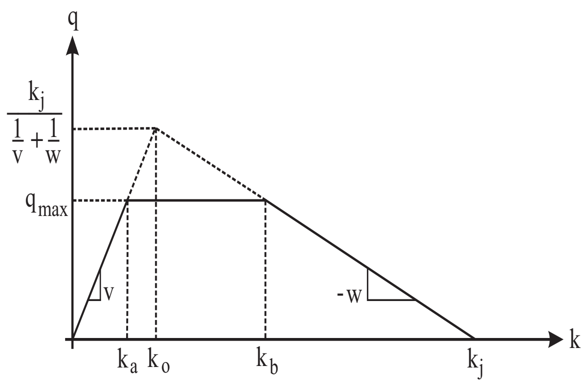

Fig. 2. Flow-density diagram

For the time evolution of the density a principle of vehicles conservation is used in (Daganzo, 1994) in such a way that

$$\dot{k}_i = \frac{1}{L_i}\left(y_i - y_{i+1}\right). \tag{2}$$

with with L_i the length of cell i.

If all cells are assumed in free flow traffic and notation in Fig. 1 is used, the dynamics of the density can be described in matrix form by

$$\dot{k} = L^{-1}\, F_L\, V k + B_L\, u \tag{3a}$$
$$y = C_L(v)k \tag{3b}$$

where k is the vector of cell densities, $V = diag\{v\}$, $v = [v_1, \cdots, v_8]^T$, $L^{-1} = diag\{1/L_1, \cdots, 1/L_8\}$, $u = \begin{bmatrix} y_1 & r_1 & d_1 & r_2 & d_2 \end{bmatrix}^T$, $C_L(v) = \begin{bmatrix} 0 & \cdots & 0 & v_8 \end{bmatrix}$

$$F_L = \begin{bmatrix} -1 & 0 & \cdots & 0 \\ 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{bmatrix}, \tag{4}$$

$$B_L = L^{-1}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{5}$$

Define

$$A_L(v) = L^{-1}F_L V\,. \tag{6}$$

The diagonal structure of V implies that

$$A_L(v)k = A_L(k)v \tag{7a}$$

$$C_L(v)k = C_L(k)v \tag{7b}$$

For the model in Eq. (3) it is assumed that it is not possible to measure density k and velocity v and that only flow measurements are available.

When all the cells are congested, it is possible to derive a similar model to that in Eq. (3), where there are some changes in the elements of F_L, $C_L(v)$, B_L, u and y. For economy of space, only the case of all cells with free-flow traffic will be explained in this paper. The final structure of the model still holds, and therefore the convergence analysis is also applicable. Details of the model for the congested case can be found in (Munoz *et al.*, 2003).

3. DENSITY AND VELOCITY ESTIMATION

The following density estimator is proposed

$$\dot{\hat{k}} = A_L(\hat{v})\hat{k} + B_L u + G_L(y - \hat{y}) \tag{8a}$$

$$\hat{y} = C_L(\hat{v})\hat{k} \tag{8b}$$

where $\hat{k}$ and $\hat{v}$ are respectively the estimates of k and v, and the pair $(A_L(v), C_L(v))$ is observable for all values of v that correspond to free-flow. If $\tilde{k} = k - \hat{k}$ is the density estimation error, its dynamics is given by

$$\dot{\tilde{k}} = \dot{k} - \dot{\hat{k}} = \bar{A}_L(v)k - \bar{A}_L(\hat{v})\hat{k} \tag{9}$$

where

$$\bar{A}_L(v) = A_L(v) - G_L C_L(v), \tag{10}$$

where matrix $\bar{A}_L(v)$ is Hurwitz with the proper choice of G_L. After some manipulations, Eq. (9) can be expressed as

$$\dot{\tilde{k}} = \bar{A}_L(v)\tilde{k} + \bar{A}_L(\tilde{v})\hat{k}, \tag{11}$$

with $\tilde{v} = v - \hat{v}$.

To design the velocity estimator, it is considered that the following flows can be measured: y_1, r_1, d_1, y_5, r_2, d_2 and y_9. The measured output for the free flow case is defined as $z = \begin{bmatrix} r_1 & d_1 & y_5 & r_2 & d_2 & y_9 \end{bmatrix}^T$, that can be expressed as

$$z = H_L K v = S(k)v \tag{12}$$

where $K = diag\{k\}$,

$$H_L = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

Accordingly, the estimated output $\hat{z}$ is given by

$$\hat{z} = S(\hat{k})\hat{v}, \tag{14}$$

therefore the output estimation error is

$$\tilde{z} = z - \hat{z} = S(k)v - S(\hat{k})\hat{v} = S(\hat{k})\tilde{v} + S(v)\tilde{k}. \tag{15}$$

It is again the case that $S(k)v = S(v)k$. The time evolution of $\hat{v}$ is chosen as a gradient like law given by

$$\dot{\hat{v}} = \Gamma S^T(\hat{k})\tilde{z} \tag{16}$$

where Γ is a diagonal matrix of positive gains.

4. CONVERGENCE ANALYSIS

In this section the stability of $\tilde{k} = 0$ and $\tilde{v} = 0$ is investigated. For that purpose consider the following Lyapunov function candidate

$$\mathcal{V} = \mathcal{V}_1 + \mathcal{V}_2, \tag{17}$$

with

$$\mathcal{V}_1 = \frac{1}{2}\tilde{k}^T P \tilde{k} \tag{18a}$$

$$\mathcal{V}_2 = \frac{1}{2}\tilde{v}^T \Gamma^{-1}\tilde{v} \tag{18b}$$

where $P = P^T > 0$. Taking the time derivative of Eq. (18a)

$$\begin{aligned} \dot{\mathcal{V}}_1 &= \frac{1}{2}\left(\tilde{k}^T P \dot{\tilde{k}} + \dot{\tilde{k}}^T P \tilde{k}\right) = \\ &= \frac{1}{2}\tilde{k}^T\left[P\bar{A}_L(v) + \bar{A}_L^T(v)P\right]\tilde{k} + \\ &+ \frac{1}{2}\tilde{k}^T P \bar{A}_L(\tilde{v})\hat{k} + \frac{1}{2}\hat{k}^T \bar{A}_L^T(\tilde{v}) P \tilde{k} \end{aligned} \tag{19}$$

Using Eq. (7a)

$$\begin{aligned} \dot{\mathcal{V}}_1 &= \frac{1}{2}\tilde{k}^T\left(P\bar{A}_L(v) + \bar{A}_L^T(v)P\right)\tilde{k} + \\ &+ \tilde{k}^T P \bar{A}_L(\hat{k})\tilde{v} \end{aligned} \tag{20}$$

Taking now the time derivative of Eq. (18b), using Eq. (16) and Eq. (15)

$$\dot{\mathcal{V}}_2 = -\tilde{v}^T S^T(\hat{k})\left(S(\hat{k})\tilde{v} + S(v)\tilde{k}\right) \tag{21}$$

From Eqs. (20) and (21)

$$
\begin{aligned}
\dot{\mathcal{V}} =& \dot{\mathcal{V}}_1 + \dot{\mathcal{V}}_2 = \\
=& \frac{1}{2}\tilde{k}^T \left(P\bar{A}_L(v) + \bar{A}_L^T(v)P \right) \tilde{k} \\
&+ \tilde{k}^T P\bar{A}_L(\hat{k})\tilde{v} \\
&- \tilde{v}^T S^T(\hat{k}) \left(S(\hat{k})\tilde{v} + S(v)\tilde{k} \right) \\
=& -\tilde{k}^T Q\tilde{k} - \tilde{v}^T S^T(\hat{k})S(\hat{k})\tilde{v} \\
&+ \tilde{k}^T P\bar{A}_L(\hat{k})\tilde{v} - \tilde{v}^T S^T(\hat{k})S(v)\tilde{k} \qquad (22)
\end{aligned}
$$

where P satisfies the Lyapunov equation

$$P\bar{A}_L(v) + \bar{A}_L^T(v)P = -2Q. \qquad (23)$$

Notice that the two first terms in Eq. (22) are negative definite and negative semidefinite, respectively. The other two terms in Eq. (22) can be grouped as

$$
\begin{aligned}
&\tilde{k}^T P\bar{A}_L(\hat{k})\tilde{v} - \tilde{v}^T S^T(\hat{k})S(v)\tilde{k} = \qquad (24) \\
&\tilde{k}^T \left(P\bar{A}_L(\hat{k}) - S^T(v)S(\hat{k}) \right) \tilde{v}.
\end{aligned}
$$

The idea now is to take advantage in the degree of freedom available in the selection of Q (Ghaoui *et al.*, 1995) and choose it in such a way that P in Eq. (18a) satisfies the following inequality

$$P\bar{A}_L(\hat{k}) + \bar{A}_L^T(\hat{k})P \leq -S^T(v)S(\hat{k}) - S^T(\hat{k})S(v). \qquad (25)$$

Using Eq. (25), it is possible to rewrite Eq. (22) as

$$
\begin{aligned}
\dot{\mathcal{V}} \leq& -\tilde{k}^T Q\tilde{k} - \tilde{v}^T S^T(\hat{k})S(\hat{k})\tilde{v} \\
&- \tilde{k}^T \left(S^T(v)S(\hat{k}) + S^T(\hat{k})S(v) \right) \tilde{v} \qquad (26)
\end{aligned}
$$

If the choice of matrix Q satisfies $Q \geq S^T(v)S(v)$, then Eq. (26) satisfies

$$\dot{\mathcal{V}} \leq -\left(S(v)\tilde{k} + S(\hat{k})\tilde{v} \right)^T \left(S(v)\tilde{k} + S(\hat{k})\tilde{v} \right)^T \leq 0, \qquad (27)$$

that implies the stability of $\tilde{k} = 0$ and $\tilde{v} = 0$. Using Barbalat's Lemma (Khalil, 1996), it is possible to prove that $S(v)\tilde{k} \to 0$ and $S(\hat{k})\tilde{v} \to 0$ as $t \to \infty$.

To verify that the selection of P in Eq. (25) is appropriate, its value is substituted in the left hand side of the Lyapunov equation (23) to obtain that $P\bar{A}_L(v) + \bar{A}_L^T(v)P \leq -2S^T(v)S(v) \leq 0$. This shows the feasibility of the linear inequality matrix problem.

5. RESULTS

To verify the performance of the estimation scheme, it was applied to a set of traffic data obtained for the stretch of highway in Fig. 1. For this highway, there is timed information for some days of operation for the following flows y_1, r_1, d_1, y_5, r_2, d_2 and y_9 (see (PEMS, 2003)). There is also local density information, inferred from occupancy measurements, for points at the beginning of the stretch, between cells 4 and 5 and at the end of the stretch. The highway has four lanes and the total length of the stretch is about 3000 m ($L = \left[373\ 373\ 603\ 225\ 225\ 444\ 444\ 396 \right]^T$). The jam density k_j was set to 0.4225 [veh/m].

There are two different estimators, one for all cells in free-flow and the other for all cells congested. The scheme switches between the two estimators depending on the density at the last cell of the stretch (cell 8). When the density k_8 is below a given threshold k_0, the estimator for free-flow is used and when k_8 is above k_0, the estimator for congestion is employed. When one estimation scheme is being used, the estimation is frozen in the other. k_0 chosen to be close to the that of Fig. 2 for nominal traffic. A value of $k_0 = 0.08$ [veh/m] was used in the simulations. The value of the observer gain matrix G_L was set to yield the eigenvalues of matrix $\bar{A}_L(v)$ to be $20 - 30\%$ faster than those of matrix $A_L(v)$.

Figs. 3-5 show the measured flow and the estimated flow at the beginning, middle point and the end of the stretch, while Figs. 6-8 show the estimated densities at cell 1, the average of cells 4 and 5, and cell 8, compared with the point density at the beginning, middle point and the end of the stretch. The agreement between real and predicted flows in Figs. 3-5 is very good. The similarity between average densities and point densities in Figs. 6-8 is also appropriate, although not as good as the similarity in flows estimates. This is due, partially to the fact that measured densities are inferred from occupancy measurements using "g" factors and the estimated densities correspond to average densities.

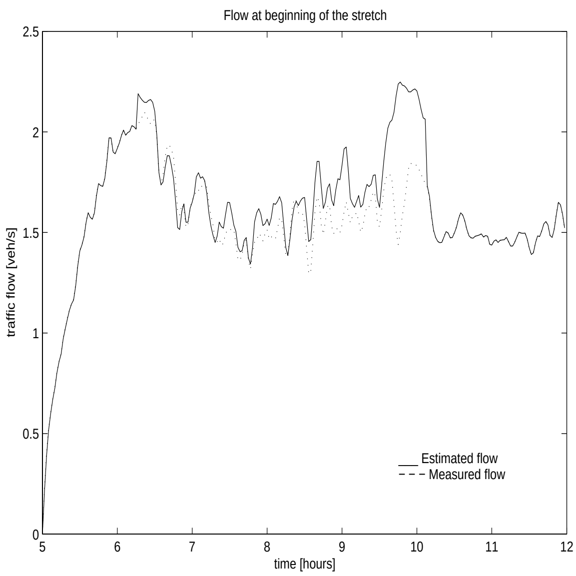

Fig. 3. Estimated and real flow at the beginning of stretch (y_1).

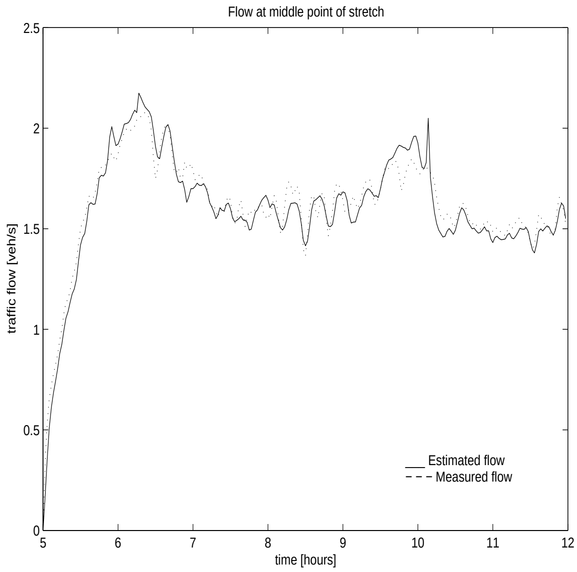

Fig. 4. Estimated and real flow at the middle of stretch (y_5).

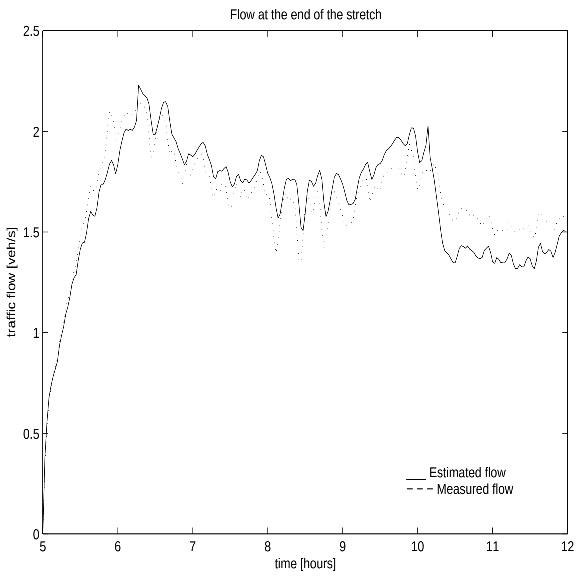

Fig. 5. Estimated and real flow at the end of the stretch (y_9).

Finally, Figs. 9-10 show the estimated wave velocities for cells 1, 5 and 8. Plots show the complementary behavior of process of velocities identification, as one set remains constant while the other varies. There are not, unfortunately, real velocity measurements to use for comparison purposes. The obtained values, however, are in good agreement with field observations performed in that stretch of highway.

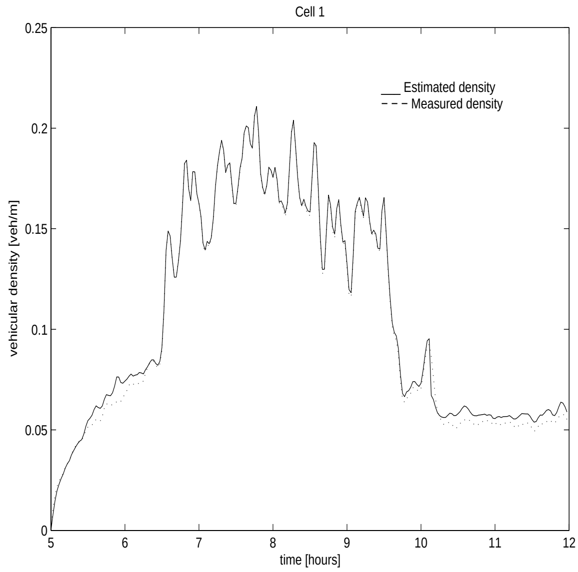

Fig. 6. Estimated density at cell 1 and point density at the beginning of stretch.

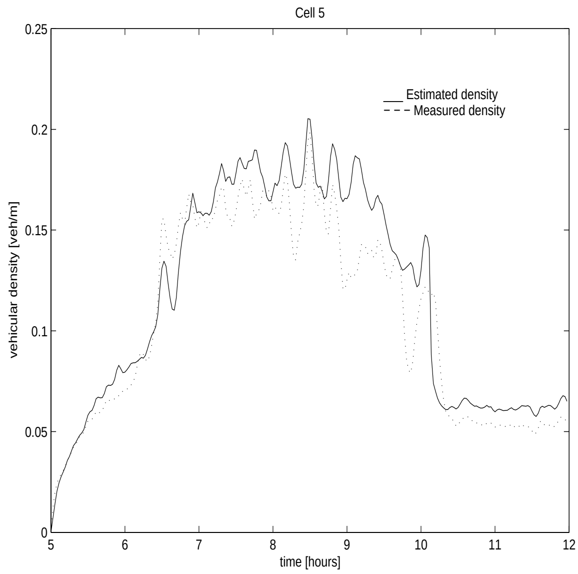

Fig. 7. Average estimated density of cells 4 and 5 and point density between cells 4 and 5.

6. CONCLUSION

An estimator for vehicles density and velocity in a stretch of highway was presented. The estimation scheme was based on the cell transmission traffic model proposed by (Daganzo, 1994; Daganzo, 1995) and later modified in (Munoz *et al.*, 2003). Two estimators were designed, one for cells in free flow and the other for cells with congested traffic. The convergence analysis of the estimation of velocities and densities was based on Lyaupunov and linear matrix inequalities techniques. Results of the use of the scheme are compared with real traffic data were presented, that showed good performance. It it still necessary to improve gains tuning.

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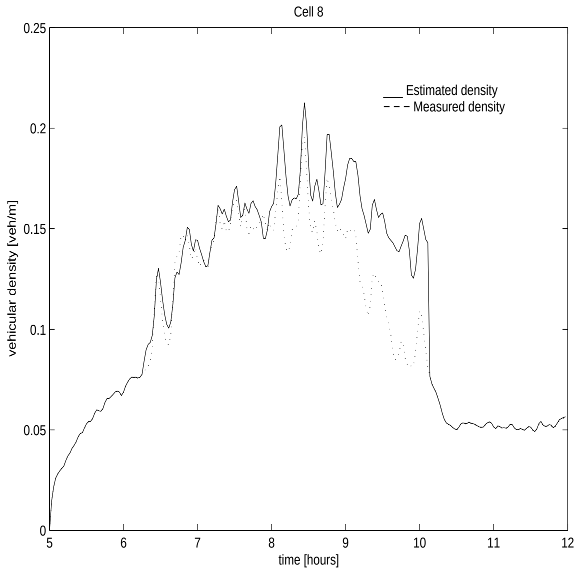

Fig. 8. Estimated density at cell 8 and point density at the end of stretch.

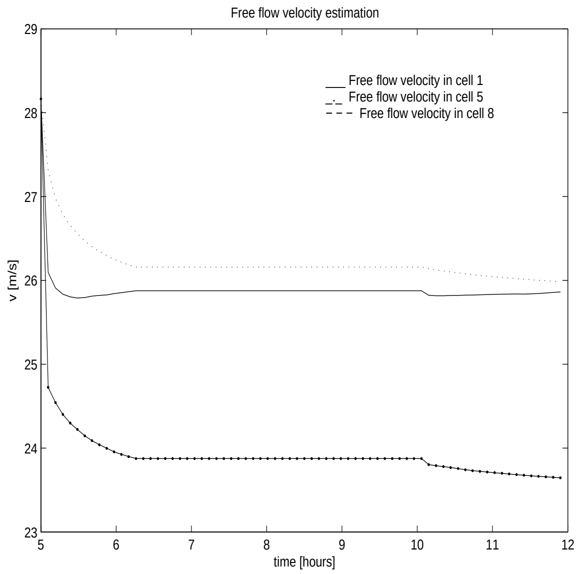

Fig. 9. Estimated free flow traffic wave velocity for cells 1, 5 and 8.

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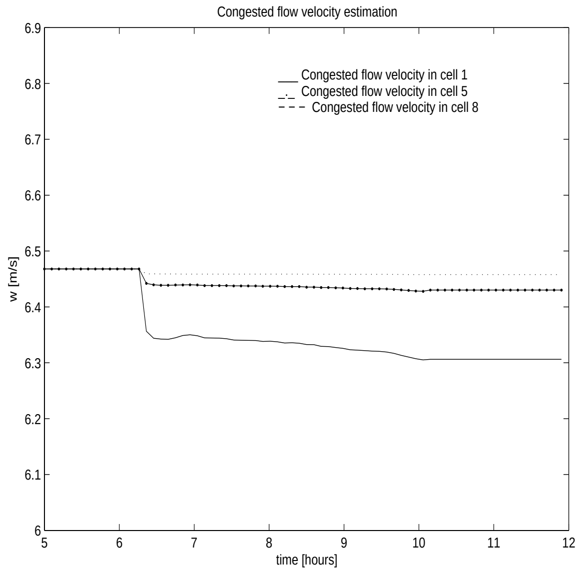

Fig. 10. Estimated congested traffic wave velocity for cells 1, 5 and 8.

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