ON SLIDING-MODE CONTROL FOR INVERSE RESPONSE PROCESSES

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Abstract: A sliding-mode controller (SMCr) synthesised using concepts of internal model control (IMC) is presented. An advantage of this method is a reduction of the inverse response effect by a feedback compensation element, proper of the IMC scheme. The performance and robustness of the proposed controller was tested and compared with a previous SMCr designed for inverse response systems (Camacho et al., 1999). The test was done by means of computer simulations through a linear model and the Van de Vusse reactor.

Keywords: Sliding-mode Control, Non-minimum Phase Systems, Robustness.

1. INTRODUCTION

A process is said to have inverse response (IR) when it shows an initial step response in a direction opposite to the direction of the final steady state. This kind of non-minimum phase processes, characterised by their contrary input-output behaviour, is not uncommon in the process industries and it is well-known from linear control theory that results in a reduction of the achievable closed-loop performance because its behaviour limits the frequency bandwidth of the controller and thus makes the plant response slow (Skogestad et al., 1996).

Furthermore, there exist system uncertainties including modelling errors, unmodelled dynamics and disturbances that cause degradation of the control in chemical process regulation, becoming a challenging control problem in many industrial processes. A robust technique like Sliding Mode Control (SMC) seems appropriate to deal with this kind of problems. In a previous work, a Sliding Mode Controller for processes with inverse response was designed from an FOPDT model. Although, that controller showed to be robust against modelling errors, disturbances and presence of noise, it did not reduce enough the settling time and the effect of the inverse response (Camacho et al., 1999).

On the other hand, recent papers have shown the viability to mix predictive structures with SMC, to improve the controllers' performance and robustness characteristics for processes with long dead time and inverse response systems (García-Gabín and Camacho, 2003). This approach simplifies the controller synthesis and makes possible use SMC for different kind of processes that contains non invertible terms, such as inverse response systems.

This paper extends the previous work and explores the viability of using the IMC concepts to synthesise a SMCr the overall idea is to design a SMCr capable to predict the inverse response by IMC using a second order model of the process. This model is separated into two models connected in series. One of them is used to design the controller and the other to compensate the modelling errors, used to design the controller. This paper is organised as follows: section 2 gives a brief description of internal model control structures and SMC. Section 3 describes the procedure to synthesise the controller. Section 4 shows the computer simulations studies to test and compare the performance and robustness of the internal model sliding-mode controller (IM-SMCr) against the previous SMCr. Finally, some conclusions are presented.

2. BASIC CONCEPTS

2.1 Internal Model Control Structure for inverse response processes.

The Internal Model Control is based on the structure shown in Figure 1. The idea behind this approach is to obtain a model of the process, and then decompose the model into two components, an invertible one for the controller design, and other noninvertible (García and Morari, 1984). For the inverse response case, the model is decomposed by:

$$G_m(s) = G_M(s) \cdot G_O(s) \tag{1}$$

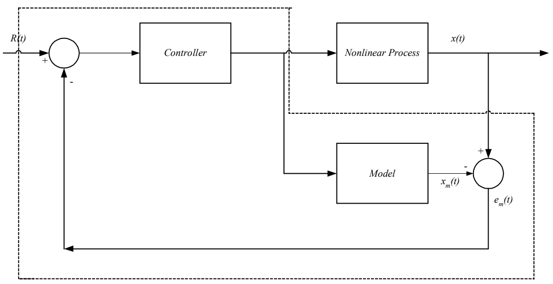

Fig. 1. Internal Model Structure

where G_M(s) corresponds to the invertible component or main function, and G_O(s) is the noninvertible or opposite function. While the main function contains the static gain and the terms related to slow motion, the opposite function presents a unit static gain, the noninvertible and faster terms. So, the IMC design procedure eliminates all elements in the process model that can produce an unrealisable controller taking into consideration the realisable ones.

2.2 Sliding Mode Control (SMC)

The sliding-mode control approach is based on the controller composed by a control law and a sliding surface. The control law contains two parts: the sliding-mode control law and the reaching mode control law. The first of these is responsible for maintaining the controlled system dynamic on the sliding surface. The second control law is designed in order to reach the desired surface (Utkin, 1977).

The sliding surface represents the desired closed loop behaviour. It is defined as a first step in SMC approach, $S(t) = 0$, and is usually formulated as a linear function of the system states. Generally, the sliding equation is a function of the reference signal, the model output, and the modelling error. Therefore, $S(t)$ can be represented by

$$S(t) = f(R(t), y_m(t), \varepsilon_1(t), \varepsilon_2(t)) \tag{2}$$

where $R(t)$ is the reference, $y_m(t)$ is the model output, and $\varepsilon(t)$ are errors. The sliding-mode control law, $U_C(t)$, is normally obtained by a method based on the Filippov's construction of the equivalent dynamics, usually called the equivalent control method (Edwards and Spurgeon, 1998). It can be summarised as follows, first the sliding condition given by (3), must be satisfied

$$\frac{dS(t)}{dt} = 0 \tag{3}$$

Then substituting it into the system dynamic equations, the control law is thereby obtained.

The reaching mode control law is basically obtained as a relay-like function of $S(t)$ affected by a constant gain can be used (Edwards and Spurgeon, 1998; Camacho et al., 1999). However, this produces the undesirable effect of chattering around the sliding surface, normally not tolerated by the actuators. To avoid chattering, smooth the discontinuity and obtain a continuous approximation to the surface behaviour other solution is to use the sigmoid-like function (Utkin, 1977; Edwards and Spurgeon, 1998; Young, 1999). Generally, the reaching control law can be written as follows

$$U_{reach}(t) = K_D \cdot \Theta(S(t)) \tag{4}$$

where K_D is the tuning parameter responsible for the speed to reach the sliding surface, and $\Theta(S(t))$ represents usually a nonlinear function of $S(t)$. Finally, the control law can be written as follows,

$$U(t) = U_C(t) + U_{reach}(t) \tag{5}$$

2.3 Previously SMCr designed for IR Processes

The SMCr designed for inverse processes by Camacho et al. (1999), was based on the assumption that the process can be approximated by a first order plus dead time (FOPDT) model:

$$G(s) = \frac{Ke^{-t_0 s}}{\tau s + 1} \tag{6}$$

Note that the inverse response is considered as the dead time term, t_0, as is depicted in Fig. 2. So the SMcr design problem is reduced to the same problem solved for minimum phase systems (Camacho and Smith, 2000).

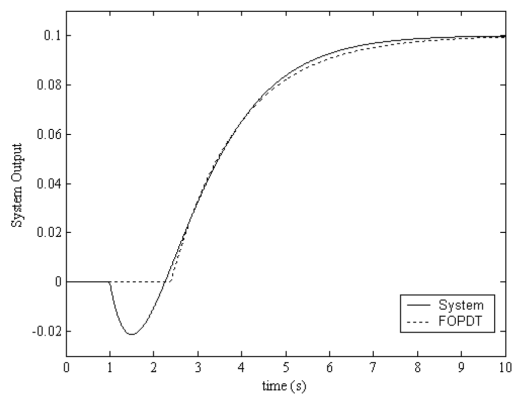

Fig. 2. Approximation of an inverse response system by a FOPDT model.

Then, the SMCr equations are summarised as follows:

$$U(t) = \frac{t_0\tau}{K}\left(\frac{X(t)}{t_0\tau} + \lambda_0 e(t)\right) + K_D\frac{S(t)}{|S(t)|+\delta} \tag{7}$$

$$S(t) = sign(K)\left[-\frac{dX(t)}{dt} + \lambda_1 e(t) + \lambda_0\int_0^t e(t)dt\right] \tag{8}$$

With a set of initial tuning parameters given by

$$\lambda_1 = \frac{t_0+\tau}{t_0\tau}\,[=]\,[time]^{-1} \tag{9}$$

$$0 < \lambda_0 < \min\left[\frac{\lambda_1}{\tau_1}, \frac{\lambda_1^2}{4}\right][=][time]^{-2} \tag{10}$$

$$K_D = \frac{0.51}{K}\left[\frac{\tau}{t_0}\right]^{0.76}\,[=]\,[CO] \tag{11}$$

$$\delta = 0.68 + 0.12(KK_D\lambda_1)\,[=]\left[\frac{TO}{time}\right] \tag{12}$$

The parameters (t_0, τ, τ_1 and K), needed to calculate the initial tuning of the controller, are obtained from the open loop step response (Camacho et al., 1999).

3. IM-SMCr SYNTHESIS

As it was shown previously that obtaining a SMCr directly from a non-minimum phase model of the process generates an unstable controller (Camacho et al., 1999). This creates the need for a different approach to obtain a stable controller. By considering the IMC approach, if we used the basic second order linear model for an inverse response process given by

$$G(s) = K\frac{-\eta s+1}{(\tau_1 s+1)(\tau_2 s+1)} \tag{13}$$

it can be divided into two parts, which are connected in cascade. The first part, of the model, contains the overall process gain and a time constant closest to the dominant time constant of the process, while the second part includes a first order model with the inverse behaviour. Therefore Eq. (13) can be divided as follows:

$$G(s) = G_1(s)\cdot G_2(s) \tag{14}$$

where

$$G_1(s) = \frac{K}{\tau_1 s+1} \quad and \quad G_2(s) = \frac{-\eta s+1}{\tau_2 s+1} \tag{15}$$

Then, a IMC scheme with SMC controller can be proposed as shown in Fig. 3. First, $G_1(s)$ is taken for the synthesis of controller which can be described in differential equation form, as follows:

$$\frac{dy_1(t)}{dt} + \frac{y_1(t)}{\tau_1} = \frac{K}{\tau_1}u(t) \tag{16}$$

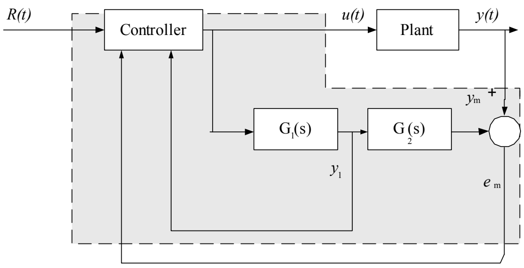

Fig. 3. IM-SMCr structure approach

where $y_1(t)$ is the direct process response and $u(t)$ is the control signal. For design purposes an integral-differential sliding surface acting on the tracking error was considered (Slotine and Lee, 1991):

$$S(t) = \left(\frac{d}{dt}+\lambda\right)^n\int_0^t e(t)dt \tag{17}$$

and considering that $G_1(s)$ is a first order model, the resulting sliding surface is:

$$S(t) = e(t) + \lambda\int_0^t e(t)dt \tag{18}$$

Following the equivalent control procedure, the continuous part of the controller is obtained:

$$U_c(t) = \frac{\tau_1}{K}\left(\frac{y_1(t)}{\tau_1} + \lambda\varepsilon(t)\right) \tag{19}$$

Now, by using a sigma function as the nonlinear function $\Theta(S(t))$ of the reaching control law, it can be written as follows,

$$U_{reach}(t) = K_D\frac{S(t)}{|S(t)|+\delta} \tag{20}$$

and the complete control law is summarised as follows

$$U(t) = \frac{\tau_1}{K}\left(\frac{y_1(t)}{\tau_1} + \lambda\varepsilon(t)\right) + K_D\frac{S(t)}{|S(t)|+\delta} \tag{21}$$

With a set of initial tuning parameters given by

$$0 < \lambda < \frac{1}{\tau_1}\,[=]\,[time]^{-1} \tag{22}$$

$$K_D = \frac{0.51}{K}\left[\frac{\tau_1}{\eta}\right]^{0.76}\,[=]\,[CO] \tag{23}$$

$$\delta = 0.68 + 0.12\frac{(KK_D)}{\tau_1}\,[=]\left[\frac{TO}{time}\right] \tag{24}$$

The parameters (η, τ_1 and K), needed to calculate the initial tuning of the controller, are obtained from the open loop step response (Povy, 1975).

4. COMPUTER SIMULATIONS

In this section, two examples are used. The first one is a linear second order non-minimum phase system. The idea behind this simulation test was to show the performance and robustness of both the SMCr and the IM-SMCr controllers against modelling errors, the range of this errors varies between -20% and 20%. The second one is the Van de Vusse reactor which was used to test the performance of both the SMCr and the IM-SMCr controllers against changes in set point and disturbances when the process is not linear.

4.1 Controllers performance and robustness to modelling errors.

To test the controllers' robustness against modelling errors, the following non-minimum phase linear model of the process was used

$$G(s) = \frac{-s+1}{(s+1)^2} \tag{25}$$

For the process model, a step change of +10% in set point was introduce at t = 1 s, and the parameters of the open loop step response were obtained (K = 1.00 [*TO/CO*], τ = 1.53 [*s*] and t_0 = 1.39 [*s*] for Eq. (6) and K = 1.00 [*TO/CO*], $\tau_1 = \tau_2$ = 1.005 [*s*] and η = 0.995 [*s*] for Eq. (13), respectively). Using the tuning equations given previously, the initial adjustment of the SMCr (λ_0 = 0.47, λ_1 = 1.37, K_D = 0.55, δ = 0.77), and the IM-SMCr (λ = 0.00, K_D = 0.512, δ = 0.741) parameters were done.

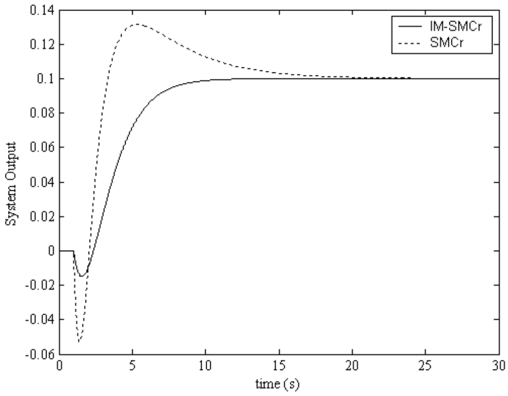

Fig. 4. System step responses when the controllers were applied.

Fig. 4 shows the closed-loop responses obtained for the set point change when the controllers were applied. From the figure, it is clear that the IM-SMCr controller is better decreasing the inverse response effect and produces a shorter settling time than the SMCr. Then maintaining the same parameter adjustments, modelling errors in static gain of ± 20% were simulated (see Fig. 5). Although, the rising times and overshoots were slightly different when the process static gain was changed, almost the same settling times of Fig.4, were obtained for the SMCr but for the IM-SMCr when the error is by excess the control action is more aggressive obtaining a shorter settling time. The opposite is obtained when the error is by defect.

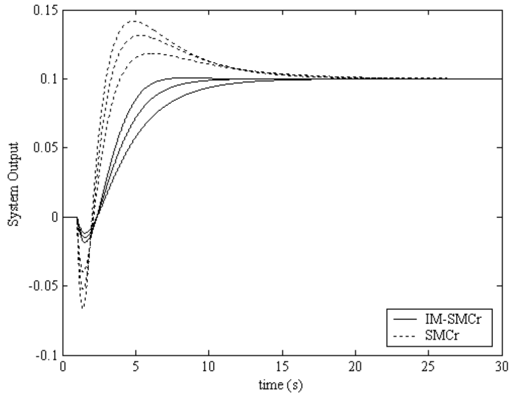

Fig. 5. System step responses when (± 20%) modelling errors in static gain were introduced.

Fig. 6 shows the closed-loop response obtained for the set point change when modelling errors of ± 20% in the system zero, η, were simulated. From the figure, it is clear that the settling times were the same that those obtained without η modelling errors with slightly different transient responses showing that the controllers' actions are robust against significant modelling errors in the system zero.

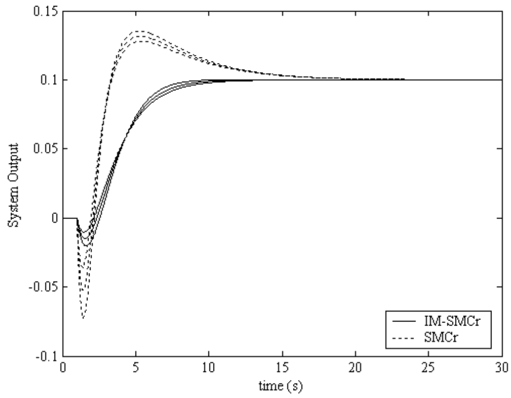

Fig. 6. System step responses when (± 20%) modelling errors in the system zero, η, were introduced.

In contrast with the previous cases the SMCr showed constant instability oscillations when a -20% error in time constant was introduce (see, Fig. 7).

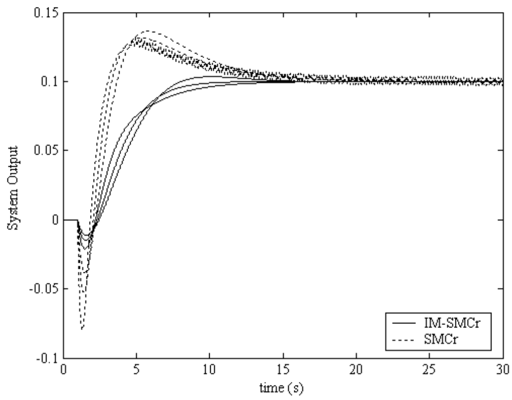

Fig. 7. System step responses when (± 20%) modelling errors in the time constant, τ were introduced.

This means, when the estimated time constant is shorter than the real one this controller present a more aggressive action than the necessary causing an increasing effect on the inverse response with the respective performance degradation. This does not occur in the same proportion with the IM-SMCr.

The second aspect to consider is robustness respect to simultaneous modelling errors. In order to make a fair comparison, the used SMCr is readjusted (fine-tuning is done), to obtain the same performance of the IM-SMCr (both controllers are set so that the obtained ISE performance index present approximately the same value). Table 1 shows its new values, it is important to mention that the IM-SMCr is kept with its original values.

Table 1. Fine tuning to obtain same ISE as IM-SMCr

Parameter	Value
λ_0	0.38
λ_1	1.37
K_D	0.00
δ	0.77

Fig. 8 shows the responses when the SMCr is readjusted, as is depicted the system present similar responses for both controllers, thus a test to generate a robustness plot can be done in a fair condition.

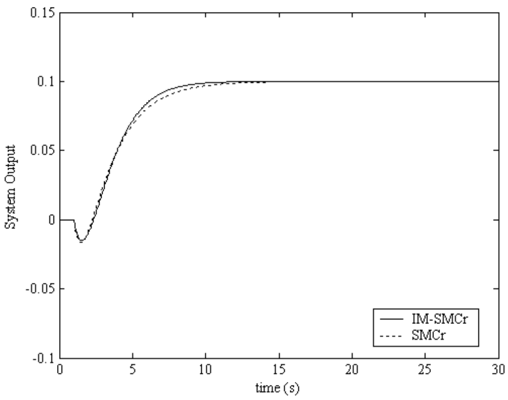

Fig. 8. Process response for set point and disturbance changes for approximately same ISE performance index.

Simultaneous changes of (± 20%) in time constant, τ, and static gain, K, were introduced. In this case robustness plots, originally used for processes modelled by FOPDT models (Shinskey, 1990), did not show how really the controlled system will behave when the process time constant and static gain change, because the readjustment in the SMCr kill K_D (which is responsible for the reaching condition). This originates a steady-state error against static gain changes when the SMCr is used (Fig. 9).

On the other hand, Fig. 10 shows how the IM-SMCr is capable to deal with simultaneous modelling error maintaining both stability and zero steady state error. This is not guaranteed by the SMCr even if the original adjustment is used (see Fig. 11).

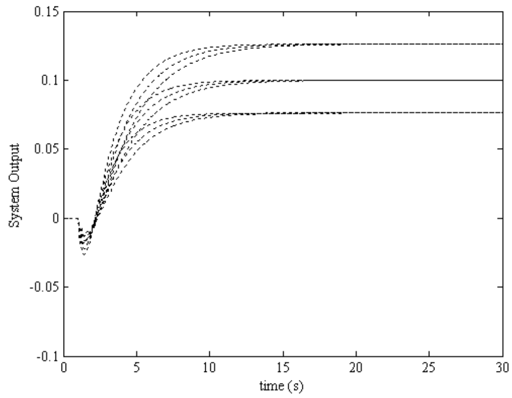

Fig. 9. Step responses of the SMCr controlled system when simultaneous (± 20%) modelling errors in the time constant, τ, and static gain were introduced.

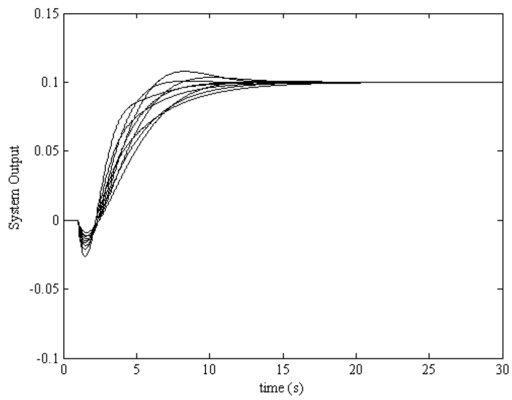

Fig. 10. Step responses of the IM-SMCr controlled system when simultaneous (± 20%) modelling errors in the time constant, τ, and static gain were introduced.

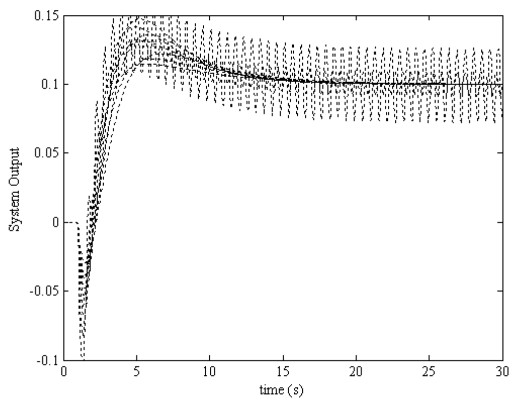

Fig. 11. Step responses of the original SMCr controlled system when simultaneous (± 20%) modelling errors in the time constant, τ, and static gain were introduced.

4.2 Controllers Performance when they are applied to a nonlinear Model

To test the controllers behaviour against set point changes, and the presence of disturbances, the Van de Vusse non linear model was used (A. Aoyama et al., 1995). The isothermal series/parallel reactions which take place in the reactor are:

$$A \rightarrow B \rightarrow C \tag{26}$$

$$2A \rightarrow D \tag{27}$$

The process model consists of two mol mass balances:

$$\frac{dC_A}{dt} = -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A) \tag{28}$$

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B - \frac{F}{V} C_B \tag{29}$$

Where C_A is the effluent concentration of component *A*, C_B is the effluent concentration of *B*, *F* is the input flow and *V* is the reactor volume. The operating values for this study are k_1 =0.833 min^{-1}, k_2 = 1.667 min^{-1} and k_3 = 0.167 L·mol^{-1}·min^{-1}. The concentration of *A* in the feed stream is given by C_{Af} and equal to 10 mol·L^{-1}. In steady-state the process concentrations present the following values CA = 3.0 mol·L^{-1} and C_B = 1.117 mol·L^{-1}. For the process, the parameters of the FOPDT and second order model approximations were obtained (*K* = 0.56 [mol·L^{-1} ·min], τ = 0.70 [min] and t_0 = 1.50 [min] for Eq. (6) and *K* = 0.59 [mol·L^{-1}·min], τ_1 = 0.447 [min], τ_2 = 0.416 [min] and η = 0.355 [min] for Eq. (13), respectively). Using the tuning equations, the adjustment of the SMCr (λ_0 = 0.99, λ_1= 2.10, K_D = 0.87, δ = 0.75), and the IM-SMCr (λ = 0.00, K_D = 1.46, δ= 0.70) parameters were done.

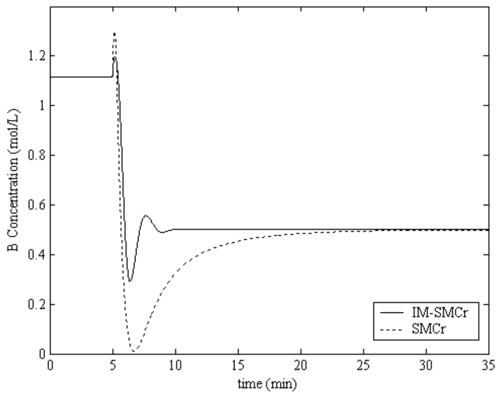

Fig. 12. IM-SMCr and original SMCr controlled system responses when a set point change was introduced.

The control objective is to regulate C_B by manipulating the input flow *F*. Fig. 12 shows the concentration output, C_B, using both controllers when the set point is changed from 1.117 to 0.5 molL^{-1}. The figure depicts inverse response characteristics with a smooth behaviour, and zero steady-state error for both controllers. The inverse response effect is attenuated by the IM-SMCr and produces a shorter settling time than the SMCr as was predicted by linear model performance test.

In the presence of a step disturbance of +20% in the inlet concentration, C_{Af}, the system responses were smooth with zero steady-state error in both cases. The IM-SMCr showed a shorter settling time than the SMCr (see Fig. 13). In spite of the IM-SMCr controller not being derived for nonlinear inverse response systems, based on the performance and robustness shown against step changes, modelling errors and disturbances, it seems to be a good alternative to control nonlinear inverse response systems.

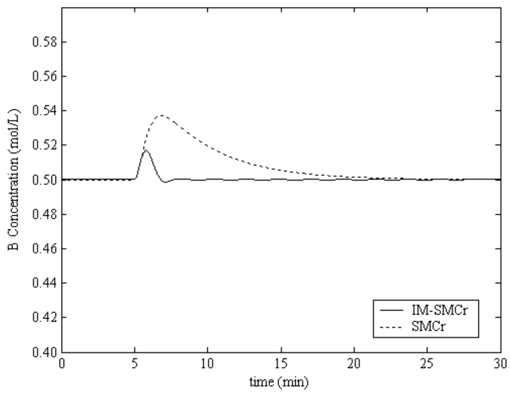

Fig. 13. IM-SMCr and original SMCr controlled system responses when a step disturbance of +20% in the inlet concentration, C_{Af}, was introduced.

5. CONCLUSIONS

This paper showed by simulations that the sliding mode controller synthesized using concepts of internal model control IM-SMCr works well for inverse response systems. The obtained responses showed that the proposed controller has the potential of being used to control more complex or nonlinear systems with inverse response, such as distillation columns, reactors among others. The robustness of the controller against modelling errors, and disturbances was clearly shown. The results showed demonstrate that this new approach can be easily implemented and its performance and robustness are superior to the original SMCr.

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