NETWORKED CONTROL SYSTEMS AND COMMUNICATION NETWORKS: INTEGRATED MODEL AND STABILITY ANALYSIS

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Abstract: An extended model for networked control systems is developed by integrating a networked control systems and a communication network model. This results in a deterministic switching system.
Stability analysis of the integrated system is then investigated using switched system theory. Specifically, sufficient conditions for global exponential stability of the systems are given in terms of plant and network dynamics. A simulation example is presented to study the properties of the extended networked control systems.

Keywords: Networked Control Systems, switched systems, congestion control.

1. INTRODUCTION

The problem of networked control systems (NCS) deals with the possibility of controlling a system via a communication network and as such, instantaneous and perfect signals between controller and plant may no longer be achievable (see Figure 1). This casts classical control problems into a setting that provides control solutions to remotely located systems such as: assembling space structures, exploring hazardous environment, executing tele-surgery, and many others. The network however introduces delays, packet losses that degrade the performance of the system and possibly destabilize it; furthermore, the limited bandwidth of the network compromises our otherwise achievable control objectives. In existing studies in this area (Ling and Lemmon, 2003; Montestruque and Antsaklis, 2001; Montestruque and Antsaklis, 2003; Teel and sicć, 2003) the packet dropout is modeled as a random or deterministic variable independent to the network dynamics.

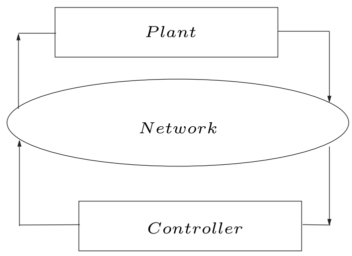

Fig. 1. Networked Control System.

On the other hand communication networks and their complex dynamics have been studied by several researchers, (Ying *et al.*, 2004; Srikant, 2000; Low *et al.*, 2002) in fact, due to the Internet growth in size and complexity, and with the ad-

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vent of industrial networks, an understanding of the organization and efficiency of communication networks has actually become necessary.

Although NCS and communication network dynamics are separated into two different areas of study, they are tightly coupled. It is then reasonable to give shape to this virtual link, by studying the effects communication network dynamics have on the performance of NCS and vice versa.

In this paper we propose a model for deterministic packet dropout in a networked control system in which the dynamics of the communication network appear explicitly, thus achieving a connection between both areas. This will actually allow us to use the congestion controller of the communication network to help in achieving the goals of the networked control system.

In Section 2, we review the various NCS concepts (Azimi-Sadjadi, 2003; Zhang *et al.*, 2001; Branicky *et al.*, 2000; Seiler and Sengupta, 2001) and describe a model for packet dropout based on the analysis of congestion control proposed in (Low *et al.*, 2002). In Section 3, a model for a congestion control network is presented (Walsh *et al.*, 2002). Then in Section 4, we merge the two models of Sections 2 and 3 leading into a complete system-network-controller model. In Section 5 an analysis of the combined model is developed by using the framework of switched systems theory. Finally we present in Section 6 simulation results to illustrate the viability of our approach.

2. PROBLEM FORMULATION AND NCS MODEL

Consider the networked control scheme from (Zhang *et al.*, 2001) depicted in Figure 2. For the remainder of this paper, the plant is a linear discrete-time system, with sampling time h (from now on we will assume for simplicity to be $h = 1$). It is assumed that the state is available for measurements. The network is placed between the sensors and the controller, while the signal between controller and actuator is directly connected. The network is initially modeled as a two-state switch: whenever a state sent across a network is received, it is used for feedback by the controller, and is placed in a buffer. In the case where the state is not received (i.e. dropped by the network), the earlier state value in the buffer is used for feedback. The dynamical model for such a system is thus:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ \bar{x}(k) &= \theta_k x(k) + (1-\theta_k)\bar{x}(k-1) \\ u(k) &= -K\bar{x}(k). \end{aligned} \quad (1)$$

In the above model $\theta_k \in \{0, 1\}$ is called a receiving sequence and indicates the reception ($\theta_k = 1$) or the loss (θ_k=0) of the packet containing the state measurement at time-step k, $x(k)$. We assume that at each time k, a state is sent across the network in one packet, but will re-visit this assumption in our example. We

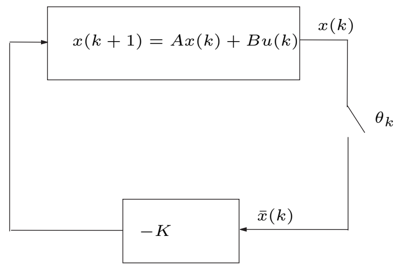

Fig. 2. Networked Control System Model.

then consider the augmented state vector $z(k) = [x^T(k)\ \bar{x}^T(k)]^T$, for which the closed-loop system affected by packet dropout evolves according to:

$$z(k+1) = \begin{bmatrix} A & -BK \\ A\theta_k & I-(I+BK)\theta_k \end{bmatrix} z(k) \quad (2)$$

which can be rewritten as a discrete-time switched system

$$z(k+1) = \tilde{A}_{\theta_k} z(k) \quad (3)$$

where the receiving sequence θ_k represents the switching signal, so that whenever a packet is received the following subsystem is activated

$$\tilde{A}_1 = \begin{bmatrix} A & -BK \\ A & -BK \end{bmatrix}. \quad (4)$$

Since the matrix $\tilde{A}_1$ is Schur stable, by the assumption that the system is closed-loop stable in the case of available full information, the corresponding subsystem is stable. Whenever a packet (a state measurement) is dropped, we have the following subsystem:

$$\tilde{A}_0 = \begin{bmatrix} A & -BK \\ 0 & I \end{bmatrix} \quad (5)$$

in which the matrix $\tilde{A}_0$ is Schur unstable and therefore, in the case of dropped packets, the resulting subsystem is unstable.

We have not yet provided any specifics about the switching signal, which is the receiving sequence θ_k. In particular, the switching rate is not constant as is assumed for example in (Zhang *et al.*, 2001). In what follows, we discuss in detail how θ_k, the switching variable, is driven by the network dynamics, and in fact by relating the switching signal to the network's dynamics, we are able to provide a complete model of the system once connected through the communication network.

3. INTERNET CONGESTION CONTROL ANALYTICAL MODEL

After the problem description from the NCS point of view, we now move on the communication network side to describe its dynamics and how they affect link congestion and therefore the dropping sequence in a NCS connected to such network. We describe an analytical model for a communication network developed in (Low *et al.*, 2002), in particular we discretize the continuous-time model by using a sampling time $T = 10^{-8} sec$.

In (Low *et al.*, 2002) the network is modeled as a dynamical system and the congestion control problem is reformulated as an optimization problem. Two main aspects of congestion control are highlighted; first the characterization of the equilibrium conditions from the point of view of fairness, efficiency in resource usage, the dependence on network parameters, etc. Second, the stability of the equilibria is studied in terms of performance metrics such as speed of convergence, capacity tracking, etc.

A network is modeled as a set L of N_l links with finite capacities $c = (c_l, l \in L)$. They are shared by a set S of N_s sources, some of which may be plants to be controlled. Each source i uses a set $L_i \subseteq L$ of links to transmit data. The sets L_i define an $N_l \times N_s$ routing matrix $R_{li} = \begin{cases} 1 & l \in L_i \\ 0 & l \notin L_i \end{cases}$.

Each source i has an associated transmission rate $r_i(k)$; the set of transmission rates determines the aggregate flow at each link by $y_l(k) = \sum_i R_{li} r_i(k)$.

In this study we neglect any transmission delays between the system and its controller. We assume each link has a capacity c_l packets per second. The next step is to model the feedback mechanism that communicates to sources the congestion information about the network. The key idea in this line of work is to associate with each link l a congestion measure $p_l(k)$, called price, which is a positive real-valued quantity. The fundamental assumption made is that sources have access to the aggregate prices of all links in their route, that is $q_i(k) = \sum_l R_{li} p_l(k)$. In order to specify the congestion control scheme, it is necessary to define how the sources adjust their rates based on their aggregate prices and how the links adjust their prices based on their aggregate rates. For more detail about the algorithms the reader can refer to (Low *et al.*, 2002). At the source side, it is possible in general to postulate a dynamic model of the form $r_i(k+1) = TG_i(q_i(k)) + r_i(k)$. Similarly, at the link level it is possible to postulate a dynamic law

$$v_l(k+1) = TH_l(y_l(k), v_l(k)) + v_l(k) \quad (6)$$

$$p_l(k+1) = TK_l(y_l(k), v_l(k)) + p_l(k). \quad (7)$$

The key restriction in the above control laws is

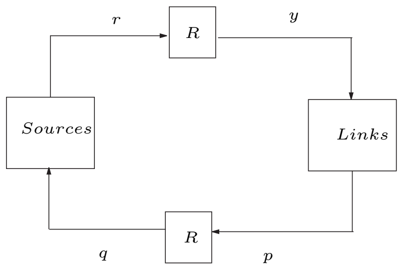

Fig. 3. Communication Network Dynamics.

that they must be decentralized, i.e., sources and links may only have access to their local information. The overall structure of the congestion control system is now depicted in Figure 3, where the diagonal structure of the source and link matrices enforces the decentralization requirement.

We take source rates as the primal variable r and link loss frequencies as prices p. We then recall the Reno average model (Low *et al.*, 2002)

$$r_i(k+1) = T\frac{1 - q_i(k)}{\tau} - \frac{1}{2} q_i(k) r_i(k)^2 + r_i(k)$$

where τ is the transmission delay that we consider negligible ($\tau \approx 0.0001$). We model queues as integrators to obtain simple models of loss frequencies. Let $b_l(k)$ denote the instantaneous queue length at time k; its dynamics is then modeled by

$$b_l(k+1) = \begin{cases} T(y_l(k) - c_l) + b_l(k), & b_l(k) > 0 \\ T\max\{0, (y_l(k) - c_l)\} + b_l(k), & \\ & b_l(k) = 0 \end{cases}$$

The averaged queue length can be modelled as $a_l(k+1) = -T\alpha_l c_l(a_l(k) - b_l(k)) + a_l(k)$, for some constant $0 < \alpha < 1$. Given the average queue length $a_l(k)$, the marking (or dropping) frequency is given by a static function

$$p_l(k) = \begin{cases} 0, & a_l(k) \leq \underline{b}_l \\ \rho_l(a_l(k) - \underline{b}_l), & \underline{b}_l \leq a_l(k) \leq \bar{b}_l \\ \nu_l a_l(k) - (1 - 2\bar{p}_l), & \bar{b}_l \leq a_l(k) \leq 2\bar{b}_l \\ 1, & a_l(k) \geq 2\bar{b}_l \end{cases}$$

where $(\bar{b}_l, \underline{b}_l, \bar{p}_l)$ are RED parameters and $\rho_l = \frac{\bar{p}_l}{\bar{b}_l - \underline{b}_l}$, $\eta_l = \frac{1-\bar{p}_l}{\bar{b}_l}$. We remark again that the preceding analysis refers to the averaged model of TCP. With the described network, and for continuous time dynamics as proven in (Low *et al.*, 2002), the sources and links adjust their rates and prices respectively in order to achieve the optimal equilibrium point at which no packet dropping occurs. A question arises whether this equilibrium is still valid in the case an external greedy source (such as a plant to be controlled), considered as a perturbation, uses the links without participating to the "game of optimization", i.e. affect the links prices without modifying its rate as the price increase. We thus introduce in the set of sources, the plant to be controlled assuming that it is the greedy source. In particular the model is modified by replacing $y_l(k)$ with $y_T(k)$, where $y_T(k) = y_l(k) + r_{gs}(k)$, in which r_{gs} is the rate of the greedy source. We can infer from the above setting that a link is congested and therefore a packet is dropped at time k, if the instantaneous queue length $b_l(k)$ in a link become greater than the queue capacity $\bar{b}_l$, where the instantaneous queue length $b_l(k) = f(c_l, b_l(k-1), y_l)$ is function of link capacity, length at previous step time, link price and cumulative sources rates, which on the other side is function of link prices at previous steps.

4. NETWORK-CONTROL SYSTEM CONNECTED VIA COMMUNICATION NETWORK

We explored both NCS side and communication network side. Now equipped with this framework we want to study how the loss of packets on the network side affects the stability of the overall system. In particular this will allow us to explicitly relate the stability of the system to the capacity of the links involved in the path used by the system, and to the rate of the sources that are accessing such a path. This relation gives us the possibility of designing for the stability of the system by controlling the rate of the sources accessing the path.

Let (L, S) be a network in which each source s_i has an associated rate $r_i(k)$ that is a function of time at which it sends packets trough a set $L_i \subset L$ of links. So through every link l_j a cumulative that is the sum of all the rates of n_s sources is given by $y_l(k) = \sum_{i=1}^{n_s} r_i(k)$. The cumulative rate at each link $y_l(k)$ affects the queue length b_l associated with each link. A link has a limiting capacity and a queue of limited length beyond which it will drop packets. In particular there is a critical queue length $\bar{b}_l$ above which the link will accommodate packets, and below which it will start dropping them. The packet drop will be modeled by the binary value variable θ_k, as discussed earlier. In particular we have that at every time k

$$\theta_k = \prod_{l=1}^{n_l} \left[\frac{sign(\bar{b}_l - b_l(k)) + 1}{2} \right] \tag{8}$$

where the function $sign : \mathbb{R} \rightarrow \{-1, 1\}$ is defined as: $sign(a) = \begin{cases} 1 & a \geq 0 \\ -1 & a < 0 \end{cases}$.

We therefore have $\theta_k = f(\bar{b}_l, b_l)$. With the provided framework we are now able to study the

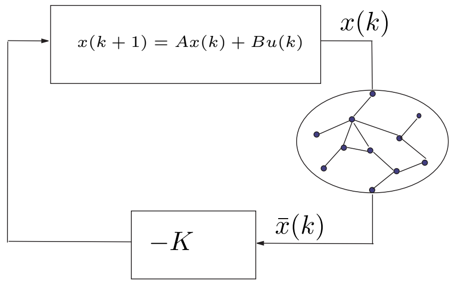

Fig. 4. NCS connected to a communication network.

stability of the following linear time varying system (Figure 4):

$$z(k+1) = \tilde{A} z(k) \tag{9}$$

where

$$\tilde{A}_{11} = A, \;\; \tilde{A}_{12} = -BK$$

$$\tilde{A}_{21} = A \left[\prod_{l=1}^{n_l} \left[\frac{sign(\bar{b}_l - b_l(k)) + 1}{2} \right] \right]$$

$$\tilde{A}_{22} = I - (I + BK) \left[\prod_{l=1}^{n_l} \left[\frac{sign(\bar{b}_l - b_l(k)) + 1}{2} \right] \right]$$

where $b_l(k)$ are the instantaneous queue length. This model of NCS is a discrete-time, time-varying dynamical system that incorporates the system state, and the network dynamics $c_i(k), r_j(k)$. The network is therefore an integral part of the overall system, therefore achieving our modeling goal.

5. STABILITY ANALYSIS FOR INTERCONNECTED NETWORKED-CONTROL SYSTEMS

We now proceed to study stability of the system (9), by using a framework from switching system theory. Consider the switching system (3), in which $\theta_k : \mathbb{N}^+ \rightarrow \{0, 1\}$ is the switching signal, A_i are constant matrices of appropriate dimensions denoting the subsystems (5) and (4), in section 2.

Definition 1. (Zhai *et al.*, 2001) The system (3) is said to be globally exponentially stable, with stability degree $\lambda < 1$, if for all $k \geq 0$ and a constant c satisfy $||x(k)|| \leq c\lambda^k ||x_0||$.

▲

Since the two matrices $\tilde{A}_1$, $\tilde{A}_0$ are Schur stable and unstable respectively, then for $0 < \lambda_1 < 1$, $\lambda_0 \geq 1$, h_0, h_1 and for all $k \geq 1$ the following conditions hold

$$||\tilde{A}_0^k|| \leq h_0 \lambda_0^k, \;\; ||\tilde{A}_1^k|| \leq h_1 \lambda_1^k. \tag{10}$$

We consider the total activation time of the unstable and stable system and denote them

$K^+(k)$, $K^-(k)$ respectively, then for the network dropping sequence we have that:

$$K^+(k) = \sum_{j=0}^{k} \left[1 - \prod_{l=1}^{n_l} \left[\frac{sign(\bar{b}_l - b_l(j)) + 1}{2} \right] \right]$$
$$K^-(k) = \sum_{j=0}^{k} \prod_{l=1}^{n_l} \left[\frac{sign(\bar{b}_l - b_l(j)) + 1}{2} \right]. \quad (11)$$

Also denote with $N_\theta(0,k)$, the number of switches in the interval $[0,k)$ where

$$N_\theta(0,k) = \sum_{j=0}^{k} |\theta_j - \theta_{j-1}|. \quad (12)$$

and the average dwelling time τ_a as the average interval between any two consecutive switching times.

Definition 2. (Zhai *et al.*, 2001) Consider the switching system (3), in which the subsystems are such that (10) holds. We say that the switching law θ_k belongs to class $SW[\tau_a^*]$ if for $\lambda \in (\lambda_1, 1)$ the following conditions are satisfied

- $\inf_{k>0} \frac{K^-(k)}{K^+(k)} \geq \frac{ln\lambda_0 - ln\lambda^*}{ln\lambda^* - ln\lambda_1}$, $\lambda^* \in (\lambda_1, \lambda)$
- The average dwelling time $\tau_a \geq \tau_a^*$

▲

Next we give sufficient condition for global exponential stability of the switching system (3), in term of network dynamics.

Theorem 1. (Zhai *et al.*, 2001) Consider the discrete-time switching system (3) with switching law (8), then for any $\lambda \in (\lambda_1, 1)$, the system is globally exponentially stable with stability degree λ if there exists a constant $\tau_a^* < \infty$, such that the switching law $\theta_k \in SW[\tau_a^*]$.

The proof for the generalized case of multiple subsystems can be found in (Zhai *et al.*, 2001). The main idea in the theorem is that the switching system will be stable if the switching is sufficiently slow and the total activation time of the stable system is sufficiently large with respect to the unstable one.

6. SIMULATION RESULTS

For our simulation we consider the feedback linear system.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -3 & 7 \\ 3 & -7 & 9 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix} \quad (13)$$
$$K = [-0.3846 \quad 0.8671 \quad -0.3283]. \quad (14)$$

Link capacity	$c_l = 9\,[packets/sec]$
Maximum queue length	$\underline{b}_l = 50[packets]$
Maximum average queue length	$\bar{b}_l = 550[packets]$
Minimum price	$\bar{p}_l = .1$
Weight for queue averaging	$\alpha = 10^{-4}$
Number of Sources/ Links	$N_s = 50,\ N_l = 10;$

Table 1. Network parameters.

For the network we consider the parameters described in table 1

At each link we associate a set of sources following law for the routing matrix $R_{l,i} = \frac{(1+sign(i-l))}{2}$. The initial conditions for queue length, average queue length, and initial prices for each link are: $b_l(1) = 0$, $r_l(1) = 0$, $p_l(1) = 0.5$. For the provided network setting the resulting dropping sequence satisfy the conditions of theorem 1 on the average dwelling time with $\tau_a^* = 3$ and total activation time of the stable and unstable system. After modifying the conditions on the initial rate and introducing a greedy source the conditions will no longer be satisfied. We will be operating at a sampling time $T = 10^{-8}sec$. In order to model the rate for the greedy source, we consider that in a typical TCP protocol a packet in general contains 20 bytes for TCP address, 20 bytes for IP address, and $8n$ bytes of data, where n is the number of data to be included in a packet. We then conclude that a packet contains $40+8n$ bytes. Assuming that we are sending at a rate of 60,000 bytes per second, we can then send 1,000 packets per second.

In the first experiment we assume the "greedy plant" is sending at a slow rate of $r_{gs} = 50\,[packets/s]$, through the network, while sending the rest of its data across a secure channel. Also, assume each of the remaining sources is sending at an initial rate of $r_i(1) = 60,000$, $i = 1, \ldots, n_s$. We plot the plant's state versus time. As we see in Figure 5, the greedy source does not affect the traffic in any significant way and the overall system remains stable.

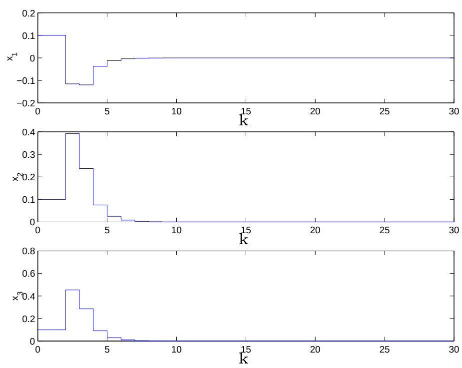

Fig. 5. NCS stabilized across a network with optimal configuration, no loss of information and $r_{gs} = 50$.

We repeat the experiment while increasing the greedy source rate to $r_{gs} = 60,000$, and as we see in Figure 6 the system is no longer stable due to the fact that the network starts dropping packets because of the excess traffic.

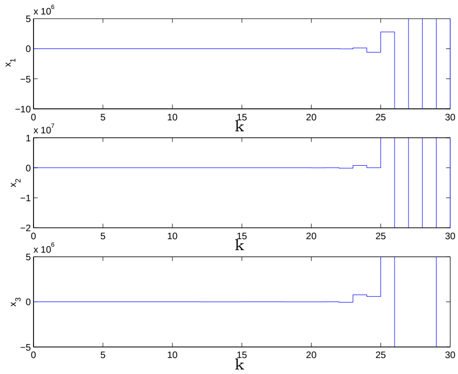

Fig. 6. NCS stabilized across a network with optimal configuration, loss of information and $r_{gs} = 60,000$.

Finally, we consider again the case of a source sending at a rate of $r_{gs} = 50$ but allow the sources involved in the optimization problem to start with a rate $r_i(1) = 1,000$. In Figure 7, we observe that the system goes unstable due to the fact that the network cannot reach the stable equilibria because of the initial excess in the sources rates.

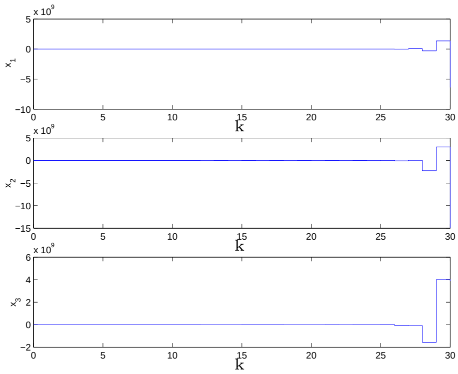

Fig. 7. NCS stabilized across a network with optimal configuration, no loss of information and $r_{gs} = 50$, $r_i(1) = 10,000$.

We then conclude that the stability of the system is sensitive to the loss of information. Moreover, the network stability is sensitive to the intrusion of external sources that affect the traffic without regulating their rate. The network stability is also sensitive to the initial sources rate.

7. CONCLUSION

In this paper we presented a NCS model affected by packet dropping, in which the communication network is explicit in the model. The model provides for the first time a study of the combined effects of the congestion controller and of the networked control system. We are currently studying the effects of time delays in combination with the packets dropping.

REFERENCES

Azimi-Sadjadi, B. (2003). Stability of networked control systems in the presence of packet losses. In: *IEEE Conference on Decision and Control.*

Branicky, M., S. Phillips and W. Zhang (2000). Stability of networked control systems: Explicit analysis of delay. In: *Proceeding of ACC.* pp. 2352–2357.

Ling, Q. and M.D. Lemmon (2003). Soft real-time scheduling of networked control systems with dropouts governed by a markov chain. In: *IEEE American Control Conference.* pp. 4845–4850.

Low, S., F. Paganini and J. Doyle (2002). Internet congestion control: An analytical perspective. *IEEE Control Systems Magazine* **22**(1), 28–43.

Montestruque, L.A. and P.J. Antsaklis (2001). Model-based networked control system - stability. Technical report. ISIS-2002-001, Notre Dame, IN,.

Montestruque, L.A. and P.J. Antsaklis (2003). Stochastic stability for model-based networked control systems. In: *Proceedings of the American Control Conference.* Vol. 2. pp. 4119–4124.

Seiler, P. and R. Sengupta (2001). Analysis of communication losses in veicle control problems. In: *Proceeding of ACC.* pp. 1491–1496.

Srikant, R. (2000). *Perspectives in Control Engineering: Technologies, Applications, New Directions.* Chap. Control of Communication Networks, pp. 462–488. IEEE Press.

Teel, A.R. and D. Nešicć (2003). Input-output stability properties of networked control systems. In: *IEEE Conference on Decision and Control.*

Walsh, G.C., H. Ye and L. Bushnell (2002). Stability analysis of networked control systems. *IEEE Transactions on Control Systems Technology* **10**(3), 438–446.

Ying, L., G. Dullerud and R. Srikant (2004). Global stability of internet congestion controllers with heterogeneous delays. In: *Proc. American Control Conference.*

Zhai, G., B. Hu, K.Yasuda and A.N. Michel (2001). Qualitative analysis of discrete- time switched systems. In: *Proceedings of the 45-th Annual Conference of the Institute of Systems, Control and Information Engineering.* number 3009. pp. 237–238.

Zhang, W., M.S. Branicky and S.M. Phillips (2001). Stability of networked control systems. *IEEE Control Systems Magazine* **21**(1), 84–99.