CONTROL OF TIME-DELAYED LPV SYSTEMS USING DELAYED FEEDBACK

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Abstract: In this paper, the delayed (memory) feedback synthesis problem for linear parameter varying (LPV) systems with parameter-varying time delays is introduced and addressed. It is assumed that the state-space data and the time-delay depend continuously on the parameters which are measurable in real-time and vary in a compact set with bounded variation rates. Synthesis conditions for stabilization and $\mathcal{L}_2$ norm performance using delayed state feedback and delayed output feedback are formulated in terms of Linear Matrix Inequalities (LMIs) that can be solved efficiently. It is shown that time-delayed feedback control provides advantages in terms of reduced conservatism, improved performance and ease of controller implementation. Numerical examples are used to demonstrate the improved performance of the proposed delayed feedback configuration compared with that of the memoryless feedback schemes.

Keywords: Time-delay, Time-varying systems, Memory, State feedback, Dynamic output feedback, Lyapunov methods, Convex optimization

1. INTRODUCTION

The dynamics of several practical engineering systems often depend on varying system parameters. Such systems have come to be known as linear parameter-varying (LPV) systems (Shamma and Athans, 1991). This paper will be concerned with LPV time-delayed systems with parameter-varying delays of the form

$$\dot{x}(t) = A(\rho)x(t) + A_h(\rho)x(t-h(\rho)) + B_1(\rho)w(t) + B_2(\rho)u(t), \quad \text{(1a)}$$

$$z(t) = C_1(\rho)x(t) + C_{1h}(\rho)x(t-h(\rho)) + D_{11}(\rho)w(t) + D_{12}(\rho)u(t), \quad \text{(1b)}$$

$$y(t) = C_2(\rho)x(t) + C_{2h}(\rho)x(t-h(\rho)) + D_{21}(\rho)w(t), \quad \text{(1c)}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $w(t) \in \mathbf{R}^{n_d}$ is the vector of exogenous inputs, $u(t) \in R^{n_u}$ is the control input, $z(t) \in R^{n_z}$ the error output, $y(t) \in \mathbf{R}^{n_y}$ denotes the measurement vector and $h(\cdot)$ is a differentiable scalar function representing the parameter-varying time delay. It is assumed that the delay is bounded and the function $t-h(t)$ is monotonically increasing, that is h lies in the set

$$\mathcal{H} \triangleq \left\{ \begin{array}{c} h \in \mathcal{C}(\mathbf{R},\mathbf{R}) : 0 \leq h(t) \leq H < \infty, \\ \dot{h}(t) \leq \tau < 1, \forall t \in \mathbf{R}_+ \end{array} \right\}. \quad (2)$$

The initial data function

$$x(\theta) = \phi(\theta), \theta \in [-h(\rho(0)), 0], \quad (3)$$

is a given function in the set of continuous functions $\mathcal{C}([-H,0],\mathbf{R}^n)$. It is assumed that all the state-space matrices and the time-delay function $h(\cdot)$ are known continuous functions of a time-varying parameter vector $\rho(\cdot) \in \mathcal{F}^{\nu}_{\mathcal{P}}$, where $\mathcal{F}^{\nu}_{\mathcal{P}}$ is

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the set of allowable parameter trajectories defined as

$$\mathcal{F}_{\mathcal{P}}^{\nu} \triangleq \left\{ \begin{array}{c} \rho \in \mathcal{C}(\mathbf{R}, \mathbf{R}^s) : \rho(t) \in \mathcal{P}, \\ |\dot{\rho}_i(t)| \le \nu_i, i = 1, 2, \ldots, s, \forall t \in \mathbf{R}_+ \end{array} \right\}, \tag{4}$$

where $\mathcal{P}$ is a compact subset of $\mathbf{R}^s$, $\{\nu_i\}_{i=1}^{s}$ are non-negative numbers and $\nu = [\nu_1, \nu_2, \ldots, \nu_s]^T$. It should be noted that the parameter-dependence will be suppressed in the following presentation whenever it is obvious.

Time-delayed linear systems has been a fertile area of research, see (Niculescu and Gu, 2004; Dugard and Verriest, 1998) and the numerous references therein. There is a large body of work concerned with analysis and control for time-delay systems in the time domain, more specifically using Lyapunov's second method. The stability property of time-delayed systems is usually assumed to lie in one of the following two categories: *delay-independent* or *delay-dependent* stability. There are two generally accepted ways of developing the second method of Lyapunov for time-delayed systems, one using the *Lyapunov-Krasovskii functional* approach and the other via *Lyapunov-Razumikhin functions* (Hale and Lunel, 1993). Delay-dependent properties are derived by using a transformed system on $[t-2h, t]$ which is obtained by applying the Leibnitz-Newton formula for the original system. It is usual practice to use the Lyapunov-Razumikhin theory to obtain delay-dependent properties and Lyapunov-Krasovskii functionals for delay-independent properties.

The development of results for LPV time-delayed systems have followed corresponding methods developed for linear time-invariant (LTI) time-delay systems with initial work being reported in (Wu and Grigoriadis, 2001) where state feedback control design for desired $\mathcal{L}_2$-gain performance was presented. The extension to output feedback control for LPV time-delayed systems with $\mathcal{L}_2$ and $\mathcal{L}_2 - \mathcal{L}_\infty$ performance requirements was carried out in (Tan and Grigoriadis, 2000; Tan *et al.*, 2003). Other than these results on output feedback control, most of the related research has focused on obtaining less conservative stability properties and better estimates of maximum time delays for delay-dependent properties using progressively more complex Lyapunov functionals (Zhang *et al.*, 2002).

This paper is concerned with delay-independent analysis and feedback control of LPV time-delayed systems wherein the desired property holds for all positive (and finite) values for the delays. Particularly, the new idea is the use of state feedback and output feedback induced $\mathcal{L}_2$-gain controllers having *memory* in the control action. Synthesis conditions are obtained for stabilization and $\mathcal{L}_2$ norm performance using such delayed control in terms of LMIs. The designed controllers are also time-delayed and hence are infinite-dimensional systems. In contrast to memoryless controllers proposed so far in time-delayed systems theory (Niculescu, 1998), the advantages in terms of reduced conservatism, improved performance and implementation ease due to the explicit inclusion of *time-delayed feedback* terms in the control law are illustrated.

The paper is organized as follows. Section 2 reviews the sufficient analysis conditions for a time-delayed LPV system to be stable and provide a prescribed level of induced $\mathcal{L}_2$ performance gain γ. The delayed state feedback and the delayed output feedback control synthesis conditions are developed in Sections 3 and 4 respectively. Section 5 demonstrates the improved performance achieved by the use of delayed feedback via numerical examples. Section 6 concludes the paper.

2. ANALYSIS OF TIME-DELAYED LPV SYSTEMS

Consider the time-delayed LPV system described by the state space equations (1). The following result (Tan *et al.*, 2003) provides a sufficient condition for the induced $\mathcal{L}_2$ gain performance of the uncontrolled time-delayed LPV system ($u \equiv 0$) to be less than a given bound γ, that is

$$\sup_{\rho \in \mathcal{F}_{\mathcal{P}}^{\nu}} \sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2} \le \gamma$$

Theorem 1. Consider the uncontrolled time-delayed system (1a)-(1b) with initial data $\phi \equiv 0$. If there exist continuously differentiable matrix functions $P, Q : \mathbf{R}^s \longrightarrow \mathbf{S}_+^{n \times n}$, $P(\rho) > 0, Q(\rho) > 0$ such that the functional linear matrix inequality

$$\Lambda(P, Q, \rho, r) < 0 \tag{5}$$

where

$$\begin{aligned}
\Lambda_{11} &= A^T(\rho)P(\rho) + P(\rho)A(\rho) \\
&\quad + \sum_{i=1}^{s} \pm \left(\nu_i \frac{\partial P}{\partial \rho_i} \right) + Q(\rho) \\
\Lambda_{21} &= A_h^T(\rho)P(\rho) \\
\Lambda_{22} &= -\left[1 - \sum_{i=1}^{s} \pm \left(\nu_i \frac{\partial h}{\partial \rho_i} \right) \right] Q(r) \\
\Lambda_{31} &= B_1^T(\rho)P(\rho), \Lambda_{32} = 0 \\
\Lambda_{41} &= C_1(\rho), \Lambda_{42} = C_{1h}(\rho) \\
\Lambda_{33} &= \Lambda_{44} = -\gamma I, \Lambda_{43} = D_{11}(\rho)
\end{aligned}$$

holds for all $\rho, r \in \mathcal{P}, |\dot{\rho}_i| \le \nu_i$, then the time-delayed system (1a),(1b) is asymptotically stable and has induced $\mathcal{L}_2$ gain less than γ.

3. STATE FEEDBACK CONTROL

In this section, the problem of designing a parameter-dependent state feedback controller for a time-delayed LPV system, which minimizes the induced $\mathcal{L}_2$ gain of the system is investigated. First, a result from (Wu and Grigoriadis, 2001) which deals with the design of *memoryless* state feedback controllers is reviewed. Next, an extension of the synthesis result to *delayed state feedback* is provided. It is shown that delayed feedback control results in reduced conservatism, improved performance and ease of implementation compared to memoryless control.

Consider the time-delayed LPV system (1) where the measurement equation is now

$$y(t) = x(t). \tag{6}$$

In addition, it is assumed that for all $\rho \in \mathcal{P}$

A1 $D_{12}(\rho)$ has full column rank.
A2 $(A(\cdot), B_2(\cdot))$ is asymptotically stabilizable.
A3 $C_1(\rho), C_{1h}(\rho)$ and $D_{12}(\rho)$ have the following normalized structure:

$$C_1 = \begin{bmatrix} C_{11} \\ C_{12} \end{bmatrix}, C_{1h} = \begin{bmatrix} C_{11h} \\ C_{12h} \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

3.1 Memoryless state feedback

A parameter-dependent state feedback controller

$$u(t) = F(\rho(t), \dot{\rho}(t))x(t) \tag{7}$$

is to be designed for the LPV time-delayed system (1) such that the closed-loop system is asymptotically stable and has induced $\mathcal{L}_2$ norm less than a specified bound γ. Using the state feedback control law (7) the closed-loop system becomes:

$$\dot{x}(t) = A_F x(t) + A_h x(t - h(\rho)) + B_1 w(t), \tag{8}$$
$$z(t) = C_F x(t) + C_{1h} x(t - h(\rho)) \tag{9}$$

where $A_F \triangleq A(\rho) + B_2(\rho)F(\rho, \dot{\rho})$ and $C_F \triangleq C_1(\rho) + D_{12}(\rho)F(\rho, \dot{\rho})$. The following result (Wu and Grigoriadis, 2001) provides conditions for the closed-loop system (8),(9) to be asymptotically stable and have induced $\mathcal{L}_2$ gain less than γ.

Theorem 2. Consider the time-delayed LPV system (1a),(1b). There exists a parameter-dependent memoryless state feedback controller (7) such that the closed-loop system is asymptotically stable and has induced $\mathcal{L}_2$ gain less than γ if there exists a continuously differentiable matrix function $R : \mathbf{R}^s \longrightarrow \mathbf{S}_+^{n\times n}$ and a matrix $S \in \mathbf{S}_+^{n\times n}$, such that for all $\rho \in \mathcal{P}$

$$\Phi(R, S, \rho) < 0 \tag{10}$$
$$-\gamma I + \psi C_{1h}(\rho) S C_{1h}^T(\rho) < 0, \tag{11}$$

where

$$\Phi_{11} = R(\rho)\hat{A}^T(\rho) + \hat{A}(\rho)R(\rho) - \sum_{i=1}^{s} \pm \left(\nu_i \frac{\partial R}{\partial \rho_i}\right)$$
$$+\psi \hat{A}_h(\rho) S \hat{A}_h^T(\rho) - \gamma B_2(\rho) B_2^T(\rho)$$
$$\Phi_{21} = R(\rho), \Phi_{22} = -S, \Phi_{31} = B_1^T(\rho),$$
$$\Phi_{32} = 0, \Phi_{33} = -\gamma I, \Phi_{42} = 0, \Phi_{43} = 0,$$
$$\Phi_{41} = C_{11}(\rho)R(\rho) + \psi C_{11h}(\rho) S \hat{A}_h(\rho),$$
$$\Phi_{44} = -\gamma I + \psi C_{11h}(\rho) S C_{11h}^T(\rho),$$
$$\psi \triangleq \left[1 - \sum_{i=1}^{s} \pm \left(\nu_i \frac{\partial h}{\partial \rho_i}\right)\right]^{-1},$$
$$\hat{A} \triangleq A - B_2 C_{12}, \hat{A}_h \triangleq A_h - B_2 C_{12h}.$$

Moreover one such memoryless state feedback control law that provides a guaranteed $\mathcal{L}_2$ gain performance γ is given by

$$F(\rho(t), \dot{\rho}(t)) = -F_1^{-1}(\rho(t), \dot{\rho}(t)) \times F_2(\rho(t), \dot{\rho}(t)) \tag{12}$$

where

$$F_1 = I + \gamma^{-1} C_{12h} \begin{bmatrix} \left(1 - \sum_{i=1}^{s} \dot{\rho}_i \frac{\partial h}{\partial \rho_i}\right) S^{-1} \\ -\gamma^{-1} C_{1h}^T C_{1h} \end{bmatrix}^{-1} C_{12h}^T$$

and

$$F_2 = C_{12h} \times \left[-\gamma^{-1} C_{1h}^T C_{1h} + \left(1 - \sum_{i=1}^{s} \dot{\rho}_i \frac{\partial h}{\partial \rho_i}\right) S^{-1}\right]^{-1}$$
$$\times \left[A_h^T R^{-1} + \gamma^{-1} C_{1h}^T C_1\right] + \gamma B_2^T R^{-1} + C_{12}$$

3.2 Delayed state feedback

In this section, the analysis result in Section 2 is used to design a *delayed* state feedback controller for LPV systems with parameter-dependent state delays. Consider again the open loop system given by (1a),(1b) with the measurement equation (6), and assume that the assumptions **A1 - A3** hold. The goal is to design a parameter-dependent delayed state feedback law

$$u(t) = F(\rho, \dot{\rho})x(t) + F_h(\rho, \dot{\rho})x(t - h(\rho)) \tag{13}$$

to stabilize the closed-loop systems and provide a desired closed-loop $\mathcal{L}_2$ gain performance γ. Although, the measurement gives the current state vector $x(t)$, memory is introduced in the feedback term so that it has the form (13). The closed-loop system with the feedback law in (13) is

$$\dot{x}(t) = A_F x(t) + A_{hF} x(t - h) + B_1 w(t), \tag{14a}$$
$$z(t) = C_F x(t) + C_{hF} x(t - h), \tag{14b}$$

where A_F, C_F are as before and $A_{hF} = A(\rho) + B_2(\rho)F_h(\rho, \dot{\rho})$ and $C_{hF} \triangleq C_{1h}(\rho) + D_{12}(\rho)F_h(\rho, \dot{\rho})$. The following result provides sufficient conditions for the closed-loop system (14) to be asymptotically stable and have induced $\mathcal{L}_2$ gain less than γ.

Theorem 3. Consider the time-delayed LPV system (1a),(1b). There exists a parameter-dependent delayed state feedback controller (13) such that the closed-loop system (14) is asymptotically stable and has induced $\mathcal{L}_2$ norm less than γ if there exists a continuously differentiable matrix function $R : \mathbf{R}^s \longrightarrow \mathbf{S}_+^{n\times n}$ and a matrix $S \in \mathbf{S}_+^{n\times n}$, such that the inequality (10) holds for all $\rho \in \mathcal{P}$. Moreover one such delayed state feedback control law (13) that provides a guaranteed $\mathcal{L}_2$ gain performance γ is given by

$$F(\rho) = -C_{12}(\rho) - \gamma B_2^T(\rho)R^{-1}(\rho) \quad (15a)$$
$$F_h(\rho) = -C_{12h}(\rho). \quad (15b)$$

Proof. Omitted. ■

Remark 1. The synthesis LMIs obtained using the delayed state feedback have a simplified structure as the inequality (11) drops out, leading to less conservative results and less computational burden due to reduced number of constraints. The state feedback gains for the delayed state feedback, given by (15) are simpler to implement when compared to the memoryless state feedback gains given by (12) since the measurement of the rate of variation of the parameter vector $\dot{\rho}$ is not required for computing delayed state feedback gains at any instant of time. Hence, delayed state feedback results in improved performance and practically implementable LPV controllers.

Remark 2. Note that, in deriving the above results Q was fixed to be a constant matrix as the main idea in this paper is motivating the use of delayed feedback. However, the extension to use of parameter-dependent Q is straightforward and will be used in the output feedback problem.

Remark 3. The delayed state feedback term in (13) vanishes when $C_{12h} = 0$ and the memoryless state feedback control law is recovered.

4. DELAYED OUTPUT FEEDBACK CONTROL

Consider the time-delayed LPV plant given by (1). The memoryless output feedback induced $\mathcal{L}_2$-gain performance problem for such a plant has been investigated in (Tan and Grigoriadis, 2000; Tan *et al.*, 2003). In this paper, the following form for the *time-delayed LPV controller* is introduced

$$\begin{aligned}\dot{x}_K(t) &= A_K(\rho)x_K(t) + A_{Kh}(\rho)x_K(t-h(\rho)) \\ &\quad + B_K(\rho)y(t), \quad (16a)\\ u(t) &= C_K(\rho)x_K(t) + C_{Kh}(\rho)x_K(t-h(\rho)) \\ &\quad + D_K(\rho)y(t). \quad (16b)\end{aligned}$$

Theorem 4. If there exist continuously differentiable matrix functions $X > 0, Y > 0$, and matrix functions $\Sigma = (Q_{11}, Q_{21}, Q_{22}, K, K_h, L, M, M_h, N)$ such that the inequalities

$$\Delta(X, Y, \Sigma, \rho, r) < 0 \quad (17)$$
$$\begin{bmatrix} Y & I \\ I & X \end{bmatrix} > 0, \begin{bmatrix} Q_{11} & Q_{21}^T \\ Q_{21} & Q_{22} \end{bmatrix} > 0 \quad (18)$$

hold for all $\rho, r \in \mathcal{P}, |\dot{\rho}_i| \leq \nu_i$ (all matrix functions, where not indicated, are assumed to be functions of $\rho \in \mathcal{P}$), where

$$\begin{aligned}
&\Delta_{11} = -\dot{Y} + AY + YA^T + B_2M + M^TB_2^T + Q_{11},\\
&\Delta_{21} = (A + B_2NC_2)^T + K + Q_{21}\\
&\Delta_{22} = \dot{X} + XA + A^TX + LC_2 + C_2^TL^T + Q_{22}\\
&\Delta_{31} = (A_hY + B_2M_h)^T, \Delta_{32} = K_h^T, \Delta_{33} = \tau Q_{11}(r),\\
&\Delta_{41} = (A_h + B_2NC_{2h})^T, \Delta_{42} = (XA_h + LC_{2h})^T,\\
&\Delta_{43} = \tau Q_{21}(r), \Delta_{44} = \tau Q_{22}(r)\\
&\Delta_{51} = (B_1 + B_2ND_{21})^T, \Delta_{52} = (XB_1 + LD_{21})^T,\\
&\Delta_{53} = 0, \Delta_{54} = 0, \Delta_{55} = -\gamma I,\\
&\Delta_{61} = C_1Y + D_{12}M, \Delta_{62} = C_1 + D_{12}NC_2,\\
&\Delta_{63} = C_{1h}Y + D_{12}M_h, \Delta_{64} = C_{1h} + D_{12}NC_{2h},\\
&\Delta_{65} = D_{11} + D_{12}ND_{21}, \Delta_{66} = -\gamma I,\\
&\tau = -\left[1 - \sum_{i=1}^{s} \pm\left(\nu_i \frac{\partial h}{\partial \rho_i}\right)\right],
\end{aligned}$$

then the closed-loop system formed by the interconnection of (1) and (16) is asymptotically stable and has induced $\mathcal{L}_2$ gain less than γ. Moreover, if matrix functions are determined that satisfy the conditions of the theorem, then a delayed output feedback controller of the form (16) can be computed by reversing the transformations defined by the following equations

$$\begin{aligned}
N &= D_K \quad (19a)\\
M &= C_KV^T + D_KC_2Y \quad (19b)\\
L &= UB_K + XB_2D_K \quad (19c)\\
K &= UA_KV^T + UB_KC_2Y + XB_2C_KV^T \quad (19d)\\
&\quad + XB_2D_KC_2Y + XAY + \dot{X}Y + \dot{U}V^T\\
M_h &= C_{Kh}V^T + D_KC_{2h}Y \quad (19e)\\
K_h &= UA_{Kh}V^T + UB_KC_{2h}Y + XB_2C_{Kh}V^T\\
&\quad + XB_2D_KC_{2h}Y + XA_hY. \quad (19f)
\end{aligned}$$

where the nonsingular matrix functions U, V are computed from the relation

$$I - XY = UV^T.$$

Proof. Omitted. ■

5. NUMERICAL EXAMPLES

5.1 Delayed state feedback

Consider the time-delayed linear parameter-varying system (1) adopted from (Mahmoud and Al-Muthairi, 1994) and modified to demonstrate the advantages of delayed state feedback with system data as follows

$$A = \begin{bmatrix} 0 & 1+0.2\rho_1 \\ -2 & -3+0.1\rho_1 \end{bmatrix}, A_h = \begin{bmatrix} 0.2\rho_1 & 0.1 \\ -0.2+0.1\rho_1 & -0.3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2\rho_1 \\ 0.1+0.1\rho_1 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C_{1h} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, h = 0.9\rho_2.$$

The system is a state-delayed LPV system with parameters $\rho_1(t)$ and $\rho_2(t)$. The parameter space is $[-1,1] \times [0,1]$ and $|d\rho_i/dt| \leq 1, i = 1,2$. The synthesis problem is solved both for memoryless state feedback and delayed state feedback and the results are compared. From Theorem 2 an induced $\mathcal{L}_2$ performance bound $\gamma^m_{\mathbf{LPV}} = 1.4265$ is obtained for memoryless control. Using Theorem 3 an induced $\mathcal{L}_2$ performance bound $\gamma^d_{\mathbf{LPV}} = 0.3838$ is achieved for delayed control which denotes more than 70% improvement in achievable performance compared to memoryless control. For an initial condition $(x_1(0), x_2(0)) = (-2,1)$, $\rho_1(t) = \sin t$, $\rho_2(t) = |\cos t|$ and a unit step disturbance $w(t)$, the closed-loop behavior of the system using both memoryless and delayed state feedback is simulated. The system states are shown in Figure 1. The dotted line corresponds to closed-loop state response with memoryless state-feedback and the solid line is the response with the delayed state-feedback. The control input profile is shown in Figure 2. Both states x_1 and x_2 converge equally rapidly (Figure 1). However the control effort using the delayed state-feedback is significantly less than that of the memoryless state-feedback (Figure 2).

5.2 Delayed output feedback

Consider the same time-delayed LPV system as before with $\rho_2 = \rho_1$ and the system data modified as follows

$$B_2 = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C_{1h} = 0$$
$$C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_{2h} = 0, h = 0.5\rho_1$$

It is assumed that $\rho_1(t) \in [0,1], |\dot{\rho}_1| \leq 1$. A parameter-dependent memoryless output feedback controller is designed using the results in

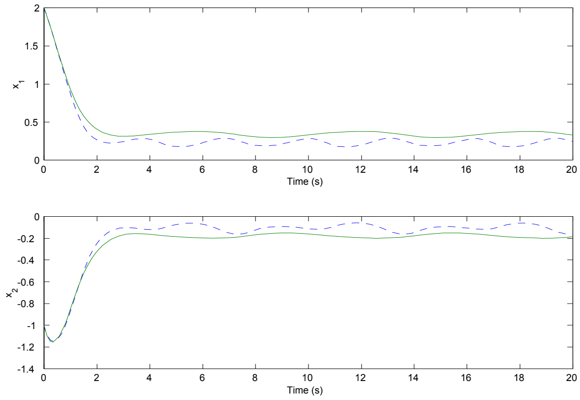

Fig. 1. State response with memoryless feedback (dashed) and delayed feedback (solid).

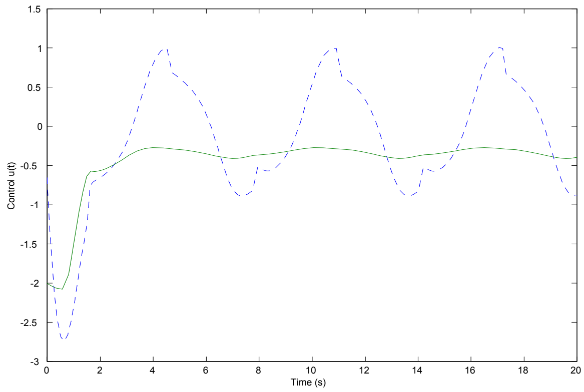

Fig. 2. Control input: Mermoryless f/b (dashed), delayed f/b (solid).

(Tan *et al.*, 2003) and a LPV time-delayed controller using Theorem 4. All matrix functions in the synthesis LMIs are assumed to be affine functions of the scheduling parameter ρ_1. The induced $\mathcal{L}_2$ performance level achieved by the memoryless output feedback controller is $\gamma^m_{\mathbf{LPV}} = 2.5078$ compared to $\gamma^d_{\mathbf{LPV}} = 0.2687$ achieved by the delayed output feedback controller which represents an almost 90% improvement in achievable performance bounds with the use of delayed output feedback. The disturbance signal $w(t)$ shown in Figure 3 is applied to the time-delayed LPV system with initial states $(x_1(0), x_2(0)) = (-2,1)$, $\rho_1(t) = \sin t$ and the time domain responses with memoryless output feedback (dashed line) and delayed output feedback (solid line) are compared in Figures 4 and 5. The error signal $z(t)$ plotted in Figure 4 shows that delayed feedback clearly achieves superior performance (better disturbance attenuation) when compared to memoryless output feedback with similar control effort (Figure 5).

6. CONCLUSIONS

In this paper a delayed feedback control problem for LPV systems with time-varying state delays is proposed. A new control structure is introduced

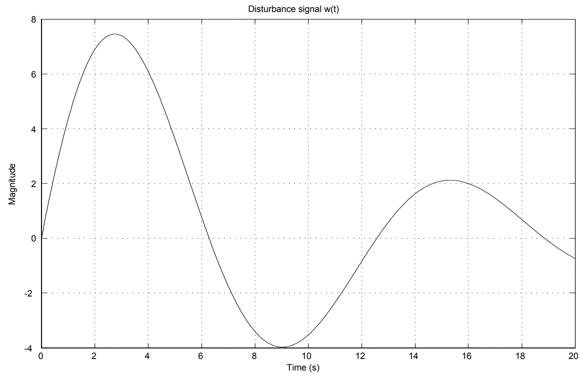

Fig. 3. Disturbance signal $w(t)$

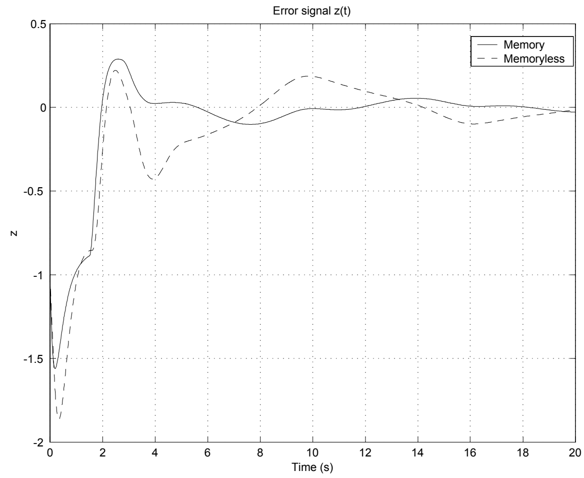

Fig. 4. Error output $z(t)$

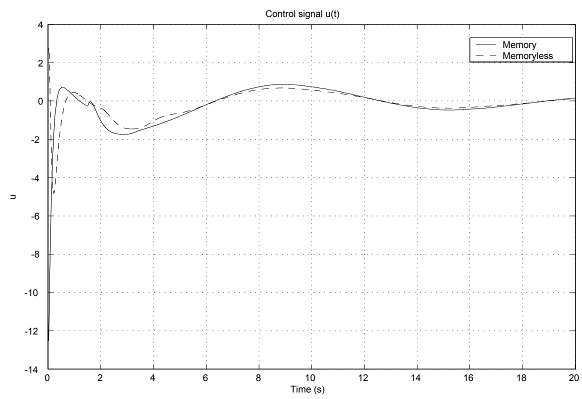

Fig. 5. Control effort $u(t)$

for state feedback and output feedback control of time-delayed systems wherein the controller is also time-delayed (infinite dimensional). The corresponding synthesis conditions for stabilization and induced $\mathcal{L}_2$ norm performance are derived in terms of LMIs. The proposed time-delayed control structure offers more degrees of freedom to reduce conservatism and obtain better closed-loop performance compared to memoryless control. In addition, for the state feedback case practically implementable controllers that are independent of parameter variation rate are obtained. Numerical examples are used to illustrate the advantage of using delayed states in the feedback.

It is straightforward to specialize all the results to the case of LTI time-delayed systems with constant or varying delays by making all functions/matrices constant (independent of any scheduling parameter). Further, new (less conservative) conditions for stability/stabilizability of time-delayed systems using delayed feedback can be obtained. Applications of the proposed time-delayed control structure for obtaining less conservative delay-dependent closed-loop properties will be considered in the future. However, it should be noted that for time-varying delays the control implementation will require more memory compared to that for the case of constant delays.

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