HYBRID MODEL PREDICTIVE CONTROL IN SUPERMARKET REFRIGERATION SYSTEMS

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Abstract: This paper presents a Model Predictive Control scheme in connection with the Mixed Logical Dynamical framework, which tackles the modelling and optimal control problem of supermarket refrigeration systems. The latter are hybrid systems with switched nonlinear dynamics and discrete-valued input variables such as valves. Simulation results indicate the potential increase in efficiency and reduced wear with respect to traditional control schemes used nowadays.

Keywords: Supermarket refrigeration systems, hybrid systems, MPC.

1. INTRODUCTION

A supermarket refrigeration system consists of a central compressor bank that maintains the required flow of refrigerant to the refrigerated display cases located in the supermarket sales area. Each display case has an inlet valve for refrigerant that needs to be opened and closed such that the air temperature in the display case is kept within tight bounds to ensure a high quality of the goods.

For many years, the control of supermarket refrigeration systems has been based on distributed control systems, which are flexible and simple. In particular, each display case used to be equipped with an independent hysteresis controller that regulates the air temperature in the display case by manipulating the inlet valve.

The major drawback, however, is that the control loops are vulnerable to self-inflicted disturbances caused by the interaction between the distributed control loops. In particular, practice and simulations show that the distributed hysteresis controllers have the tendency to synchronize (Larsen, 2004*a*), meaning that the opening and closing actions of the valves coincide. Consequently, the compressor periodically has to work hard to keep up the required flow of refrigerant, which results in low efficiency, inferior control performance and a high wear on the compressor.

The control problem is significantly complicated by the fact that many of the control inputs are restricted to discrete values, such as the opening/closing of the inlet valves and the stepwise control of the compressor. Furthermore, the system features switched dynamics turning the supermarket refrigeration system into a hybrid system.

Motivated by these difficulties, we present in this paper a novel approach to the modelling and controller design problem for supermarket refrigeration systems. The refrigeration system is modelled as a hybrid system using the Mixed Logical Dynamical (MLD) (Bemporad and Morari, 1999) framework allowing one to capture the switched dynamics and the discrete-valued control inputs. Based on the MLD model, we propose a centralized constrained finite-time optimal controller, specifically Model Predictive Control (MPC) (Maciejowski, 2002), which accommodates the multivariate nature of the refrigeration system, handles the constraints explicitly, and most important, allows for a systematic controller design.

The paper is structured in the following way. Section 2 describes the basic layout of a supermarket refrigeration system, and Section 3 provides an overview of the traditional control setup. Section 4 summarizes the nonlinear model of the refrigeration system, the MLD framework and the derivation of the MLD model. In Section 5, the control objectives are formulated and an objective function is set up. The proposed hybrid MPC scheme is compared with the traditional control approach in Section 6 through simulations. Conclusions are drawn in Section 7.

2. SYSTEM DESCRIPTION

In a supermarket many goods need to be refrigerated to ensure preservation for consumption. These goods are normally placed in open refrigerated display cases that are located in the supermarket sales area for self service.

A simplified supermarket refrigeration circuit is shown in Figure 1. The heart of the system are the compressors. In most supermarkets, the compressors are configured as compressor racks, which consist of a number of compressors connected in parallel. The compressors supply the flow of refrigerant in the system by compressing the low pressure refrigerant from the suction manifold, which is returning from the display cases. The compressors keep a certain constant pressure in the suction manifold, thus ensuring the desired evaporation temperature. From the compressors, the refrigerant flows to the condenser and further on to the liquid manifold. The evaporators inside the display cases are fed in parallel from the liquid manifold through an expansion valve. The outlets of the evaporators lead to the suction manifold and back to the compressors thus closing the circuit.

In Figure 2, a cross section of an open display case is depicted. The refrigerant is led into the evaporator located at the bottom of the display case, where the refrigerant evaporates while absorbing heat from the surrounding air circulating through the evaporator. The resulting air flow creates an air-curtain at the front of the display case. The fact that the air-curtain is colder than the goods leads to a heat transfer from the goods $\dot{Q}_{goods-air}$ and - as a side effect - from the surrounding $\dot{Q}_{airload}$ to the air-curtain. Inside the display cases, a temperature sensor is located, which measures the air temperature close to the goods. This temperature measurement is used in the control loop as an indirect measure of the goods' temperature. Furthermore, an on/off inlet valve is located at the refrigerant inlet of

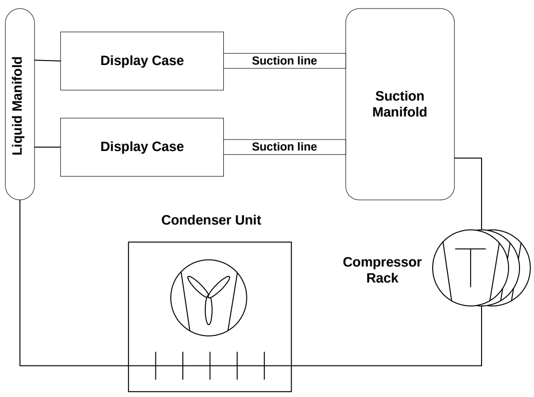

Fig. 1. A simplified layout of a supermarket refrigeration system.

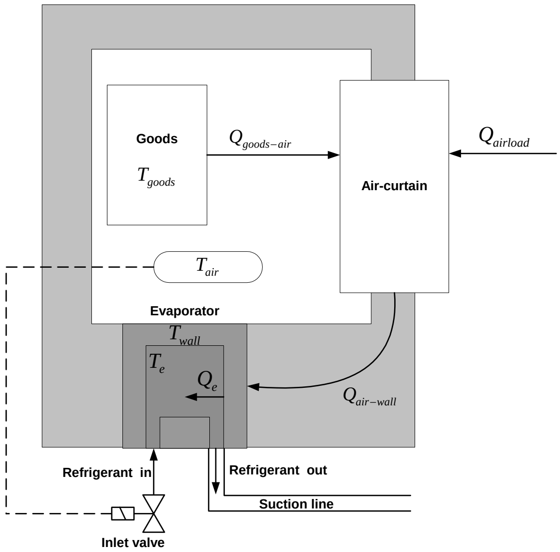

Fig. 2. Cross section of a refrigerated display case.

the evaporator, which is used to control the temperature in the display case.

3. TRADITIONAL CONTROL

The control systems used in today's supermarket refrigeration systems are decentralized as mentioned in Section 1. Each of the display cases is equipped with a temperature controller and a superheat controller that controls the filling level of the evaporator. The compressor rack is equipped with a suction pressure controller. Furthermore, the condenser is equipped with a condenser pressure controller, and on top, various kinds of supervisory controllers may be used that help adjusting the set-points. In this paper, we will only consider the display case temperature controller and the compressor controller.

The display case temperature is controlled by a hysteresis controller that opens and closes the inlet valve. This means that when the temperature T_{air} reaches a certain upper temperature bound the valve is opened and T_{air} decreases until the lower temperature bound is reached and the valve is closed again. Unfortunately, when the valve is closed, the air temperature continues to decrease below the lower bound for two reasons. Firstly, the remaining refrigerant contained in the evaporator evaporates, and secondly, the thermal capacity of the evaporator wall acts as a low-pass filter. In the supermarket many of the display cases are alike in design and they are working under uniform conditions. As a result, the inlet valves of the display cases are switched with very similar switching frequencies. Therefore, the valves have a tendency to synchronize leading to periodic high and low flow of evaporated refrigerant into the suction manifold thus creating large variations in the suction pressure.

Turning on and off the compressors in the compressor rack controls the suction pressure. To avoid excessive compressor switching, a dead band around the reference of the suction pressure is commonly used.

When the pressure exceeds the upper bound, one or more additional compressors are turned on to reduce the pressure, and vice versa when the pressure falls below the lower bound. In this way, moderate changes in the suction pressure do not initiate a compressor switching. Nevertheless, pronounced synchronization effects lead to frequent compressor switchings causing large fluctuations in the suction pressure and a high wear on the compressors.

4. MODELLING

We start by summarizing the model of the supermarket refrigeration system in continuous-time derived in (Larsen, 2004*b*). After recalling the MLD framework, we derive an MLD model of low complexity and sufficient accuracy to serve as a prediction model for MPC in the next section.

4.1 Nonlinear Continuous-time Model

The nonlinear model of the supermarket refrigeration system is composed of individual models of the display cases, the suction manifold, the compressor rack and the condensing unit. The main emphasis in the modelling is laid on the suction manifold and the display cases such that the dynamics relevant for controlling the compressor(s) and display cases are captured. The condensing unit is not considered here since it only has minor effects on the considered control.

The Display Cases

The dynamics in the display case can be described by three states, namely the goods' temperature T_{goods}, the temperature of the evaporator wall T_{wall} and the mass of liquified refrigerant in the evaporator M_{ref}. The model of the display case encompasses three parts, namely the goods, the evaporator and the air-curtain in between (see Figure 2). By setting up the energy balance for the goods and the evaporator, the following two state equations can be derived, assuming a lumped temperature model.

$$\frac{dT_{goods}}{dt} = -\frac{UA_{goods-air}}{M_{goods}\cdot Cp_{goods}}\cdot(T_{goods}-T_{air}) \quad (1)$$

$$\begin{aligned}\frac{dT_{wall}}{dt} =& \frac{UA_{air-wall}}{M_{wall}\cdot Cp_{wall}}\cdot(T_{air}-T_{wall})\\ &-\frac{UA_{wall-ref}(M_{ref})}{M_{wall}\cdot Cp_{wall}}\cdot(T_{wall}-T_e),\end{aligned} \quad (2)$$

where UA is the heat transfer coefficient with the subscript denoting the media between which the heat is transferred, M denotes the mass and Cp the heat capacity, where the subscript denotes the media the parameter refers to. Moreover T_e is the evaporation temperature.

As indicated in Eq. (2), the heat transfer coefficient between the refrigerant and the evaporator wall $UA_{wall-ref}$ is a function of the mass M_{ref} of the liquified refrigerant in the evaporator. It is assumed here that this relation can be described by the following linear function:

$$UA_{wall-ref}(M_{ref}) = UA_{wall-refrig,max}\cdot\frac{M_{ref}}{M_{ref,max}}, \quad (3)$$

where $M_{ref} = M_{ref,max}$, when the evaporator is completely filled with liquid refrigerant.

The accumulation of refrigerant in the evaporator is described by:

$$\frac{dM_{ref}}{dt} = \begin{cases} 0 & \text{if } valve = 1,\\ -\frac{\dot{Q}_e}{\Delta h_{lg}} & \text{if } valve = 0 \text{ and } M_{ref} > 0,\\ 0 & \text{if } valve = 0 \text{ and } M_{ref} = 0.\end{cases} \quad (4)$$

where Δh_{lg} is the specific latent heat of the remaining liquified refrigerant in the evaporator, which is a nonlinear function of the suction pressure. $\dot{Q}_e$ is the cooling capacity that can be found by setting up the energy balance for the evaporator:

$$\dot{Q}_e = (T_{wall}-T_e)\cdot UA_{wall-ref}(M_{ref}) \quad (5)$$

The amount of liquid refrigerant in the evaporator M_{ref} follows a switched nonlinear dynamic governed by Eq. (4). It is assumed that the evaporator is filled instantaneously when the valve is opened ($valve = 1$). Furthermore, when the valve is closed ($valve = 0$) and all of the enclosed refrigerant has evaporated ($M_{ref} = 0$), then $\dot{M}_{ref} = 0$.

Finally, the air temperature T_{air} can be found by setting up the energy balance for the air-curtain.

$$T_{air} = \dot{Q}_{airload} + T_{goods}\cdot UA_{goods-air} + T_{wall}\cdot UA_{air-wall}, \quad (6)$$

where $\dot{Q}_{airload}$ is a given external heat load on the air-curtain.

The Suction Manifold

The dynamic in the suction manifold is modelled by two states, namely the density of the refrigerant in the suction manifold ρ_{suc} and the suction pressure P_{suc}. Setting up the mass and energy balance, and assuming an ideal gas, the corresponding state equations are obtained:

$$\frac{d\rho_{suc}}{dt} = \frac{\dot{m}_{in-suc} - \dot{V}_{comp}\cdot\rho_{suc}}{V_{suc}}, \quad (7)$$

$$\frac{dP_{suc}}{dt} = R\frac{h_{in-suc}\dot{m}_{in-suc} - h_{out-suc}\dot{V}_{comp}\rho_{suc}}{C_v\cdot V_{suc}}, \quad (8)$$

where $\dot{V}_{comp}$ is the volume flow out of the suction manifold determined by the compressor(s), $\dot{m}_{in-suc} = \sum_{i=1}^{n}\dot{m}_i$ is the total mass flow from the display cases to the suction manifold, h_{in-suc} is the inlet enthalpy to the suction manifold which is a refrigerant specific nonlinear function of P_{suc} and the superheat, n is the number of display cases, and m_i is the mass flow of the refrigerant out of the i'th display case that is determined by the position of the valve and the amount of enclosed refrigerant when the valve is closed.

The Compressor

In most refrigeration systems, the compressor capacity is discrete-valued, as compressors can be switched only either on or off. Let q denote the total number of compressors. The compressor bank is modelled using a constant volumetric efficiency η_{vol} and the maximal displacement volume V_{sl}. Thus, the volume flow $\dot{V}_{comp,i}$ out of the suction manifold created by the i'th compressor can be determined as follows

$$\dot{V}_{comp,i} = comp_i \cdot \frac{1}{100} \cdot \eta_{vol} \cdot V_{sl} \qquad i = 1,..,q \quad (9)$$

where $comp_i$ is the i'th compressor capacity, and $\dot{V}_{comp} = \sum_{i=1}^{p} \dot{V}_{comp,i}$ is the total volume flow. For a more elaborate model derivation, the reader is referred to (Larsen, 2004*b*).

4.2 MLD Framework

The general MLD form of a hybrid system introduced in (Bemporad and Morari, 1999) is

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) \quad (10a)$$
$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) \quad (10b)$$
$$E_2\delta(k) + E_3z(k) \leq E_4x(k) + E_1u(k) + E_5, \quad (10c)$$

where $k \in \mathbb{N}$ is the discrete time-instant, and $x \in \mathbb{R}^{n_c} \times \{0,1\}^{n_\ell}$ denotes the states, $u \in \mathbb{R}^{m_c} \times \{0,1\}^{m_\ell}$ the inputs and $y \in \mathbb{R}^{p_c} \times \{0,1\}^{p_\ell}$ the outputs, with both continuous and binary components. Furthermore, $\delta \in \{0,1\}^{r_\ell}$ and $z \in \mathbb{R}^{r_c}$ represent binary and auxiliary continuous variables, respectively. These variables are introduced when translating propositional logic or PWA functions into mixed integer linear inequalities. All constraints on states, inputs and auxiliary variables are summarized in the inequality (10c). Note that the equations (10a) and (10b) are linear; the nonlinearity is hidden in the integrality constraints over the binary variables.

4.3 MLD Model

Before transforming the nonlinear model into MLD form, we simplify the model and disregard the emptying phenomena of the evaporator in Eq. (4), meaning that the evaporator empties instantaneously when the valve closes. Instead, the emptying dynamics are approximated by low-pass filtering T_{air}. Therefore, T_{air} is a state in the simplified model rather than M_{ref}. Moreover, the density of the refrigerant in the suction manifold described by Eq.(7) is assumed constant.

Several nonlinearities are present in the model, namely in the second term in Eq. (2), and in the Eqs. (5), (7) and (8). One might approximate these nonlinearities by piecewise affine functions to obtain an arbitrarily accurate representation of the nonlinear model. However, as the control experiments in the last section will show, linearizing the nonlinearities around the operating point yields for our control purposes a sufficiently accurate model. The resulting hybrid model is a switched linear system as detailed in (Larsen, 2004*b*). Specifically, depending on the position of the inlet valves (open or closed) and the number of compressor capacities running, different linear dynamics are active.

By subsequently discretizing the model in time an MLD formulation is obtained. Choosing a suitable sampling interval T_{samp} is difficult, as the switched dynamic in the display case exhibit greatly differing time constants. Specifically, the decrease of T_{air} when the valve is open is significantly faster than the increase when the valve is closed. To avoid long prediction horizons (in terms of steps) in MPC, a long sampling interval needs to be chosen. In general, however, this implies that the valve should be opened for *less* time than the sampling interval in order to bring the temperature down from the upper to the lower temperature bound without excessive undershoot. The necessary additional flexibility is achieved by introducing the intermediate opening time t_{open} as a continuous-valued control input that allows one to vary the opening *time* of the valve within the sampling interval. This behavior is well approximated by introducing t_{open} as a variable that varies the opening *degree* of the valve thus avoiding one additional nonlinearity.

Summing up, each display case has the three states $x_{disp} = [T_{air}, T_{goods}, T_{wall}]$, and the input $u_{disp} = [d_{valve}, t_{open}]$, where $d_{valve} \in \{0,1\}$ describes whether the valve is open or closed, and $t_{open} \in [t_{min}, T_{samp}]$ denotes the opening time of the valve. The opening time is constrained to be less than the sampling interval and larger than some minimum opening time t_{min}. The latter constraint is introduced to ensure a proper filling of the evaporator whenever the valve is opened. The control input to the compressor, the compressor capacity, is given by $comp \in \{0, \frac{100}{q}, \frac{2\cdot 100}{q}, \ldots, 100\}$.

4.4 Example Refrigeration System

In the following, we focus on a supermarket refrigeration system consisting of two display cases and one compressor with the discrete capacities $comp \in \{0, 50, 100\}$. The input vector is defined as $u = [d_{valve,1}, t_{open,1}, d_{valve,2}, t_{open,2}, \Delta comp]^T$, where $\Delta comp(k) = comp(k) - comp(k-1)$. Note that the $\Delta comp$ formulation will be needed in the next section to penalize the switching of the compressor. Thus we need to add the state-update function $comp(k) = comp(k-1) + \Delta comp(k)$ to the MLD model. The sampling interval is $T_{samp} = 60$ sec. Assuming that it takes 20 sec to ensure that the evaporator is completely filled after opening the valve, we set $t_{open} \in [20, 60]$.

The procedure in Section 4.3 yields an MLD system with 8 states (7 states from the system and one additional state for $\Delta comp(k-1)$), 2 z-variables, 4 δ-variables and 52 inequality constraints. The derivation of the MLD system is performed by the compiler HYSDEL (Torrisi and Bemporad, 2004) generating the matrices of the MLD system starting from a high-level textual description of the system.

5. OPTIMAL CONTROL PROBLEM

5.1 Control Objectives

The control objectives are to bring the suction pressure P_{suc} close to its reference value of 4.2 bar while fulfilling the soft constraints on the air temperatures in the display cases $T_{air} \in [0,4]$, and while switching the compressors as little as possible in order to minimize wear. Switch transitions in the inlet valves of the display cases are by far less critical concerning wear. Furthermore in some cases, it is even desired that the air temperature has a zigzag behavior as experience indicates that this gives a more compact icing on the evaporator improving the heat transfer between the evaporator and the surrounding air.

5.2 MPC

As introduced in (Tyler and Morari, 1999), *Model Predictive Control* (MPC) is well suited for finding control laws for hybrid systems described in the MLD framework. Here, an objective function is used that penalizes with the ∞-norm over a finite horizon the following three terms. (i) the deviation of the suction pressure P_{suc} from its reference, (ii) the switching of the manipulated variables (the compressor and valve control actions) and (iii) the violation of the soft constraints on T_{air}. The control law is then obtained by minimizing the objective function subject to the evolution of the MLD model (10) and the physical constraints on the manipulated variables. As we are using the ∞-norm, this minimization problem amounts to solving a *Mixed-Integer Linear Program* (MILP). For details concerning the set up of the MPC formulation in connection with MLD models, the reader is referred to (Bemporad and Morari, 1999) and (Bemporad *et al.*, 2000). Details about MPC can be found in (Maciejowski, 2002).

5.3 Objective Function

According to (Bemporad *et al.*, 2000) and Section 5.1, the following optimal control problem is considered:

$$\min_{u(0),..,u(N-1)} \quad J = \sum_{k=0}^{N-1} \left(\|P_{suc}(k) - 4.2\|^{\infty}_{Q_1} + \|u(k)\|^{\infty}_{Q_2} \right) + p \cdot \sum_{k=1}^{N-1} \left(\underline{S}(k) + \overline{S}(k) \right)$$

subject to the evolution of the MLD model (10) over the prediction horizon N and taking into account the discrete-valued nature of some of the manipulated variables ($d_{valve,i}$ and $\Delta comp$).

The deviation of P_{suc} from its reference is weighted by Q_1. To keep the energy consumption low, the variations on P_{suc} should be kept at a minimum. Therefore, a large weight is chosen, i.e. $Q_1 = 500$. The weight matrix on the manipulated variables is given by $Q_2 = diag(q_1,q_2,q_3,q_4,q_5)$. Recall that step changes in the compressor capacity have a magnitude of 50 ($comp \in \{0,50,100\}$). To avoid compressor switching when the suction pressure lies within a reasonable dead band of ± 0.2 bar, we set q_5 to 2. Thus, switching parts of the compressor on or off costs $2 \cdot 50 = 100$, and the deviation in P_{suc} has to amount to $\frac{100}{500} = 0.2$ bar before a change in the compressor capacity is initiated. Assuming that it is 100 times less expensive (in terms of wear) to open or close the valves than to change the compressor capacity, we set the weights on $d_{valve,1}$ and $d_{valve,2}$ to $q_1 = q_3 = \frac{100}{100} = 1$. The weights on t_{open} are set to $q_2 = q_4 = 0.1$.

The soft constraints on the temperature bounds are taken into account by introducing slack variables for the upper $\overline{S}$ and the lower $\underline{S}$ bounds and a large penalty, e.g. $p = 10^4$.

6. SIMULATION RESULTS

This section presents control experiments simulated on the nonlinear model described in Section 4.1. To illustrate the performance improvements that can be achieved by using an MPC scheme, the control performance resulting from a traditional controller (as described in Section 3) is compared with the MPC scheme. To illustrate the problems that often arise with the traditional control scheme, we have chosen a refrigeration system with two equally sized display cases that has a pronounced tendency for synchronization.

The traditional controller comprises a hysteresis based temperature controller with $[T_{lower}, T_{upper}] = [0,4]$ and a PI-type suction pressure controller with a dead band of ± 0.2 bar around the reference of 4.2 bar. MPC uses a horizon of $N = 10$.

The lower part of Figure 3 depicts the air temperatures in the two display cases when using the traditional control scheme. Initially, the two temperatures exhibit an offset which vanishes within the first hour due to the control actions. In other words, the valves of the display cases get synchronized leading to large variations in the suction pressure as shown in the lower part of Figure 4. As depicted in Figure 5, the compressor controller tries in vain to suppress these variations when they exceed the dead band of ± 0.2 bar causing excessive switching and wear. Unless something is done to de-synchronize the valves, they will remain synchronized.

The upper part of Figure 3 shows the temperatures in the display cases when employing MPC. The two temperatures coincide in the beginning and the valves are thus synchronized. However, after only 20 min, the switching of the valves is de-synchronized, resulting in smaller variations in the suction pressure as can be seen in the upper part of Figure 4. These reduced variations not only result from the de-synchronization, but also from the significantly reduced undershoot in the air temperatures [1]. The traditional controller fails to respect the lower bound as it cannot predict the undershoot in contrast to MPC. Finally, as can be seen in Figure 5, the large penalty on the compressor switching reduces the number of switch transitions considerably with respect to the traditional control scheme.

To solve the optimal control problem online at each time step, CPLEX 9.0 run on a Pentium IV 2.0 GHz computer. For $T_{samp} = 1$ min and a horizon of $N = 10$, the computation time is in average 3.9 sec and always

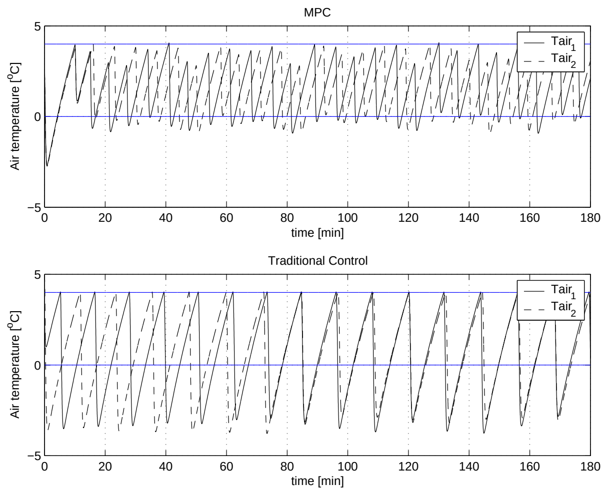

Fig. 3. T_{air} in the 2 display cases, controlled respectively with MPC and a traditional hyst. controller.

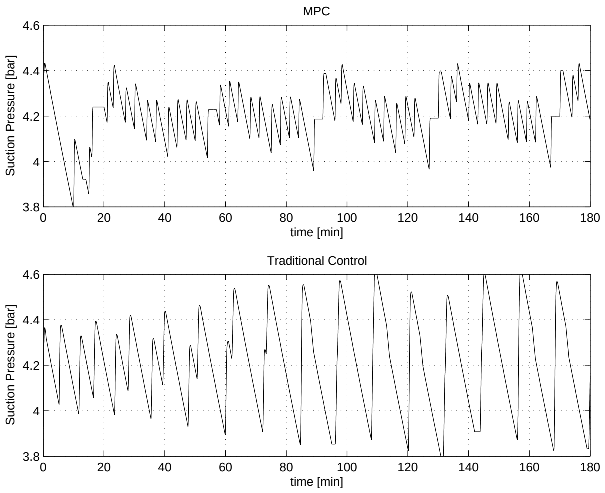

Fig. 4. P_{suc} in the suction manifold, controlled respectively with MPC and a traditional PI controller.

less than 9.7 sec rendering an online implementation of the MPC scheme computationally feasible.

7. CONCLUSIONS

The nonlinear system described in Section 4.1 was cast into the MLD form by approximating the switched nonlinear dynamics by switched affine dynamics, introducing integer manipulated variables to describe the discrete-valued control actions, and discretizing the model in time. The result is a hybrid model of low complexity that is sufficiently accurate to serve as a prediction model for MPC and which is computationally tractable.

The case study of a supermarket refrigeration system illustrated the performance limitations of traditional control schemes. MPC in connection with the MLD

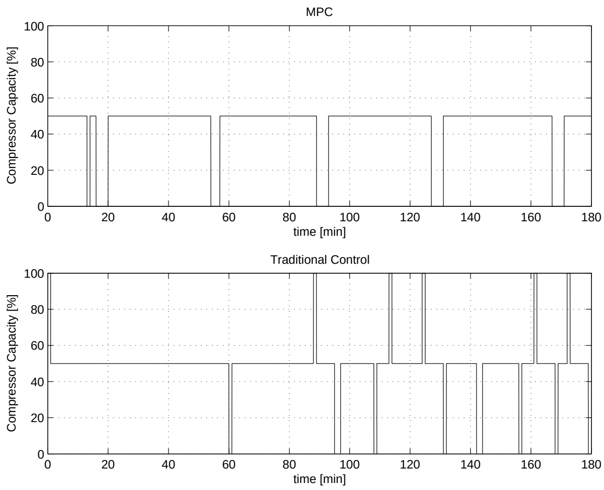

Fig. 5. The compressor control input (*comp*) using respectively MPC and a traditional PI controller.

model proved to be better suited for handling the discrete control actions, taking into account the interactions between the display cases and the compressor, respecting the temperature constraints, minimizing the variations in the suction pressure, and reducing the switching in the compressor bank. This led to a lower wear in the compressor and a higher energy efficiency of the supermarket refrigeration system. Most important, the design and tuning of the cost function was easy and intuitive, and the extension of the case study to larger refrigeration systems is expected to be straightforward. Moreover, if needed, the explicit piecewise affine state feedback controller might be derived by pre-solving the optimal control problem offline for all states.

[1] The small undershoot in T_{air} for MPC, despite the soft constraints, is mainly caused by the coarse sampling.

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