ITERATIVE LEARNING CONTROL OF NONHOLONOMIC HAMILTONIAN SYSTEMS: APPLICATION TO A VEHICLE SYSTEM

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Abstract: This paper is concerned with iterative learning control of Hamiltonian systems with nonholonomic constraints. The author has proposed a novel iterative learning control method based on the symmetric property of Hamiltonian control systems. This paper shows its application to a four-wheeled vehicle for which we need to employ an approximation of the control law. A numerical example demonstrates the effectiveness of the proposed method.

Keywords: iterative learning control, nonholonomic systems, vehicle systems, nonlinear control.

1. INTRODUCTION

In research area on control of physical systems, most of the existing results focus on feedback stabilization and related topics such as trajectory tracking control, output feedback control and so on. On the other hand, it is also an important task to generate an appropriate feedforward input for a designed control system. Usually such a problem is formulated by an optimal control problem, see e.g. (Young, 1969), and solved numerically by optimization based iteration algorithm. This algorithm requires the precise information of the plant system to be controlled and it is sensitive to unknown modeling errors. Since some of the feedback control methods stated above derive control systems which are robustly stable against unknown modeling errors (parameter variations), a robust feedforward input generation algorithm is needed.

Iterative learning control method proposed in (Arimoto *et al.*, 1984) is an algorithm to generate a feedforward input achieving a trajectory tracking control (on a finite time interval) without using the precise information of the system. See also, e.g. (Sugie and Ono, 1991; Kurek and Zaremba, 1993; Xu *et al.*, 1999; Hamamoto and Sugie, 1999; Ghosh and Paden, 2000). Since this algorithm does not require the precise model of the plant system, it is robust against modeling errors. So far, however, this algorithm was only applicable to trajectory tracking control problems.

Recently, the authors proposed a novel iterative learning control method based on the symmetric properties of Hamiltonian systems (Fujimoto and Sugie, 2002*b*; Fujimoto and Sugie, 2002*a*; Fujimoto *et al.*, 2002). In this result, it is proved that the variational systems of Hamiltonian systems are symmetric and this property can be utilized for executing the iterative algorithm for optimal control problems. Furthermore, this procedure is generalized to *non-canonical* Hamiltonian systems, that is, Hamiltonian systems with *nonholonomic* velocity constraints. But this procedure requires a restrictive assumption and we need to employ an approximation to apply this method to systems in the real world.

The present paper evaluates the effectiveness of our preliminary result by employing an appropriate approximation. The proposed method is applied to a four-wheeled vehicle system which is a typical example of real world nonholonomic systems. Moreover, some simulation of the wheeled vehicle demonstrate the advantage of the proposed method.

2. PRELIMINARIES

2.1 Optimal control of Hamiltonian systems via iterative learning

This section briefly refers to some preliminary backgrounds.

Symmetric properties Our target system is a Hamiltonian system with dissipation Σ with a controlled Hamiltonian $H(x, u, t)$ as $(x^1, y) = \Sigma(x^0, u)$:

$$\begin{cases} \dot{x} &= (J - R)\,\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}},\ x(t^0) = x^0 \\ y &= -\dfrac{\partial H(x,u,t)}{\partial u}^{\mathrm{T}} \\ x^1 &= x(t^1) \end{cases} . \quad (1)$$

Here the structure matrix $J \in \mathbb{R}^{n\times n}$ and the dissipation matrix $R \in \mathbb{R}^{n\times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The matrix R represents dissipative elements such as friction of mechanical systems and resistance of electric circuits. For this system, the following theorem holds.

Theorem 1. (Fujimoto and Sugie, 2003) Consider the Hamiltonian system with dissipation and the controlled Hamiltonian Σ in (1). Suppose that J and R are constant and that there exists a nonsingular matrix $T \in \mathbb{R}^{n\times n}$ satisfying

$$J = -T J\, T^{-1} \qquad R = T R\, T^{-1}$$
$$\frac{\partial^2 H(x,u,t)}{\partial (x,u)^2} = \begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix} \frac{\partial^2 H(x,u,t)}{\partial (x,u)^2} \begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix}^{-1} . \quad (2)$$

Then the Fréchet derivative of Σ is described by another linear Hamiltonian system $(x_v^1, y_v) = \mathrm{d}\Sigma((x^0, u), (x_v^0, u_v))$:

$$\begin{cases} \dot{x} = (J - R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, & x(t^0) = x^0 \\ \dot{x}_v = (J - R)\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial x_v}^{\mathrm{T}}, & x_v(t^0) = x_v^0 \\ y_v = -\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial u_v}^{\mathrm{T}} \\ x_v^1 = x_v(t^1) \end{cases}$$

with a controlled Hamiltonian $H_v(x, u, x_v, u_v, t)$

$$H_v(x,u,x_v,u_v,t) = \frac{1}{2}\begin{pmatrix} x_v \\ u_v \end{pmatrix}^{\mathrm{T}} \frac{\partial^2 H(x,u,t)}{\partial (x,u)^2} \begin{pmatrix} x_v \\ u_v \end{pmatrix} .$$

Furthermore, the adjoint of the variational system with zero initial state $u_a \mapsto y_a = (\mathrm{d}\Sigma^{x^0}(u))^*(u_a)$ is given by

$$\begin{cases} \dot{x} &= (J - R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}} \\ \dot{x}_v &= -(J - R)\dfrac{\partial H_v(x,u,x_v,u_a,t)}{\partial x_v}^{\mathrm{T}} \\ y_a &= -\dfrac{\partial H_v(x,u,x_v,u_a,t)}{\partial u_a}^{\mathrm{T}} \end{cases}$$

with the terminal states $x(t^0) = x^0$ and $x_v(t^1) = 0$. Suppose moreover that $J - R$ is nonsingular. Then the adjoint $(x_a^1, u_a) \mapsto (x_a^0, y_a) = (\mathrm{d}\Sigma(x^0, u))^*(x_a^1, u_a)$ is given by the same state-space realization (3) with the terminal states $x(t^0) = x^0$, $x_v(t^1) = -(J - R)T\ x_a^1$ and $x_a^0 = -T^{-1}(J - R)^{-1}x_v(t^0)$.

This theorem reveals that the variational system and its adjoint of a Hamiltonian system in the form (1) have almost the same state-space realizations. This means that the input-output mapping of the adjoint can be produced by the input-output data of the original Hamiltonian system as

$$\mathcal{R}\circ(\mathrm{d}\Sigma(u))^*\circ\mathcal{R}(v) = \mathrm{d}\Sigma(\bar{u})(v) \approx \Sigma(\bar{u}+v) - \Sigma(\bar{u}) \quad (3)$$

provided appropriate boundary conditions are selected, where $\mathcal{R}$ is the time reversal operator defined by

$$\mathcal{R}(u)(t - t^0) = u(t^1 - t), \quad \forall t \in [t^0, t^1]. \quad (4)$$

This property is utilized for solving the optimal control problems in which the adjoint operator plays an important role.

Remark 2. It is noted that if the system is a *gradient system* (Crouch, 1981) which is a nonlinear generalization of a linear symmetric system, that is, $J = 0$, then the assumption (2) in Theorem 1 is automatically satisfied with $T = I$. On the other hand, if the system is conservative, that is, $R = 0$ then it is self-adjoint in the usual sense (Fujimoto and Sugie, 2003).

Optimal control via iterative learning Let us consider the system $\Sigma : U \to Y$ in (1) and a cost function $\Gamma : X^2 \times U \times Y \to \mathbb{R}$. The objective is to find the optimal input $(x^0_\star, u_\star)$ minimizing the cost function $\Gamma(x^0, u, x^1, y)$. In general, however, it is difficult to obtain a global minimum since the cost function Γ is not convex. Hence we try to obtain a local minimum here. Here we can calculate

$$\begin{aligned}
&\mathrm{d}\left(\Gamma((x^0, u), \Sigma(x^0, u))\right)(\mathrm{d}x^0, \mathrm{d}u) \\
&= \mathrm{d}\Gamma((x^0, u), \Sigma(x^0, u))\left((\mathrm{d}x^0, \mathrm{d}u), \mathrm{d}\Sigma(x^0, u)(\mathrm{d}x^0, \mathrm{d}u)\right) \\
&= \langle \Gamma'((x^0, u), \Sigma(x^0, u)), \begin{pmatrix} \mathrm{id}_{X\times U} \\ \mathrm{d}\Sigma(x^0, u) \end{pmatrix} (\mathrm{d}x^0, \mathrm{d}u)\rangle_{X^2\times U\times Y} \\
&= \langle \left(\mathrm{id}_{X\times U}, (\mathrm{d}\Sigma(x^0, u))^*\right) \Gamma'(x^0, u, x^1, y), (\mathrm{d}x^0, \mathrm{d}u)\rangle_{X\times U}.
\end{aligned}$$

Therefore, if the adjoint $(\mathrm{d}\Sigma(x^0, u))^*$ is available, we can reduce the cost function Γ down at least to a local minimum by an iteration law with a $K_{(i)} > 0$.

$$\begin{aligned}
&u_{(i+1)} \\
&= u_{(i)} - K_{(i)}\,(0_{UX}, \mathrm{id}_U)\left(\mathrm{id}_{X\times U},\ (\mathrm{d}\Sigma(x^0_{(i)}, u_{(i)}))^*\right) \\
&\times \Gamma'(x^0_{(i)}, u_{(i)}, x^1_{(i)}, y_{(i)}) \qquad (5)
\end{aligned}$$

The results in the previous section enable one to execute this procedure without using the parameters of the original operator Σ by the relation (3), provided Σ is a Hamiltonian system and the boundary conditions are selected appropriately. In (Fujimoto and Sugie, 2003), this framework is effectively utilized for iterative learning control (of trajectory tracking) for a 'round trip' type trajectory. More precise discussion for optimal control will be made in the following sections.

2.2 Application of iterative learning control into Nonholonomic systems

The result obtained above is quite useful for feedforward optimal control of mechanical systems with unknown modeling errors. In this section, we will mention about a problem upon applying the former method of iterative learning control into Nonholonomical systems and a simple transformation method to overcome it.

Problem to solve It is proved in (Maschke and van der Schaft, 1994) that simple Hamiltonian systems with nonholonomic constraints which is linear in the velocity of the configuration states are represented by the following form

$$\begin{pmatrix} \dot q \\ \dot p \end{pmatrix} = \begin{pmatrix} 0 & J_{12}(q) \\ -J_{12}(q)^{\mathrm{T}} & J_{22}(q,p) \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial q}^{\mathrm{T}} \\ \frac{\partial H}{\partial p}^{\mathrm{T}} \end{pmatrix} + \begin{pmatrix} 0 \\ G(q) \end{pmatrix} u \qquad (6)$$

with a Hamiltonian

$$H(q,p) = \frac{1}{2} p^{\mathrm{T}} M(q)^{-1} p. \qquad (7)$$

This system is called a port-controlled Hamiltonian system. Here $q(t) \in \mathbb{R}^n$ and $p(t), u(t) \in \mathbb{R}^m$ with $n > m$. Note that the structure matrix J depends on the state $x = (q, p)$ and is not constant. From this notice, it is obvious that the proposed method in section 2.1 is *not* applicable to Hamiltonian systems with nonholonomic velocity constraints whose structure matrix depends on the state, where the former method requires the assumption that the structure and dissipation matrices J and R are constant.

Application to nonholonomic Hamiltonian systems To apply the iterative learning method, proposed in section 2.1, to nonholonomic Hamiltonian system, we have to convert the system (6) into the form (1). In order to do this, we will use the following result which is a variation of the result in (Fujimoto and Sugie, 1999).

Lemma 3. (Fujimoto and Sugie, 1999) If a driftless system

$$\dot q = J_{12}(q) v \qquad (8)$$

can be transformed into a form

$$\dot{\bar q} = \bar J_{12}(\bar q) \bar v \qquad (9)$$

by a set of feedback and coordinate transformations

$$v = N(q)\bar v, \quad \bar q = \Psi(q), \qquad (10)$$

then a coordinate transformation for the port-controlled Hamiltonian system

$$\bar x = \begin{pmatrix} \bar q \\ \bar p \end{pmatrix} = \Phi(x) := \begin{pmatrix} \Psi(q) \\ N(q)^{\mathrm{T}} M(q)^{-1} p \end{pmatrix}$$

converts the corresponding port-controlled Hamiltonian system (6) into another one whose structure matrix has a form

$$\bar J(\bar q, \bar p) = \begin{pmatrix} 0 & \bar J_{12}(\bar q) \\ -\bar J_{12}(\bar q)^{\mathrm{T}} & \bar J_{22}(\bar q, \bar p) \end{pmatrix}.$$

When the system is controllable, it is easy to find out from (8) to (10) that if the transformation matrices $\Psi(q), N(q)$ satisfies

$$\frac{\partial \Psi(q)}{\partial q} J_{12}(q) N(q) = \begin{pmatrix} I \\ \bar J_{12}^2 \end{pmatrix}$$

it is always possible to find a transformation (10) such that the corresponding matrix $\bar J_{12}(\bar q)$ has a form

$$\bar J_{12}(\bar q) = \begin{pmatrix} I \\ \bar J_{12}^2(\bar q) \end{pmatrix}$$

with a matrix $\bar{J}_{12}^2(\bar{q}) \in \mathbb{R}^{m\times(n-m)}$, for any port-controlled Hamiltonian system (6).

Since $\bar{J}_{12}^2(\bar{q})$ is not constant, the proposed iterative learning method is not applicable. So from now on we consider a division state of the $\bar{q}$ as

$$\bar{q} = (\bar{q}^1, \bar{q}^2), \quad \bar{q}^1 \in \mathbb{R}^m,\ \bar{q}^2 \in \mathbb{R}^{n-m}.$$

Finally, by applying a feedback

$$\begin{aligned}\bar{u} &= G(q)u + \gamma(\bar{q}, \bar{p}) \qquad (11)\\ &:= G(q)u - \bar{J}_{12}^2(\bar{q},\bar{p})^{\mathrm{T}} \frac{\partial H}{\partial \bar{q}}^{\mathrm{T}} + \bar{J}_{22}(\bar{q},\bar{p}) \frac{\partial H}{\partial \bar{p}}^{\mathrm{T}}\end{aligned}$$

the dynamics of the partial state $(\bar{q}^1, \bar{p})$ can be described by a Hamiltonian control system

$$\begin{cases} \begin{pmatrix} \dot{\bar{q}}^1 \\ \dot{\bar{p}} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial \bar{H}}{\partial \bar{q}^1}^{\mathrm{T}} \\ \frac{\partial \bar{H}}{\partial \bar{p}}^{\mathrm{T}} \end{pmatrix} \\ y = -\dfrac{\partial \bar{H}}{\partial \bar{u}}^{\mathrm{T}} = \bar{q}^1 \end{cases} \qquad (12)$$

with a Hamiltonian

$$\bar{H}(\bar{x}, \bar{u}) = H(\Phi^{-1}(\bar{x})) - \bar{q}^1 \bar{u}^{\mathrm{T}}.$$

By using the *generalized canonical transformation*[1] and converting (6) into a system described as (12) that has a constant structure matrix, we are able to apply the iterative learning method to nonholonomic Hamiltonian system.

Ignoring the feedback As for iterative learning control, it is not proper to use an particular preliminary information of the system. From this point of view, the feedback γ applied in (11) is not always appropriate. But the feedback in (11) has the following property.

Proposition 4. (Fujimoto, 2004) The function γ in (11) satisfies

$$\gamma(\bar{q}, \bar{p}) = o(\|\bar{p}\|).$$

This proposition implies that if $\bar{p}$ is small enough, that is, the velocity (momentum) is sufficiently small, then the feedback in (11) can be approximated by just

$$\bar{u} = G(q)u. \qquad (13)$$

Therefore, what we need for iterative learning control of the port-controlled Hamiltonian system (6) is the information on $G(q)$ and $\Psi(q)$ in (10), if the prescribed desired trajectory y^d moves sufficiently slow.

[1] A generalized canonical transformation is a set of coordinate and feedback transformations preserving the Hamiltonian structure (Fujimoto and Sugie, 2001).

3. APPLICATION TO A VEHICLE SYSTEM

In this section, the proposed method is applied to a four-wheeled vehicle system which is a typical and practical example of port-controlled Hamiltonian systems with nonholonomic constraints.

3.1 Modeling

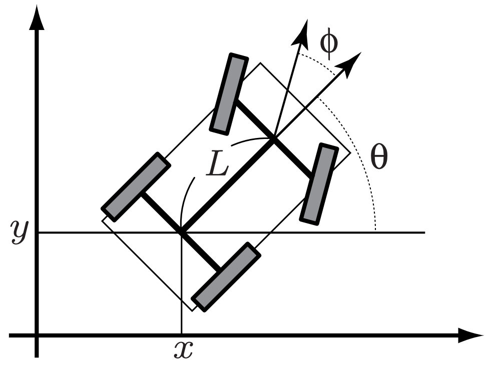


Fig. 1. A four-wheeled vehicle model

Let us consider a four-wheeled vehicle as depicted in Figure 1. Suppose this vehicle as a driftless system, then it's nonholonomic constraints are described by

$$\begin{aligned}&\dot{x}\sin\theta - \dot{y}\cos\theta = 0\\ &(\dot{x}\cos\theta + \dot{y}\sin\theta)\tan\phi = L\dot{\theta}\end{aligned}.$$

The first equation describes the constraints of the rear wheels and the second equation describes those of the front ones.

Using this equation, the dynamics of this system is

$$M\ddot{q} = A(q)\lambda + B(q)u$$

where $A(q)$, $B(q)$, M, λ, u, q are given by

$$A = \begin{pmatrix} \tan\phi\cos\theta & -\cos\theta \\ \tan\phi\cos\theta & \sin\theta \\ -L & 0 \\ 0 & 0 \end{pmatrix},\ B = \begin{pmatrix} \sin\theta & 0 \\ \cos\theta & 0 \\ \frac{1}{L}\tan\phi & 0 \\ 0 & 1 \end{pmatrix}.$$

$$M = \mathrm{diag}(m, m, J_\theta, J_\phi), \quad \lambda = (\tau, f)^T$$
$$u = (u_r, u_f)^T, \quad q = (x, y, \theta, \phi)^T$$

The parameters are defined as follows: u_r denotes the input thrust force of the rear wheels, u_f denotes the steering torque for the front wheels, m denotes the weight of the vehicle itself, J_θ denotes the inertia of the vehicle, J_ϕ denotes the inertia of the front wheels, τ and f denotes the constraint torque and the constraint force of the vehicle.

3.2 Settings for iterative learning control

As shown in (6), this system can be described by a port-controlled Hamiltonian system with

$$J_{12} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \frac{1}{L}\tan\phi & 0 \\ 0 & 1 \end{pmatrix},\ J_{22} = \begin{pmatrix} 0 & \frac{\dot{\theta}}{\cos^2\theta} \\ \frac{-\dot{\theta}}{\cos^2\theta} & 0 \end{pmatrix}.$$
$$G = \begin{pmatrix} 1 + \frac{1}{L^2}\tan^2\theta & 0 \\ 0 & 1 \end{pmatrix}$$

Here the transformation in (10) can be chosen as

$$\Psi(q) = \begin{pmatrix} x \\ \frac{\sec^3\theta\tan\phi}{L} \\ \tan\theta \\ y \end{pmatrix}$$
$$N(q) = \begin{pmatrix} \frac{1}{\cos\theta} & 0 \\ -\frac{3\tan\theta\tan^2\phi}{L\cos\theta\sec^2\phi} & \frac{1}{\sec^3\theta\sec^2\phi} \end{pmatrix}. \tag{14}$$

The corresponding transformation converts the system into another port-controlled Hamiltonian system with

$$\bar{J}_{12}(\bar{q}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\tan\phi}{\cos^3\theta} & 0 \\ \tan\theta & 0 \end{pmatrix}.$$

That is, this transformation can be used for iterative learning control for nonholonomic system by the procedure given in the previous section. For the result of the transformation which is described as in (12), the states needed to be measured in iterative learning are

$$\bar{q}^1 = \begin{pmatrix} x \\ \frac{\sec^3\theta\tan\phi}{L} \end{pmatrix}.$$

To stabilize the plant asymptotically, we employ a PD pre-feedback which preserves the Hamiltonian structure (Fujimoto and Sugie, 2003)

$$\tilde{u} = \bar{u} + K_{\mathrm{P}}\bar{q}^1 + K_{\mathrm{D}}\dot{\bar{q}}^1.$$

Note that the preliminary information used for the setting of the vehicle system for iterative learning control is only L, which denotes the distance between the front wheel axis and the rear one. Different from other parameters, this information is needed to recognize the nonholonomic constraints and to plan the desired trajectory, so we think this information as a critical one and use it in learning control.

3.3 Simulation

Since this simulation is about four-wheeled vehicle, desired trajectory is given by it's position (x, y). For the flatness of the system, other states of the system are calculated from the given desired trajectory as

$$\begin{aligned} \theta &= \mathrm{atan2}(\dot{y}, \dot{x}) \\ \phi &= \pm\mathrm{atan2}(L(\dot{x}\ddot{y} - \ddot{x}\dot{y}), (\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}) \end{aligned}. \tag{15}$$

The desired output signal $\bar{q}^{1d}$ is calculated from the desired trajectory using (15).

Take the cost function $\Gamma = \|\bar{q}^1 - \bar{q}^{1d}\|^2_{L_2}$ with the desired trajectory given by

$$\begin{aligned} x &= \pi\cos(\frac{\pi}{T}t) \\ y &= -\frac{\pi}{12}\cos\left(\pi\cos(\frac{\pi}{T}t) + \frac{3}{2}\pi\right) \end{aligned}$$

where T is the simulation time. The physical parameters like the weight of the vehicle and the inertia are all set to 1, including L. The design parameters for iteration are

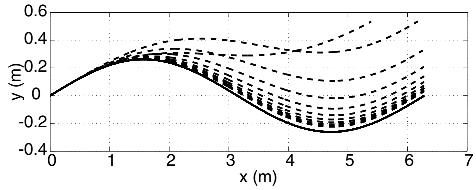
$$\begin{aligned} K_{\mathrm{P}} = \mathrm{diag}(100, 100),\quad K_{\mathrm{D}} = \mathrm{diag}(100, 100) \\ K_{(i)} = \mathrm{diag}(12000, 12000) \end{aligned}.$$

Simulation results are shown in Figure 2 ($T = 5$[s]) and Figure 3 ($T = 10$[s]). Only the parameters of the simulation time T are different between Figure 2 and Figure 3, other physical and design parameters are same. Both Figure 2(a) and Figure 3(a) depict the responses on the X-Y plane. In the figures, the solid line denotes the desired trajectory and the dashed ones are the intermediate responses in the learning procedure. Figure 2(b) and Figure 3(b) depict the transition of the cost function.

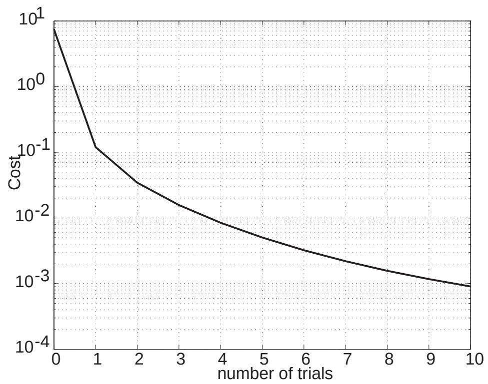
The figures show that the proposed iterative learning method works well also for nonholonomic Hamiltonian systems. Comparing the difference between Figure 2 and Figure 3, in particular, with respect to the tracking error on the X-Y plane, the method is much more effective in the latter case than that in the former, i.e., the case with the slower trajectory works better. This means that the conclusion given in the previous section, which is implied by Proposition 1 as in Equation (13), is true for this system.

4. CONCLUSION

In this paper, we have shown that our iterative learning method for port-controlled Hamiltonian systems with nonholonomic constraints works well with a wheeled vehicle example. In this method, we need to employ an approximation to derive an iterative learning control system, due to the non-canonical Hamiltonian structure. In fact this method works well with a wheeled vehicle system which was evaluated with some numerical simulations.

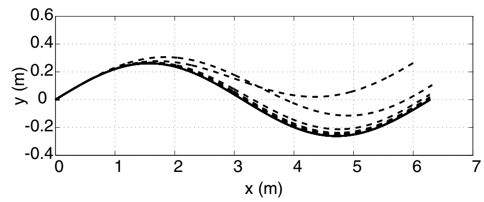


(a) Response in the X-Y plane

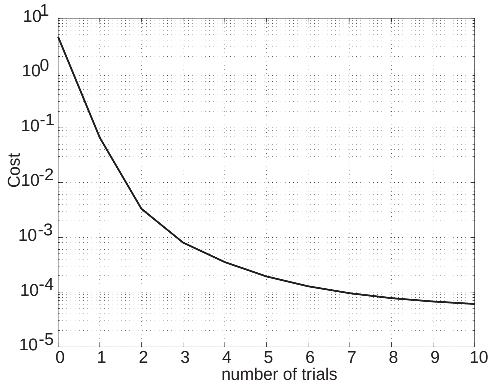


(b) Cost function

Fig. 2. 5 second simulation



(a) Response in the X-Y plane



(b) Cost function

Fig. 3. 10 second simulation

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