MULTI–PARAMETRIC H_∞ CONTROL OF A MICRO–ACTUATOR

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Abstract: In this article the control problem of a micro-actuator ($\mu - A$) is considered. The $\mu - A$ is composed of a micro-capacitor, whose one plate is clamped while its other flexible plate's motion is constrained by hinges acting as a combination of springs and dashpots. The distance of the plates is varied by the applied voltage between them. The flexibility of the moving plate coupled to the dynamics of the plate's rigid-body motion results in an unstable, nonlinear system of distributed nature. Utilization of FEM can approximate the $\mu - A$ dynamics nonlinear-PDE to a finite nonlinear-ODE. The nonlinearity stems from the plate's rigid-body motion, while all flexibility effects are considered as additive linear-terms. A controller composed of: a) a feedforward term for regulation at selected setpoints, and b) a constrained finite time optimal controller to handle any deviations from the equilibrium is synthesized. Simulation studies are used to investigate the efficacy of the suggested controller.

Keywords: MEMs, Micro–Actuator, Optimal Control.

1. INTRODUCTION

Micro-capacitor structures (Sitti, 2001; A. Menciassi and A. Eisinberg and I. Izzo and P. Dario, 2004; Ishihara *et al.*, 1996) have been used as the primitive components of single-degree of freedom linear μ-Actuators. These actuators could be utilized for positioning, orienting or applying a force (in the range of picoNewtons) in various applications (Lee *et al.*, 2003; Liu *et al.*, 2003; Zhang *et al.*, 2003). Linearity of the actuator's model is a desired key feature as it enables the usage of classical controllers. However, most nano–positioners are nonlinear, and advanced closed loop controllers are necessary.

Due to the diminution of these μ-As, there is a need to properly devise advanced control techniques for satisfying certain performance criteria (Lyshevski, 1998). These techniques primarily stem from the modelling peculiarities of these micro–actuators, including the effects that tend to be ignored in the macro-world and yet are important in the micro-domain.

Rather than relying on the design of nonlinear controllers, which require a large computational burden for their impplementation, there is a trend to utilize linear optimal controllers (Robl *et al.*, 1999) computed in an offline manner. Moreover optimal control of PieceWise Affine (PWA) systems have also received great interest in the

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[2] This work was partially funded by the European Social Fund (ESF), Operational Program for Educational and Vocational Training II (EPEAEK II) Program HERAKLEITOS, No. B238.012.

research community, since PWA systems represent a powerful tool for approximating non-linear systems (Bemporad *et al.*, 2002; Grieder *et al.*, 2004). The algorithms for computing the feedback controllers for constrained PWA systems were presented for quadratic as well as linear cost functions of finite time (Borelli *et al.*, 2003).

Even though the multi-parametric approaches rely on an off-line computation of a feedback law, the computation can quickly become prohibitive for larger problems. This is not only due to the high complexity of the multi-parametric programs involved, but mainly because of the exponential number of transitions between regions which can occur when a controller is computed in a dynamic programming fashion (Borelli *et al.*, 2003).

In this article, a multi-parametric controller in conjunction with a feedforward controller is applied in simulation studies in the positioning problem of a microactuator.

2. MICRO–ACTUATOR MODELLING

The μ-A from a dynamics point of view corresponds to a micro-capacitor whose one plate is attached to the ground while its other moving plate is floating in air. The boundary of the moving plate is either supported (pinned) or constrained by hinges (springs), as shown in Figure 1.

2.1 Dynamic Plate Model

The equation of motion for a 2-D distributed thin plate (Hong *et al.*, 1998) floating on air and supported at its boundary is expressed as follows

$$\mathcal{L}w(x,y,t)+\mathcal{C}\dot{w}(x,y,t)+m_p\ddot{w}(x,y,t)=f(x,y,t),$$

where $\mathcal{L}$ is a time-invariant, symmetric, non-negative differential operator, $\mathcal{C}$ is a damping operator, m_p is the mass density of the 2-D structure, $f(x,y,t)$ is the time-varying distributed control force acting on the thin plate at the (x,y)–coordinate, and the structural proportional damping is $C=\alpha_1+\alpha_2 m_p$.

In thin plate theory the operator $\mathcal{L}w(x,y,t)$ is

$$\frac{Eh^3}{12(1-\nu^2)}\left(w_{xxxx}+2w_{xxyy}+w_{yyyy}\right),$$

where E is the Young's modulus, ν is the Poisson's ration of the plate material, h is the thickness of the plate, and the symbol w_{xy} corresponds to $\frac{\partial}{\partial x}\frac{\partial}{\partial y}w(x,y,t)$.

For a thin square plate of length ℓ, shown in Figure 1, the equations of motion along with its boundary conditions are:

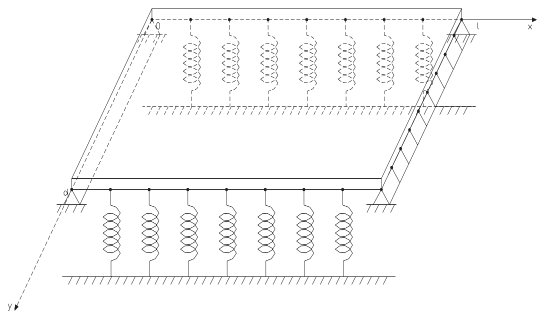

Fig. 1. μ-Actuator Thin Supporting Plate

$$\begin{aligned}
&w_{tt}+D_1(w_{xxxx}+2w_{xxyy}+w_{yyyy})=0,\\
&\qquad\qquad 0<x,y<\ell, t>0\\
&w(x,y,0)=w_0(x,y),\quad w_t(x,y,0)=w_1(x,y),\\
&\qquad\qquad 0<x,y<\ell,\\
&w(0,y,t)=w(\ell,y,t)=w_{xx}(0,y,t)=w_{xx}(\ell,y,t)=0,\\
&\qquad\qquad 0<y<\ell\\
&D(w_{yyy}+(2-\nu)w_{xxy})=-k^2w,\ y=0,0<x<\ell\\
&D(w_{yyy}+(2-\nu)w_{xxy})=k^2w,\ y=\ell,0<x<\ell\\
&w_{yy}+\nu w_{xx}=0,\ y=0,y=\ell,0<x<\ell
\end{aligned}$$

where $D_1=\frac{D}{\rho}=\frac{Eh^3}{12m_p(1-\nu^2)}$, $w_0(x,y)$ $(w_1(x,y))$ is the initial displacement (velocity) of the plate in z-direction, and k represents the linear restoring force of the springs.

Application of the assumed modes method dictates that the displacement and point control force can be expressed as

$$w(x,y,t)=\sum_{i=1}^{\infty}W_i(x,y)\eta_i(t)$$
$$f(x,y,t)=\sum_{i=1}^{p}F_i(t)\delta(x-x_i)\delta(y-y_i)$$

where $\eta_n(t)$ is the nth mode modal displacement, $F_i(t)$ is the force amplitude, p is the number of actuators, $\delta(x-x_i)$ and $\delta(y-y_i)$ are spatial Dirac delta functions.

For the given stated boundary conditions, closed form solutions can be found (Zarubinskaya and Horssen, 2003) for the free-response expressions $w(x,y,t)$. Retaining a finite number of modes the ordinary differential equation describing the motion for the nth mode is

$$\ddot{\eta}_n+\left(\alpha_1\omega_n^2+\alpha_2\right)\dot{\eta}_n+\omega_n^2\eta_n=\sum_{i=1}^{p}W_n^*(x_i,y_i)F_i.$$

When: 1) the forcing element $f(x,y,t)=f(t)$ is independent of the point of application, 2) there is no proportional damping ($\alpha_1=0$), and 3) retaining only one mode ($n=1$) the equation of motion degenerates to

$$\ddot{\eta}_1+\alpha_2\dot{\eta}_1+\omega_1^2\eta_1=W_1^*F. \tag{1}$$

In this case, the displacement of the plate $z(t) = w(x, y, t)$ is identical for all points (x, y) of the plate and equal to $\eta_1(t)$. Multiplication of both sides of (1) by $W_1^* = m$ yields the following equation of motion $m\ddot{z} + b\dot{z} + kz = F$, where m is the total mass of the plate, and k is the overall stiffness of the springs, as shown in Figure 2.

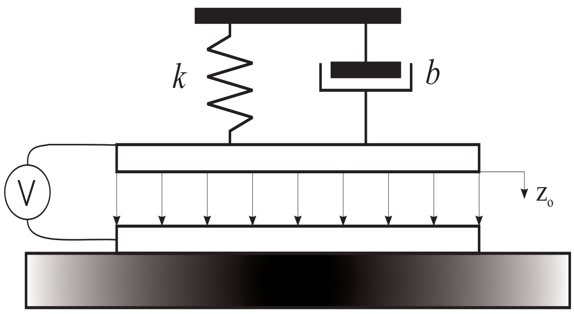

Fig. 2. Rigid body dynamical modelling

2.2 *Electric Force Model*

Application of a voltage U between the capacitor's plates generates an electrically–induced force

$$F_x = \frac{\varepsilon A U^2}{2s},$$

where A is the area of the plates, ϵ is the dielectricic constant and s is the distance between the plates when the spring is relaxed.

2.3 *μ-Actuator Model Linearization*

The nonlinear equation of motion is

$$m\ddot{z} + b\dot{z} + kz = \frac{aU^2}{(s-z)^2} \tag{2}$$

with z being the displacement, and $a = \frac{\varepsilon A}{2}$.

The "equilibria"-points z_o depend on the applied nominal voltages U_o. Equation (2) for $\ddot{z}_o = \dot{z}_o = 0$ yields

$$kz_o = \frac{aU_o^2}{(s-z_o)^2} \quad \text{or} \quad U_o = \pm \left[\frac{kz_o(s-z_o)^2}{a}\right]^{1/2} \tag{3}$$

This nominal U_o–voltage must be applied as a feedforward action if the capacitor's plate is to be moved and maintained at a distance z_o from its unstretched position. Moreover an additional feedback term is needed in order for the plate's position despite the presence of disturbances and discrepancies between the used (simplified) system model and the real system.

The linearized equations of motion can be found around the equilibria points (U_o, z_o). Perturbation theory for the variables U and z, where $U = U_o + \delta u$ and $z = z_o + \delta z$ yields the equation of the perturbed system as:

$$m\delta\ddot{z} + b\delta\dot{z} + kz_o + k\delta z = \frac{aU_o^2}{(s-z_o)^2} + \frac{2aU_o^2}{(s-z_o)^3}\delta z + \frac{2aU_o}{(s-x_o)^2}\delta u \ . \tag{4}$$

From the utilization of the perturbation dynamics from (3) into (4) we obtain

$$m\delta\ddot{z} + b\delta\dot{z} + \left[k - \frac{2aU_o^2}{(s-z_0)^3}\right]\delta z = \left[\frac{2aUo}{(s-z_0)^2}\right]\delta u \ . \tag{5}$$

If we leave the "perturbation dynamics" expressed in terms of $(\delta z, \delta u)$, we would not be able to apply the methodology of constrained optimal control, since the constraint on δz will be the same for each operating region. To overcome this problem we need an equivalent expression for (5) which will use z instead of δz. For this purpose we add the term $m\ddot{z}_o + b\dot{z}_o + kz_o$ on both sides resulting in

$$\ddot{z} + \frac{b}{m}\dot{z} + K^u z = \frac{2aU_o}{m(s-x_o)^2}\delta u + K^u z_o \ ,$$

where $K^u = \frac{k - \frac{2aU_o}{(s-z_o)^3}}{m}$. The previous equation can be written in a more compact form as

$$\ddot{z} + q_i\dot{z} + r_i z = K_i^u \delta u + K_i^o \tag{6}$$

where we used the ith subscript to denote the dependence of the previous variables on the selected equilibrium point.

The equivalent state space model accounting for small perturbations around the equilibrium point $(z_{o,i}, V_{o,i})$ is

$$\begin{bmatrix}\dot{z}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -r_i & -q_i\end{bmatrix}\begin{bmatrix}z\\ \dot{z}\end{bmatrix} + \begin{bmatrix}0\\ K_i^u\end{bmatrix}\delta u + \begin{bmatrix}0\\ K_i^o\end{bmatrix} \ . \tag{7}$$

The dependence of this PWA approximation on the specific equilibrium point is through the r_i, K_i^u, K_i^0 terms.

These linear time-invariant state space models can be transformed into their discrete equivalents under the assumption of a sampling process with sampling period T_s. The resulting discrete models can be cast in a compact form as

$$\begin{bmatrix}z(k+1)\\ \dot{z}(k+1)\end{bmatrix} = x(k+1) = A_i x(k) + B_i \delta u(k) + f_i \ . \tag{8}$$

This set of constrained PWA–subsystems from (8) will be stabilized by a multi-parametric controller, as shown in the following section.

3. CONSTRAINED FINITE TIME OPTIMAL CONTROLLER DESIGN (CFTOC)

Consider a constrained discrete piecewise affine system of the form shown in (8). The number of subsystems involved in this notation depends on the granularity of the selected equilibria points $(i = 1, \ldots, L)$.

Let the state vector and control effort be constrained within certain regions (guard functions), or

$$\begin{bmatrix} x(k) \\ \delta u(k) \end{bmatrix} \in \mathcal{P} = H_i x + J_i u \leq K_i \qquad (9)$$

These functions partition the (2+1)-dimensional space $[x(k), \delta u(k)]$ into a set of polyhedra.

The controller's objective is to generate the $\delta u(k)$ control effort by minimizing a cost over a receding horizon as

$$\delta u(k) = \min_{\delta u(k)} \left[||Px(k+N)||_\infty + \sum_{i=0}^{N-1} \left(\|R\delta u_{k+i}\|_\infty + \|Qx_{k+i}\|_\infty \right) \right], \qquad (10)$$

where N is the prediction horizon interval , Q, R and P are the weighting matrices on the states, the control effort and the desired final state, respectively.

The solution to the CFTOC–problem with $P = 0$ is a PWA state feedback control law of the form (Borelli *et al.*, 2003; Grieder *et al.*, 2004; Kvasnica *et al.*, 2004; Bemporad *et al.*, 2002)

$$\delta u = F_j^k x(k) + G_j^k, \text{ if } x(k) \in \mathcal{R}_j^k \ , \qquad (11)$$

where $\mathcal{R}_j^k$, $j =, \dots, N_k$ is a polyhedral partition of the set of feasible states $X(k)$ spanning the space affected by the prediction horizon N, the guard functions defined in (9) and the parameters P, Q, R and $x(k+N)$ involved in the formulation of the cost function in (10).

It should be noted that the $\delta u(k)$ control effort can be generated in an off line manner, thus simplifying the real-time computation of the control effort. Furthermore, the number of computed polyhedra depends on the length of the prediction horizon N and the nature of the guard functions.

The overall control framework appears in Figure 3, where it is shown that the suggested controller consists of: 1) a feedforward portion generating the control effort U_o based on the desired position z_o, and 2) the multi parametric controller generating the deviation δu to account for any perturbations along the nominal desired position.

4. SIMULATION STUDIES

Simulation studies were carried on a micro actuator's non-linear model where its SiO_2–plates have an area $A = 40\mu m \times 40\mu m = 160 \cdot 10^{-9} m^2$, with a mass $m = 7.0496 \cdot 10{-}10 Kgr$. The initial gap was set to $s = 4\mu m$ while the dielectric constant of the air was $\varepsilon = 9 \cdot 10^{-12} \frac{\text{Coulomb}^2}{N \cdot m^2}$. The allowable displacements of the micro–capacitor's plate in the z vertical axis were $z \in [0.1, 3.9]\ \mu m$. The goal of the controller was to move the capacitor's plates from an initial position to a new desired one (set-point regulation).

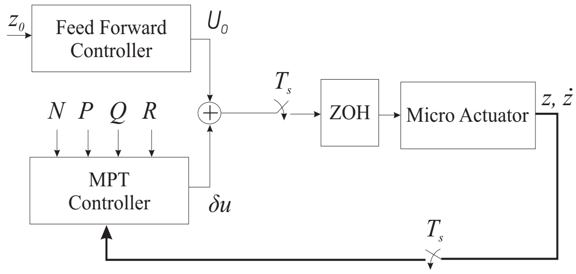

Fig. 3. Feedforward and Multi-parametric Feedback Control Framework

The derivation of the nonlinear equation (1) is obtained through the usage of a Finite-Element Model (FEM). The FEM computes the vibrational modes (Kuijpers *et al.*, 2003; Mita *et al.*, 2003; Morrell and Salisbury, 1998; L. Meirovitch, 1967) accounting for the effects of bending, torsion, axial and shear stress. The natural frequency of the 1st mode was computed from the FEM–model and is equal to $\omega_1 = 2\pi 5410$ rad/sec while the stiffness $k = 0.8146 N/m$. The damping coefficient, assuming air as the medium between the capacitor's plate, is $b = 1.4378 \cdot 10^{-5} N \cdot sec/m$.

The μ-actuator's first three modes ($\omega_2 = 2\pi 8240$, $\omega_3 = 2\pi 14597$) of vibration appear in Figures 4, 5 and 6.

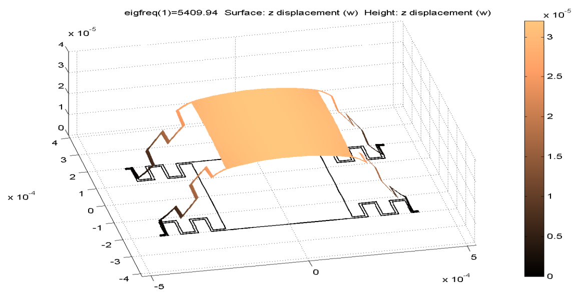

Fig. 4. μ-Actuator's 1st–Vibrational mode

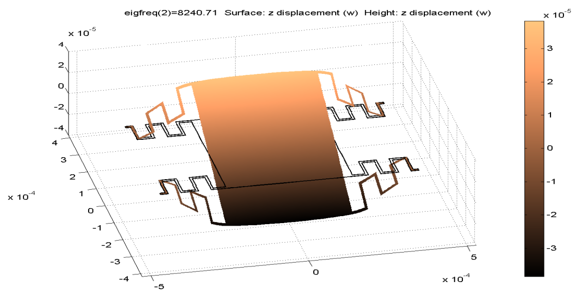

Fig. 5. μ-Actuator's 2nd–Vibrational mode

For the linearization process 5 operating points were selected with a discretization step equal to $0.88\mu m$. These operating points are $z_{o,i} = 0.49 +$

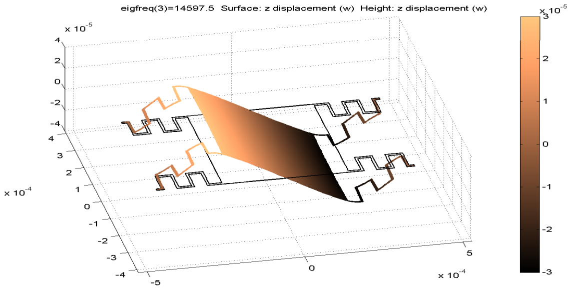

Fig. 6. μ-Actuator's 3rd–Vibrational mode

$i0.88(\mu m), i = 0, \ldots, 4$ and the selected sampling period was $T_s = \frac{2pi}{10\omega_1}$.

Given these operating points (equilibria) the space is partitioned into 5 regions $[0.1, 0.88)$, $[0.88, 1.66)$, $[1.66, 2.44)$, $[2.44, 3.22)$ and $[3.22, 3.9)$. Each linearized system is valid in only one of these regions (i.e., the 1st subsystem with $z_{o,0} = 0.49 = \frac{0.1+0.88}{2}$ is valid for all values of z within $[0.1, 0.88)\mu m$). Therefore the guard functions for δz are $-0.39 \leq \delta z < 0.39$. In this study, no constraint was posed on the velocity $\delta \dot{z}$ of the moving plate, while the control feedback effort was constrained (guard function) $-10 \leq \delta u \leq 10$.

The parameters involved in the cost function were $P = 0$, $Q = 100\ I_{2\times 2}$ and $R = 10^{-4}$. The number of polyhedra involved in the partitioning of the $[z, \dot{z}]$ space w.r.t. the prediction horizon $N = 1, \ldots, 5$ appears in Figure 7.

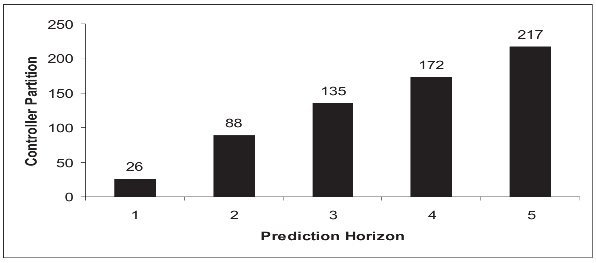

Fig. 7. Polyhedra partitioning vs. Prediction Horizon ($N = 1, \ldots, 5$)

For $N = 2$ the space is partitioned into 88 regions, as shown in Figure 8, while the generated feedback command appears in Figure 9.

The μ-actuator's step response appears in Figure 10, where the initial state was $z(0) = 0.5\mu m$ and the final desired state was set at $2\mu m$.

In a similar manner, the overall control effort $U_o|_{z=2\mu m} + \delta u$ appears in Figure 11, where in all sampling instants the δu term was constrained within $[-10, +10]$ Volt.

The effect of increasing the prediction horizon from $N = 2$ to $N = 5$ results in: a) an increase to the number of the polyhedra involved in the partitioning, and b) an overall "smoother" trajectory compared to the earlier one. Figure 12 displays the 217–polyhedra for the same cost parameters and $N = 5$.

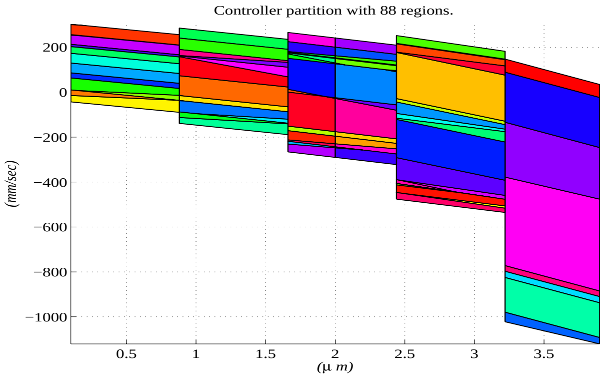

Fig. 8. Polyhedral Partition for $N = 2$

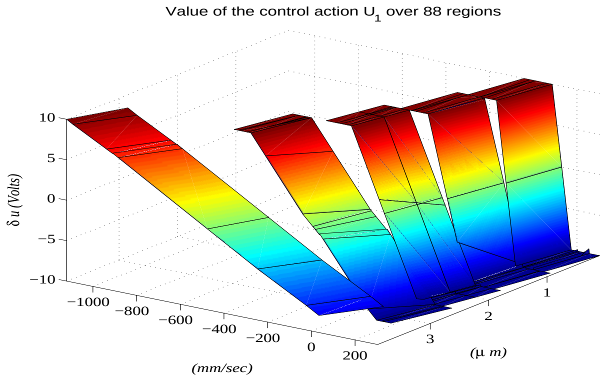

Fig. 9. Feedback Control Effort w.r.t. Polyhedral Partition ($N = 2$)

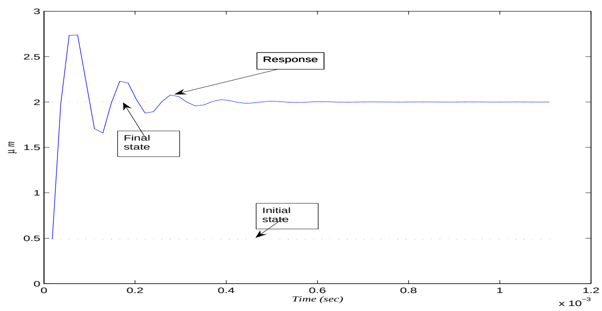

Fig. 10. μ–Actuator step response

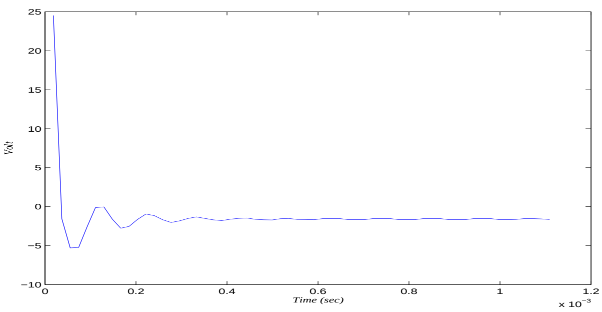

Fig. 11. Control command for step–response

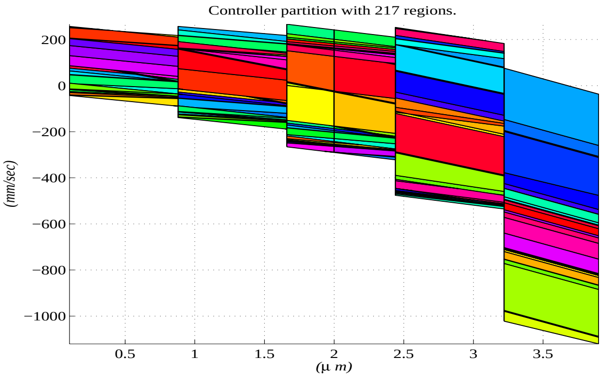

Fig. 12. Polyhedral partition for $N = 5$

5. CONCLUSIONS

In this paper a constrained finite time controller was developed for controlling the positioning of a microactuator. In principle, the linear displacement microactuator operates like a varying micro-capacitor. The displacement of the capacitor's plate is controlled by the combination of a feedforward and the multi-parametric feedback controller. The system's nonlinear model of the system is linearized around different operating points. All perturbations in the states and the system's control input were modelled in corresponding PWA dynamics. The resulting control structure was applied in simulation studies to the nonlinear model of a micro-actuator for testing the efficacy of the suggested control scheme.

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