EXACT WAVEFRONT CORRECTION IN ADAPTIVE OPTICS.

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Abstract: In this work we consider an adaptive optical system in which a deformable mirror is controlled to compensate for random wavefront disturbances. For most systems of this type, the shape of the mirror is taken as a linear function of the wavefront error, leading to satisfactory results in *linear* regimes. Here, the geometric shape of the mirror leading to a *perfect* correction of the wavefront is derived. Next, a control is designed to reach that geometric shape when the deformable mirror is a membrane mirror with electrostatic actuators. Numerical simulations illustrating the improvements supplied by the geometric approach are also reported here.

Keywords: Adaptive optics, exact wavefront correction, membrane mirror.

1. INTRODUCTION

Adaptive Optics (AO) is the technology developed for 25 years for correcting random optical wavefront distortions in real time (see e.g. (Roggeman and Welsh, 1996); (Hardy, 1998); (Tyson, 1998); (Roddier, 1999); (Plemmons and Pauca, 2000); (Luke *et al.*, 2002); (Zakynthinaki and Saridakis, 2003)). Wavefront disturbances typically appear when optical rays cross the Earth's atmosphere, since the refraction index depends on the air density, and the air density fails to be uniform in a turbulent environment.

An adaptive optics system is composed of a wavefront sensor and of a deformable mirror which is controlled in real time to compensate for random wavefront disturbances. The correction of the wavefront is said to be *perfect* (or *exact*) when the wavefront obtained after the reflection on the deformable mirror is planar. For most systems encountered in AO, the shape of the mirror is taken as a linear function of the wavefront error, leading to satisfactory results in *linear* regimes. Obviously, a perfect correction of the wavefront cannot be obtained this way, in general. In this work we investigate the possibility of achieving a perfect correction of any incident wavefront. That issue proves to be of great importance in nonlinear regimes where linear compensation does not work well.

The mathematical issue whether any incident wavefront may be corrected through the reflection on a convenient mirror is not obvious at all. The first reason is that computations based upon the Snell-Descartes first law reveal that a loss of derivative occurs, and that the Nash-Moser fixed-point theorem cannot be applied. The second reason is that the solution may fail to exist in certain circumstances: if the incident wavefront is a sphere, then we cannot find any corrector mirror passing through the center of the sphere.

Here, following a different approach we succeed in deriving a parametric representation of a deformable mirror achieving a perfect correction of any given incident wavefront. The formulas are provided in any dimension and for any incident angle. Next, a control is designed to compensate

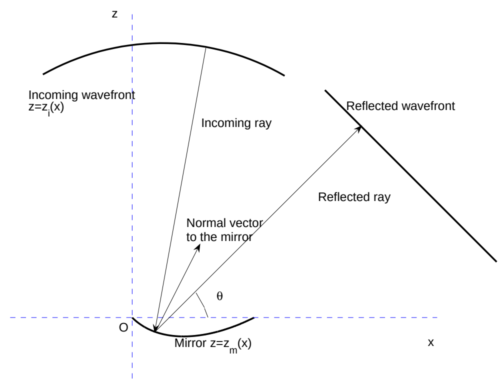

Fig. 1. Adaptive Optics device

for slowly time-varying wavefront disturbances when the deformable mirror is a membrane mirror with electrostatic actuators. Numerical simulations illustrating the improvements supplied by the geometric approach are provided.

The paper is organized as follows. In Section 2 we provide the parametric equations of the corrector mirror in the n-dimensional framework (Theorem 1), and we sketch the derivation of such formulas in dimension 2. These formulas are given when the incident wavefront is a hypersurface. Their generalization to parametrically defined incident wavefronts is also supplied in dimensions 2 and 3. Section 3 is devoted to the determination of the incident wavefront when the measured wavefront is the reflected one (closed-loop configuration). Numerical simulations are displayed in Section 4. In Section 5 we investigate the tracking problem for a membrane mirror. Section 6 is a brief conclusion.

2. OPEN LOOP

2.1 The n-dimensional problem

Let $\mathbb{R}^n$ be the Euclidean n-dimensional space, whose generic point is denoted by $(x, z) = (x_1, ..., x_{n-1}, z)$. Let $\{\boldsymbol{e}_i\}_{1\le i\le n}$ denote the canonical basis of $\mathbb{R}^n$. The incoming wavefront (resp., the deformable mirror) is assumed to be defined by the equation $z = z_i(x) = z_i(x_1, ..., x_{n-1})$ (resp., $z = z_m(x)$), where $z_i : [0,1]^{n-1} \to \mathbb{R}$ and $z_m : [0,1]^{n-1} \to \mathbb{R}$ are given functions of class C^1. The undeformed mirror is assumed to lie on the hyperplane $z = 0$, so that a planar incident wavefront of the form $\cos(\theta)x_1 - \sin(\theta)z = const$ gives rise to a planar reflected wavefront of the form $\cos(\theta)x_1 + \sin(\theta)z = const$. The wave vector associated with the reflected wavefront is defined as $\boldsymbol{f} = \cos(\theta)\boldsymbol{e}_1 + \sin(\theta)\boldsymbol{e}_n$, where $\theta \in [0, \frac{\pi}{2}]$ is some given angle. (See Figure 1.)

The following function is introduced for notational convenience

$$\Delta := \left(1 + \sum_{j=1}^{n} \left(\frac{\partial z_i}{\partial x_j}\right)^2\right)^{\frac{1}{2}}.$$

The following theorem provides a parametric representation of the mirror achieving a perfect correction of any given incoming wavefront.

Theorem 1. Assume that the function z_i is of class C^2. Then for any number C, the following parametric equations

$$X_j = x_j + \frac{\cos(\theta)x_1 + \sin(\theta)z_i(x) + C}{\Delta - \cos(\theta)\frac{\partial z_i}{\partial x_1} + \sin(\theta)} \cdot \frac{\partial z_i}{\partial x_j},$$
$$j = 1, \ldots, n-1 \tag{1}$$

$$Z = z_i(x) - \frac{\cos(\theta)x_1 + \sin(\theta)z_i(x) + C}{\Delta - \cos(\theta)\frac{\partial z_i}{\partial x_1} + \sin(\theta)} \tag{2}$$

with $(x_1, \ldots, x_{n-1}) \in [0,1]^{n-1}$, define a mirror shape leading to a perfect correction of the incoming wavefront $z = z_i(x)$.

The proof of Theorem 1 presents two steps.

- In a first step, assuming that the problem has indeed a solution, we derive the formulas (1)-(2) in using the fact that (i) all the reflected rays share the same wave vector (namely, $\boldsymbol{f}$) and (ii) the planar reflected wavefront is an equiphase surface (i.e., the time needed to reach the reflected wavefront from the incident wavefront does not depend of the ray under consideration.) Notice that the Snell-Descartes first law is not explicitly used in the computations.
- In a second step, we check that the reflected wavefront computed in applying the Snell-Descartes first law is indeed *planar* when the shape of the mirror is given by (1)-(2).

The full details of the proof will appear elsewhere. The derivation of (1)-(2) when $n = 2$ is sketched in the next section. Let us do some comments.

(1) The shape of the deformable mirror that enables a perfect correction of an arbitrary incoming wavefront is given as a parametrized surface. When the map $x \mapsto X = (X_1, ..., X_{n-1})$ is invertible, then the mirror may be defined by some equation $Z = z_m(X_1, ..., X_{n-1})$.
(2) A loss of regularity occurs: if z_i is of class C^r, then the functions $X_1(x), ..., X_{n-1}(x), Z(x)$ are expected to be of class C^{r-1} only, due to the presence of the derivatives $\partial z_i/\partial x_j$ in (1)-(2). Notice that the formulas (1)-(2) may be used when z_i is merely of class C^1, although the Snell-Descartes first law cannot

be applied to check whether the reflected wavefront is still planar. Indeed, to apply the Snell-Descartes reflection law, the existence of a normal vector to the mirror surface is required at each point of the mirror. That property is guaranteed if the functions $X_i(x)$ $(1 \leq i \leq n-1)$ and $Z(x)$ are of class C^1, but these functions are only continuous when z_i is of class C^1. Notice that a loss of regularity (expressed in a statistical framework) has also been pointed in (Le Roux, 2003).

(3) The shape of the mirror which works fine turns out not to be unique. Actually, we found a one-parameter family of solutions (C being the parameter). Geometrically, giving a value to C amounts to choosing the location where some incident ray and the mirror intersect. Notice that the different shapes of the mirror do not correspond to simple translations of one of them: the shape of the mirror change with C. On the other hand, the surface defined by (1)-(2) may reduce to a point for certain value of C (see below for some example).

2.2 The 2D problem

Here, we consider the simplest case where $n = 2$, hence $x = x_1$. We aim to derive the parametric form of the mirror shape thanks to which a perfect correction of the incident wavefront may be carried out. An incident ray issued from $(x, z_i(x))$ admits as (tangent) wave vector the vector $\boldsymbol{n} = \frac{1}{\Delta}(z_i'(x), -1)$. The intersection point of the ray with the mirror is given by

$$\begin{cases} X &= x + \dfrac{t}{\Delta} z_i'(x) \\ Z &= z_i(x) - \dfrac{t}{\Delta} \end{cases} \tag{3}$$

where $\Delta(x) = \sqrt{1 + |z_i'(x)|^2}$ and t denotes the time needed to reach the mirror. (The speed of light, whose value does not matter here, is chosen to be one.) The key point is that the wave vector of the reflected ray is $\boldsymbol{f} = (\cos(\theta), \sin(\theta))$, which amounts to saying that the reflected wavefront takes the form $g(x) := \cos(\theta)x + \sin(\theta)z = \mathit{const}$. Therefore, for T large enough, the function

$$\begin{aligned} g(x) &= \cos(\theta)(X + (T-t)\cos(\theta)) \\ &\quad + \sin(\theta)(Z + (T-t)\sin(\theta)) \\ &= (x + \frac{t}{\Delta} z_i'(x))\cos(\theta) \\ &\quad + (z_i(x) - \frac{t}{\Delta})\sin(\theta) + (T-t) \end{aligned}$$

has to be constant with respect to x. Expressing t as a function of x and plugging it in (3), we arrive to

$$\begin{cases} X &= x + \dfrac{\cos(\theta)x + \sin(\theta)z_i(x) + C}{\Delta - \cos(\theta)z_i'(x) + \sin(\theta)} z_i'(x) \\ Z &= z_i(x) - \dfrac{\cos(\theta)x + \sin(\theta)z_i(x) + C}{\Delta - \cos(\theta)z_i'(x) + \sin(\theta)}, \quad x \in [0,1]. \end{cases}$$

Let us now assume that the incident wavefront is only slightly disturbed (i.e. $z_i(x) = x + K + \delta(x)$ with $|\delta(x)| << 1$, K being some constant), and that $\theta = \pi/4$. Then taking the second order Taylor expansion of X and Z one obtains for $C = -K/\sqrt{2}$

$$\begin{cases} X &= 2x + \dfrac{\delta}{2} + x\delta' - \dfrac{1}{8}x\delta'^2 + \dfrac{\delta\delta'}{2} + o(\delta'^2 + |\delta\delta'|), \\ Z &= K + \dfrac{1}{2}\delta + \dfrac{1}{8}x\delta'^2 + o(\delta'^2). \end{cases}$$

It is then easy to derive the second order approximation of Z as a function of X: $Z = K + \frac{1}{2}\delta(X/2) - \frac{1}{16}\delta'(X/2)^2 - \frac{1}{8}\delta(X/2)\delta'(X/2)$. In particular, we obtain at the first order in δ

$$Z = K + \frac{1}{2}\delta(X/2).$$

Thus, at the first order the shapes of the mirror and of the wavefront must overlap (up to a translation and a dilatation). This is the rule used in most classical systems.

To illustrate the comments (2) and (3) of the previous section, let us assume that $\theta = \pi/4$ and that the incoming wavefront is spherical, e.g. $z_i(x) = \sqrt{1 - x^2}$, $x \in [-1, 0]$. According to Theorem 1, the shape of the mirror which achieves a perfect correction of the wavefront is given by

$$\begin{cases} X &= \dfrac{x}{1 + (x/\sqrt{2}) + (\sqrt{1-x^2}/\sqrt{2})}(1 - C) \\ Z &= \dfrac{\sqrt{1-x^2}}{1 + (x/\sqrt{2}) + (\sqrt{1-x^2}/\sqrt{2})}(1 - C), \end{cases}$$

C being any real constant. It follows that the mirror is a part of the parabola $(X - Z)^2 = \frac{2}{C-1}(X + Z) - 2$, as illustrated in Figure 2. The well-known property that a parabolic mirror transforms a planar wavefront into a spherical one (and vice-versa) is recovered. Notice that the parabola reduces to the origin when $C = 1$, and that the parabolas associated with different values of C fail to be isometric.

The result in Theorem 1 may be extended to the case where the incoming wavefront is defined in a parametric way. This extension proves to be useful when the wavefront measure is performed *after* the reflection on the deformable mirror; indeed, in that case the incident wavefront is defined by parametric equations (see below). Assume the incoming wavefront to be defined as $(x, z) = (x_i(s), z_i(s))$, where x_i and z_i are given functions

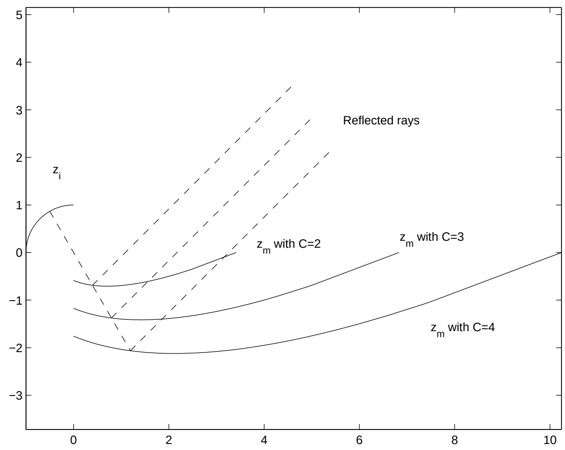

Fig. 2. Correction of a spherical wavefront

belonging to $C^1(I,\mathbb{R})$ ($I \subset \mathbb{R}$ is some interval) and set

$$\tilde{\Delta} = \left({x_i'}^2 + {z_i'}^2\right)^{\frac{1}{2}}.$$

Then the following result holds true.

Proposition 1. Assume that the functions x_i and z_i are of class C^2. Then for any number C, the following parametric equations

$$X = x_i + \frac{\cos(\theta)x_i + \sin(\theta)z_i + C}{\tilde{\Delta} - \cos(\theta)z_i' + \sin(\theta)x_i'} \cdot z_i' \quad (4)$$

$$Z = z_i - \frac{\cos(\theta)x_i + \sin(\theta)z_i + C}{\tilde{\Delta} - \cos(\theta)z_i' + \sin(\theta)x_i'} \cdot x_i' \quad (5)$$

define a mirror shape leading to a perfect correction of the incoming wavefront $x = x_i(s), z = z_i(s)$.

The proof follows the same line as for Theorem 1.

2.3 The 3D Problem

Clearly, the case $n = 3$ corresponds to the most interesting one from a physical viewpoint. An application of Theorem 1 provides the following parametric representation of the mirror

$$X_1 = x_1 + \frac{\cos(\theta)x_1 + \sin(\theta)z_i(x_1,x_2) + C}{\Delta - \cos(\theta)\frac{\partial z_i}{\partial x_1} + \sin(\theta)}\,\frac{\partial z_i}{\partial x_1}$$

$$X_2 = x_2 + \frac{\cos(\theta)x_1 + \sin(\theta)z_i(x_1,x_2) + C}{\Delta - \cos(\theta)\frac{\partial z_i}{\partial x_1} + \sin(\theta)}\,\frac{\partial z_i}{\partial x_2}$$

$$Z = z_i(x_1,x_2) - \frac{\cos(\theta)x_1 + \sin(\theta)z_i(x_1,x_2) + C}{\Delta - \cos(\theta)\frac{\partial z_i}{\partial x_1} + \sin(\theta)},$$

where (x_1, x_2) ranges over $[0,1]^2$.

In order to extend above formulas to the case where the incident wavefront is defined by parametric equations we need to introduce a few notations. Let the incident wavefront be defined as $(x, z) = (x_{i1}(s,t), x_{i2}(s,t), z_i(s,t))$, where x_{i1}, x_{i2} and z_i are given functions belonging to $C^1(I^2,\mathbb{R})$ (I is again some real interval). Let $\boldsymbol{n} = (n_1, n_2, n_3)$ denote the wave vector of any incident ray. We readily find

$$\begin{cases} n_1 = \dfrac{\partial x_{i2}}{\partial t}\dfrac{\partial z_i}{\partial s} - \dfrac{\partial x_{i2}}{\partial s}\dfrac{\partial z_i}{\partial t} \\ n_2 = \dfrac{\partial x_{i1}}{\partial s}\dfrac{\partial z_i}{\partial t} - \dfrac{\partial x_{i1}}{\partial t}\dfrac{\partial z_i}{\partial s} \\ n_3 = \dfrac{\partial x_{i1}}{\partial t}\dfrac{\partial x_{i2}}{\partial s} - \dfrac{\partial x_{i1}}{\partial s}\dfrac{\partial x_{i2}}{\partial t}. \end{cases}$$

Set

$$\tilde{\Delta} = ||\boldsymbol{n}|| = (|n_1|^2 + |n_2|^2 + |n_3|^2)^{\frac{1}{2}}.$$

The extension of Proposition 1 to the 3D case is as follows.

Proposition 2. Assume that the functions x_{i1}, x_{i2} and z_i are of class C^2. Then for any number C, the following parametric equations

$$X_1 = x_{i1} + \frac{\cos(\theta)x_{i1} + \sin(\theta)z_i + C}{\tilde{\Delta} - \cos(\theta)n_1 - \sin(\theta)n_3}\,n_1$$

$$X_2 = x_{i2} + \frac{\cos(\theta)x_{i1} + \sin(\theta)z_i + C}{\tilde{\Delta} - \cos(\theta)n_1 - \sin(\theta)n_3}\,n_2$$

$$Z = z_i - \frac{\cos(\theta)x_{i1} + \sin(\theta)z_i + C}{\tilde{\Delta} - \cos(\theta)n_1 - \sin(\theta)n_3}\,n_3,$$

define a mirror shape leading to a perfect correction of the incoming wavefront $x_1 = x_{i1}(s,t), x_2 = x_{i2}(s,t), z = z_i(s,t)$.

3. 2D CLOSED LOOP CONFIGURATION

Here we focus on the so-called *closed loop configuration* for which the wavefront sensors are located on the optical path after the reflection.

To define the shape of the corrector mirror, we first have to derive the form of the incident wavefront from the measure of the reflected wavefront. Then we may use (1)-(2) to get the parametric equations defining the mirror.

For the sake of simplicity, we restrict ourselves to the two-dimensional case. Thanks to the reversibility of Snell-Descartes laws, the incoming wavefront may be seen as the reflected wavefront associated with the measured wavefront (which is the authentic reflected wavefront). Assume that the mirror is given by the equation $z = z_m(x)$ and that the reflected wavefront is described by the system of parametric equations $x = x_r(s), z = z_r(s)$. Let us set

$$\Delta_r := \sqrt{{x'_r}^2 + {z'_r}^2},$$
$$\Delta_m := \sqrt{1 + {z'_m}^2}.$$

If $t(s)$ is the time elapsed from the reflection (i.e., the length of the optical path between the mirror and the reflected wavefront), then the incoming wavefront is defined by

$$x_i(s) = x_r(s) + \frac{t(s)}{\Delta_r} z'_r(s) - (T - t(s))\, n_{i1} \quad (6)$$
$$z_i(s) = z_r(s) - \frac{t(s)}{\Delta_r} x'_r(s) - (T - t(s))\, n_{i2} \quad (7)$$

where $T > 0$ is a given duration and

$$\begin{aligned} \boldsymbol{n_i} &= (n_{i1}, n_{i2}) \\ &= \frac{2(x'_r + z'_m z'_r)}{\Delta_m^2 \Delta_r}(z'_m, -1) + \frac{1}{\Delta_r}(-z'_r, x'_r) \end{aligned}$$

stands for the incident wave vector. Notice that to compute $t(s)$ we need to find the intersection of a ray with the mirror. This task cannot be done in an analytical way in general.

Combining (6)-(7) to (4)-(5), we obtain a parametric representation of the mirror.

4. NUMERICAL SIMULATIONS

In order to numerically compute the reflection of the wavefront on a mirror, we need to determine the intersection points of the incoming rays with the mirror together with the normal vectors to the mirror surface at these points. Using parametric representations of both the mirror surface and the rays, the intersection points are found by solving a 1D nonlinear equation by means of a variant of the secant method.

One of the burning issues is how to obtain an "efficient" shape of the mirror from the parametric representation. It turns out that only a part of the mirror may be used in practice for the wavefront correction. Indeed, a parametrized surface is generally the graph of a function only locally. An incident ray may intersect the mirror several times, and the first intersection point may be on the wrong side of the mirror. One way to overcome this problem is to impose that Z be an increasing (or slightly decreasing) function of X, so that any incident ray meets the mirror only one time. On the other hand, the parametrized surface may go beyond the zone which may be reached by the incident rays. The parameter C arising in (1)-(2) and the range of the variable x have to be chosen in such a way that the deformed mirror remains as close as possible of the undeformed mirror.

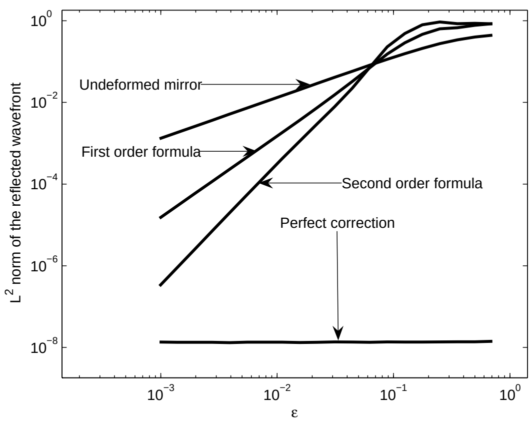

Fig. 3. Residual error versus ϵ

A sample of tests when $n = 2$ have been carried out with a planar mirror (no correction), and with mirrors defined respectively by first-order, second-order and parametric formulas.

The incident wavefront, depending on a small parameter $\varepsilon \in [0, 1]$, is defined as

$$z_i(x) = 2 + \varepsilon \sin(\pi x), \;\; x \in [0, 1]$$

The *residual error* is

$$e_r = \left(\int_0^1 |z_r(x) - z_p(x)|^2 \, dx \right)^{\frac{1}{2}}$$

where $z = z_r(x)$ (resp., $z = z_p(x)$) is the equation defining the reflected wavefront (resp., the planar wavefront).

The residual error associated to each type of correction is plotted in Figure 3. As expected, using a deformable mirror designed with a first-order or second order formula allows to correct in a convenient way the incoming wavefront in a *linear* regime ($\varepsilon < 10^{-2}$). The error is proportional to ε^2 with a mirror defined by the first-order formula. A dramatic improvement of the correction of the wavefront in a nonlinear regime ($\varepsilon \sim 10^{-1}$) is achieved with a deformable mirror designed along the parametric equations (1)-(2).

5. MEMBRANE MIRROR

In this section, we assume that the deformable mirror is a clamped *membrane mirror* (see e.g. (Welsh and Gardner, 1989)); (Tyson, 1998); (Hardy, 1998); (Fernández and Artal, 2003)). Then its dynamics is governed by the wave equation (written here in its normalized form)

$$y_{tt} - \Delta y = v \qquad \text{in } \Omega \times]0, T[$$

supplemented with the initial conditions $y(0) = y^0$, $y_t(0) = y^1$ and the boundary conditions $y = 0$ on $\Sigma := \partial\Omega \times]0, T[$. Here, Ω is an open set in $\mathbb{R}^2$ and v stands for the control input. We stress that in AO, the control input is usually assumed to be applied *everywhere*. The following result deals with the tracking problem for the wave equation.

Theorem 2. Let $(y^0, y^1) \in H_0^1(\Omega) \times L^2(\Omega)$ and let $\bar{y} \in H^2_{loc}(\mathbb{R}^+, L^2(\Omega)) \cap L^2_{loc}(\mathbb{R}^+, H^2(\Omega) \cap H_0^1(\Omega))$ be a given trajectory. Pick any number $k > 0$ and set

$$\begin{aligned} \varepsilon(t,x) &:= y - \bar{y} \\ v(t,x) &:= \bar{y}_{tt} - \Delta \bar{y} - k\,\varepsilon_t. \end{aligned}$$

Then $\varepsilon(t) \to 0$ strongly in $H_0^1(\Omega)$ as $t \to +\infty$.

Proof. Clearly, ε fulfills

$$\begin{aligned} \varepsilon_{tt} - \Delta\varepsilon &= -k\varepsilon_t && \text{in } \Omega \times (0,T) \\ \varepsilon &= 0 && \text{on } \Sigma. \end{aligned}$$

The proof is completed by applying a classical result (see e.g. (Lions, 1988)) on the internal stabilization of the wave equation. □

Assume now that the incident wavefront z_i is a smooth function of both t and x. Applying Theorem 1, we may associate to it a nominal mirror shape $\bar{y}$ achieving a perfect correction of the incident wavefront. Then it follows from Theorem 2 that a control input may be designed in such a way that the difference between the nominal mirror shape and the actual mirror shape tends to 0. As a consequence, we obtain that the residual error tends also to zero.

Remarks.

(1) Actually, the nominal trajectory $\bar{y}$ has to belong to $H_0^1(\Omega)$ (hence, $y = 0$ on $\partial\Omega$). That condition is fulfilled by taking the projection on $H_0^1(\Omega)$ of the mirror shape provided by Theorem 1. The error with respect to the exact mirror shape may be minimized by a convenient choice of the parameter C.
(2) In practice, a flat mirror (the so-called *tilt* correction mirror, see (Roggeman and Welsh, 1996); (Tyson, 1998)) is used to compensate for low frequency wavefront disturbances. With such a device at hand, the mirror shape to be reached is actually almost flat at the boundary.
(3) In a closed loop framework, the incident wavefront depends also on y. Then the nominal trajectory is a function of z_r and y.

6. CONCLUSION

A parametric representation of a mirror achieving a perfect correction of a given incident wavefront has been given here. When the deformable mirror is a membrane mirror, a control input allowing to compensate for a slowly time-varying wavefront disturbance has been proposed. Such control deserves to be numerically studied, in the open loop or in the closed loop configurations. The tracking problem may also be investigated when the deformable mirror is a bimorph mirror ((Tyson, 1998), (Lenczner and Prieur, 2004)). It will be the purpose of further works in a near future.

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