NONLINEAR MODEL BASED PREDICTIVE CONTROLLER OF A BUCK BOOST CONVERTER

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Abstract: This paper presents a Nonlinear Model Based Predictive Controller (NMBPC) of a buck-boost converter (BBC). The NMBPC uses a nonlinear prediction of the system outputs based on a discretization of the average continuous model of the BBC, assuming the duty ratio as the control variable. A classical quadratic cost function J is minimized at each sample time using a Sequential Quadratic Programming (SQP) optimization algorithm that guarantees that the obtained control action gives a local optimal value of J. The tuning of the controller parameters is defined to obtain a compromise between performance and robustness. Simulation results in a wide range of the output voltage show that the proposed control strategy yields very fast time responses even under varying load situations.

Keywords: Model Based Predictive Control, Nonlinear Systems, Optimization, Sequential Quadratic Programming, Buck-Boost Converter

1. INTRODUCTION

Switched mode DC-to-DC power converters are used in several electric power supply systems, including vehicles, illumination, control systems, computers, and others systems. Due to their intrinsic nonlinearity these systems represent a challenging field for control algorithms (Buso, 1999). Several control strategies have been proposed in the last years to control these processes. Linear control techniques are normally based on a small signal analysis of the process and their performances generally depend on the operating point (Mattavelli, 1997). To overcome this drawback and to cope with the parameter variation of the linearized model, a robust controller based on a μ-synthesis approach is presented in (Buso, 1999). Nonlinear approaches based on sliding model control have been proposed in (López, 1999) and (Shtessel, 2003), both of which use an explicit mathematical model of the process. This strategy is simple to implement but the design is complex and specific for each plant. Thus, this paper presents a different nonlinear approach that uses a mathematical model of the plant, is easy to tune and could be used for different topologies without changes in the controller.

A nonlinear model based predictive controller (NMBPC) is proposed for the output voltage regulator of a Buck-Boost converter (BBC). Model based predictive controllers (MBPC) are widely used in the process industry and have demonstrated their potentiality. Although MBPC are used to control multivariable complex processes, most of the MBPC used in industry are based on linear models (Camacho, 1998). When the process exhibits strong nonlinearities, as is the case of the BBC, NMBPC must be used if a high closed loop

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performance is desired (Chen, 2002). The main advantage of the NMBPC is the simple way in which the control law is proposed. Also, simple and intuitive time domain rules could be used for tuning the controller (Camacho, 1998) although it is computationally more demanding than other linear algorithms. However, with the current computer power available (like Digital Signal Processing based systems) computational complexity is not a real restriction (De Keyser, 2003).

The NMBPC proposed here is an extension to nonlinear systems of the well known generalized predictive controller (GPC) (Camacho, 1998). It uses a nonlinear prediction of the system outputs based on a discretization of the average continuous model of the BBC, assuming the duty cycle as the controlled variable. A classical quadratic cost function J is minimized at each sample time using a SQP optimization algorithm (Boggs, 1996) that guarantees that the obtained control action gives a local optimal value of J subject to a set of predefined constrains in the control action and plant output.

The paper is organized as follows: section 2 presents the BBC modelling and discretization. Section 3 presents the proposed NMBPC and the optimization algorithm. The tuning of the controller is presented in section 4 together with some simulation results. The paper ends with the conclusions.

2. BUCK-BOOST CONVERTER MODELING

Figure 1 shows the circuit of an ideal BBC. The

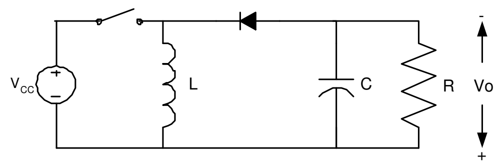

Fig. 1. Ideal Buck Boost Converter (BBC).

BBC is a typical DC-to-DC converter normally used as power supply with adjustable output voltage (Vo) than can be higher or lower than the supply voltage (Vcc). From the control point of view the objective of this system is to provide an output that can follow a desired voltage reference and reject the disturbances caused by the load variations represented in figure 1 by the resistance R. To do it, an adequate control strategy must be defined to actuate in the switch S.

The BBC can operate in two different modes. If the current in the inductor L is not zero the BBC operates in the continuous conduction mode. If not a discontinuous operation mode is considered.

Typically, two BBC models are used in the literature. The instantaneous model considers all the dynamic phenomena related to the switch operation. The average model does not consider the switch dynamics but only the dominant behavior caused by the other elements of the circuit, (Middlebrook, 1976), (Kassakian, 1991), (Vorpérian, 1990).

2.1 The instantaneous model

In order to obtain an instantaneous model of the BBC, q is defined as a signal that characterizes the dynamic behavior of the switch:

$$S : \begin{cases} q = 0, \text{ if the switch is open} \\ q = 1, \text{ if the switch is closed} \end{cases}$$

Thus, two sets of differential equations describe the behavior of the BBC. If $q = 0$:

$$\frac{di_L}{dt} = \frac{1}{L} v_C \tag{1}$$
$$\frac{dv_C}{dt} = \frac{1}{C}(-i_L - \frac{v_C}{R}) \tag{2}$$
$$V_0 = -v_C \tag{3}$$

and if $q = 1$:

$$\frac{di_L}{dt} = \frac{1}{L} V_i \tag{4}$$
$$\frac{dv_C}{dt} = -\frac{1}{C}\frac{v_C}{R} \tag{5}$$
$$V_0 = -v_C \tag{6}$$

Combining these two sub-systems the instantaneous model of the BBC is obtained:

$$\frac{di_L}{dt} = \frac{1}{L}(qV_i + (1-q)v_C) \tag{7}$$
$$\frac{dv_C}{dt} = -\frac{1}{C}(\frac{v_C}{R} + (1-q)i_L) \tag{8}$$
$$V_0 = -v_C \tag{9}$$

2.2 The average model

The average model is obtained using the instantaneous model and some simplification hypotheses. Assuming that the switch conmutes at a frequency that is much higher than the frequencies associated to the transfer of energy in the passive elements of the circuit (R, L, and C) it is possible to substitute the instantaneous control signal $q(t)$ by the signal $d(t)$ that gives, at each instant t, the average value of $q(t)$ in a switching period. Using this approximation, the model can be represented by the following set of equations (Borges, 2002):

$$\frac{d\widehat{i_L}}{dt} = \frac{1}{L}(dV_i + (1-d)\widehat{v_C}) \tag{10}$$
$$\frac{d\widehat{v_C}}{dt} = -\frac{1}{C}(\frac{\widehat{v_C}}{R} + (1-d)\widehat{i_L}) \tag{11}$$
$$V_0 = -\widehat{v_C} \tag{12}$$

3. NMBPC OF THE BBC

The proposed NMBPC algorithm consists of applying a control sequence that minimizes a multistage cost function of the form

$$J = \sum_{j=N_1}^{N_2} \delta(j)[E(y(t+j \mid t)) - w(t+j)]^2 + \sum_{j=1}^{N_u} \lambda(j)[\triangle u(t+j-1)]^2 \quad (13)$$

where $E(y(t+j \mid t))$ is the j-step ahead prediction of the system output on data up to time t, N_1 and N_2 are the minimum and maximum cost horizons, N_u is the control horizon, $\delta(j)$ and $\lambda(j)$ are weighting sequences, and $w(t+j)$ is a future set-point or reference sequence. The objective of predictive control is to compute the future incremental control sequence $\triangle u(t)$, $\triangle u(t+1)$,... in such a way that the future plant output $y(t+j)$ is driven close to $w(t+j)$. This is accomplished by minimizing J, subject to a set of constrains on the control variable and plant output.

To obtain an appropriate solution for this control problem two main aspects must be analyzed. First, a predictor set of equations must be defined in order to compute the expected future values of the plant output $\hat{y}(t+j \mid t)$ for $j = 1, ..., N_2$. After that, using the relation between the predictions and the control actions, the optimization problem is formulated. These two points are analyzed in the following sub-section. Finally, an algorithm that guarantees a local solution of the optimization problem should be used. This point is discussed in sub-section 3.2.

3.1 The NMBPC strategy

The NMBPC uses a prediction of the behavior of the process to compute the control action. To compute these predictions it is necessary to define a discrete model of the plant. Also, a disturbance signal is normally included in the model. A very common model of the disturbances is:

$$p(k) = \frac{e[k]}{1-q^{-1}}$$

where $e[k]$ is a white noise signal with zero mean and the integrator is used to produce an off-set free steady state control (Camacho, 1998). Thus, using equations 10, 11, 12 of the average model and the approximation:

$$\frac{d\zeta}{dt} = f(t) \quad \Rightarrow \quad \frac{\zeta[k+1]-\zeta[k]}{T_s} = f[k] \quad (14)$$

that is valid if the sample period T_s is adequately chosen, the following discrete model can be obtained for the plant and disturbances:

$$\widehat{i_L}[k] = \frac{T_s}{L}(d[k-1]V_i + (1-d[k-1]) \widehat{v_C}[k-1]) + \widehat{i_L}[k-1] + \frac{e[k]}{1-q^{-1}} \quad (15)$$

$$\widehat{v_C}[k] = -\frac{T_s}{C}(\frac{\widehat{v_C}[k-1]}{R} + (1-d[k-1]) \widehat{i_L}[k-1]) + \widehat{v_C}[k-1] + \frac{e[k]}{1-q^{-1}} \quad (16)$$

$$Vo[k] = -\widehat{v_C}[k] \quad (17)$$

Thus assuming that the best expected value for the future errors $e[k+i]$ is zero,$i \geq 0$, the predictions of $\widehat{i_L}$ and $\widehat{v_C}$ can be computed using:

$$E(i_L[k]) = \frac{T_s}{L}[(d[k-1]-d[k-2])V_i + (1-d[k-1])(\widehat{v_C}[k-1]-\widehat{v_C}[k-2])] + 2\widehat{i_L}[k-1] - \widehat{i_L}[k-2] \quad (18)$$

$$E(v_C[k]) = -\frac{T_s}{C}(\frac{\widehat{v_C}[k-1]-\widehat{v_C}[k-2]}{R} + (1-d[k-1])(\widehat{i_L}[k-1]-\widehat{i_L}[k-1])) + 2\widehat{v_C}[k-1] - \widehat{v_C}[k-2] \quad (19)$$

$$E(Vo[k]) = -\widehat{v_C}[k] \quad (20)$$

In the cost function the weighting factors are considered constant along the horizons ($\delta(j) = \delta$, $\lambda(j) = \lambda$) and as the controlled system is SISO, only λ will be used as a tuning parameter.

3.2 The optimization procedure

Sequential quadratic programming (SQP) is a framework for designing effective algorithms for nonlinearly constrained programming. SQP is a practical method that can tackle small or large and convex or nonconvex problems. It produces a series of iterates by solving quadratic subproblem approximations that converges from remote starting points to locally optimal solutions of nonconvex problems (*global convergence*) and, in the vicinity of an attractor, its speed of convergence approaches the quadratic convergence rate of Newton's method (*local convergence*). Take the general nonlinearly constraint optimization problem:

$$P: \ \underset{x \in \mathbb{R}^n}{\text{Minimize}} \quad f(x) \quad \text{Subject to: } c_j(x) = 0, \ j \in \mathcal{E}; \quad c_j(x) \geq 0, \ j \in \mathcal{I}$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and $c_j : \mathbb{R}^n \rightarrow \mathbb{R}$ are smooth functions, and $\mathcal{I}$ and $\mathcal{E}$ are two finite sets of indices. SQP is more of a framework for algorithm design than an algorithm, leaving a number of parameters to be chosen by the users to best tackle the problem at hand. In what follows we describe the principles of the method near to a local solution (when the starting point is within a basin of attraction) and far from it. To keep the presentation manageable, we favor a somewhat informal description of the SQP method—the interested reader can refer to the technical literature for a rigorous treatment (Boggs, 1996).

3.3 Local Method

Given an initial guess (x_0, λ_0) for the solution and Lagrangian multiplier obtained as a hot start from preceding solutions, or otherwise arbitrary vectors, SQP generates a sequence $\{(x_k, \lambda_k)\}_{k=0}^{\infty}$ of improving iterates. To improve the current iterate (x_k, λ_k), the method solves a quadratic approximation of P:

$$\begin{aligned} P_k: \quad & \underset{p_k}{\text{Minimize}} \quad m_k(p_k) = \frac{1}{2} p_k^T W_k p_k + \nabla f_k^T p_k \\ & \text{Subject to: } \nabla c_j(x)^T p_k + c_j(x_k) = 0, \ j \in \mathcal{E} \\ & \qquad\qquad\quad \nabla c_j(x)^T p_k + c_j(x_k) \geq 0, \ j \in \mathcal{I} \end{aligned}$$

where W_k is the Hessian of the Lagrangian function at the current iterate. More precisely, $\mathcal{L}(x, \lambda) = f(x) - \sum_{j \in \mathcal{E} \cup \mathcal{I}} \lambda_j c_j(x)$ is the Lagrangian, $W(x, \lambda) = \nabla_{xx}^2 \mathcal{L}(x, \lambda)$ is the Lagrangian's Hessian, and $W_k = W(x_k, \lambda_k)$ is the Hessian matrix evaluated at the current iterate. For each $k = 0, 1, \ldots$, the local SQP method begins by evaluating $f_k = f(x_k)$, $\nabla f_k = \nabla f(x_k)$, $W_k = W(x_k, \lambda_k)$, $c_j(x_k)$ and $\nabla c_j(x_k)$ for all $j \in \mathcal{I} \cup \mathcal{E}$. It then solves P_k to yield steps p_k and μ_k for the current solution and Lagrangian multiplier, respectively, obtaining the next iterate $(x_{k+1}, \lambda_{k+1}) = (x_k + p_k, \lambda_k + \mu_k)$. If a convergence criterion is satisfied, such as the minimum step $\|(p_k, \mu_k)\| < \varepsilon$, the algorithm terminates, or else it continues iterating from (x_{k+1}, λ_{k+1}). Notice that the objective function of P_k is not a Taylor's series approximation of f, but rather a quadratic approximation of the Lagrangian $\mathcal{L}(x, \lambda)$ that captures the curvature of the constraints. This choice is not incidental. It gives rise to a nice interpretation of SQP iterates that explains its fast convergence near local minima. Suppose that P is devoid of inequalities, $\mathcal{I} = \emptyset$. Then the SQP method becomes equivalent to Newton's method applied to the first-order conditions, $\nabla \mathcal{L}(x, \lambda) = 0$, which consists in solving a system of linear equations:

$$\begin{bmatrix} W_k & -A_k^T \\ A_k & 0 \end{bmatrix} \begin{bmatrix} p_k \\ \mu_k \end{bmatrix} = \begin{bmatrix} -\nabla f(x_k) + A_k^T \lambda_k \\ -c(x_k) \end{bmatrix} \quad (21)$$

where $c = [c_j : j \in \mathcal{E}]$ is the constraint vector function and $A_k = \nabla c(x_k)$ is the Jacobian matrix of the constraints evaluated at the current iterate. Assuming that (x_k, λ_k) is sufficiently close to a locally optimal pair (x^*, λ^*) and W_k is positive definite in the null space of A_k, then P_k is a convex quadratic problem that has a unique solution-Lagrangian multiplier (p_k, μ_k). This pair is precisely the solution to the linear system (21). Thus, at least locally, the SQP method defines not only a good step from x_k to a local solution x^*, but also a good step from λ_k towards the optimal Lagrangian multiplier λ^*. The advantage of the SQP framework over the application of Newton's method to solve the first-order optimality conditions is twofold. First, the SQP framework provides a tool for modifying the step if the current iterate is not close to (x^*, λ^*). Second, it can be more effectively extended to tackle inequality-constrained problems.

When the current iterate is sufficiently near to (x^*, λ^*), the SQP method identifies the optimal active set $\mathcal{A}(x^*) = \mathcal{E} \cup \{j \in \mathcal{I} : c_j(x^*) = 0\}$ for P and, from thereon, it behaves as if there were only equality constraints. Let $c_A(x^*) = [c_j(x^*) : j \in \mathcal{A}(x^*)]$ be the vector function with only active constraints and $\nabla c_A(x^*)$ be the corresponding Jacobian. Suppose that $\nabla c_A(x^*)$ has full row rank, that $d^T W(x^*, \lambda^*) d > 0$ for all $d \neq 0$ such that $\nabla c_A(x^*) d = 0$ [2], that strictly complementary holds, and that the current iterate (x_k, λ_k) is sufficiently close to (x^*, λ^*). Then, there is a local solution $\hat{x}$ to the quadratic approximation P_k whose active set $\mathcal{A}(\hat{x})$ is identical to the active $\mathcal{A}(x^*)$ of the nonlinear problem P (Nocedal, 1999). As the iterates draw closer to a local minimizer satisfying the conditions just stated, the active set remains fixed and subproblem P_k behaves as if there were no inequality constraints—the inequality constraints not appearing in $\mathcal{A}(x^*)$ are discarded. Hence, local convergence is also ensured for inequality-constrained problems.

3.4 Global Methods

For SQP to be practical, convergence should be ensured for nonconvex problems and from remote starting points. The globally convergent version of SQP behaves much like trust-region methods. In short, if W_k is positive definite on the tangent space of the active constraints, then m_k is convex implying that P_k has a unique solution and the Newton's step (p_k, μ_k) is well defined. On the other hand, if W_k is not positive definite, a number of alternatives can be implemented to promote progress towards a minimizer: linear search methods replace it by a positive definite matrix B_k; quasi-Newton methods use matrix factorization to obtain a positive definite B_k; and other methods replace W_k with the Hessian of an augmented Lagrangian having convexity properties. Alone, these techniques are not sufficient to guarantee convergence from distant starting points. A merit

[2] In other words, $W(x^*, \lambda^*)$ is positive definite on the space tangent to the active constraints

function ϕ is typically used to control the length of steps in line search methods or to determine how the size of a trust-region should be adjusted in trust-region methods. In essence, a merit function combines constraint infeasibility and the objective value of an iterate so that a descent direction for this function means progress towards a local solution. For equality-constrained problems, two merit functions are the l_1 *merit function*

$$\phi_1(x;\mu) = f(x) + \frac{1}{\mu}\|c(x)\|_1$$

where $\mu > 0$, and *Fletcher's augmented Lagrangian merit function*, which is defined by

$$\phi_F(x;\mu) = f(x) + \theta(x)^T c(x) + \frac{1}{2\mu}\|c(x)\|_2^2$$

where $\theta(x) = [\nabla c(x)\nabla c(x)^T]^{-1}\nabla c(x)\nabla f(x)$. From the above discussion, it becomes clear that the SPQ method can be implemented in a number of ways. A practical implementation of SQP uses line search and quasi-Newton approximation of the Hessian matrix. Below we describe the steps of a general SQP algorithm (Nocedal, 1999) for solving the nonlinearly constrained problem P.

SQP Algorithm

- Choose parameters $\eta \in (0, 1/2)$ and $\tau \in (0, 1)$
- Choose a starting iterate (x_0, λ_0)
- Let $B_0 \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix (initial Hessian)
- $k = 0$
- Compute $f_k = f(x_k)$, $\nabla f_k = \nabla f(x_k)$, $c_k = c(x_k)$, and $A_k = \nabla c(x_k)$
- For $k = 0, 1, \ldots, max_k$
  - If the KKT conditions are satisfied stop
  - Solve P_k to obtain a search direction p_k
  - Find step μ_k that renders p_k a descent direction for ϕ at x_k
  - $\alpha_k \leftarrow 1$
  - While $\phi(x_k + \alpha_k p_k; \mu_k) >$ $\phi(x_k; \mu_k) + \eta\alpha_k D\phi(x_k; p_k)$ do
    $\alpha_k \leftarrow \tau'\alpha_k$ for $\tau' \in (0, \tau)$
    end-while
  - $x_{k+1} = x_k + \alpha_k p_k$
  - Evaluate f_{k+1}, ∇f_{k+1}, c_{k+1}, A_{k+1}
  - Obtain λ_{k+1} by solving the linear system $[A_{k+1}A_{k+1}^T]\lambda_{k+1} = -A_{k+1}\nabla f_{k+1}$
  - Calculate $s_k = \alpha_k p_k$ and $y_k = \nabla_x\mathcal{L}(x_{k+1}, \lambda_{k+1}) - \nabla_x\mathcal{L}(x_k, \lambda_k)$
  - Obtain B_{k+1} using a quasi-Newton formula

  end-for

The operator $D\phi(x, p)$ corresponds to the directional derivative of function ϕ in the direction given by p. Algorithms to compute Hessian approximations are fully described in (Dennis, 1977). For the numerical analysis reported in this paper, the SQP algorithm followed the above main steps and was implemented using CFSQP (Lawrence, 1996).

Element	Value
Input voltage (V_i)	$48V$
Inductance (H)	$1.4 * 10^{-3}Hy$
Capacitor (C)	$10 * 10^{-6}F$
Resistance (min. value) (R_{min})	40Ω
Resistance (max. value) (R_{max})	120Ω
Output voltage (min. value) (V_{oMin})	12V
Output voltage (max. value) (V_{oMax})	$100V$
Switching frequency (T)	$50KHz$
Sampling Period (T_s)	$0.1mseg$

Table 1. BBC parameters.

4. SIMULATION RESULTS

The simulation tests were performed using a BBC model with the parameters given in table 1. The proposed control algorithm was implemented in C programming language, while the process simulation is executed in the language Ecosim (Ecosim Pro, 2004). For the simulations, the instantaneous model of the BBC is used, whose a dynamic behavior satisfactorily approximates the real plant (Borges, 2002).

The tuning of the horizons and λ is made in order to obtain a good compromise between performance and robustness in the wide range of operation of the BBC, even when high changes in the load are considered. Note that this are the only parameters that must be tuned in the controller.

The first test shows the step reference following properties of the closed loop system. Using nominal load ($R = 80\Omega$), the reference was changed from $0V$ to $12V$ at $t = 0$, from $12V$ to $100V$ at $t = 5ms$ and finally from $100V$ to $56V$ at $t = 10ms$. After that, maintaining the output voltage in its nominal value $56V$, step changes in the load were introduced. At $t = 15ms$ R changes from 80Ω to 40Ω and at $t = 20ms$ R changes from 40Ω to 120Ω. Figure 2 show the reference and the voltage output while figures 3 and 4 shows respectively the inductor current and the control action.

As can be seen in the figures the closed loop performance of the system presents and overshoot less than 10% and a settling time less than $2ms$ in all the simulated situations when changes in the set point of the BBC were introduced. The same dynamics properties can be observed in the disturbance rejection response when step changes in the load were considered.

5. CONCLUSIONS

This paper presented the application of a NMBPC to a BBC. The controller performs well both for set point changes and disturbance rejection. The

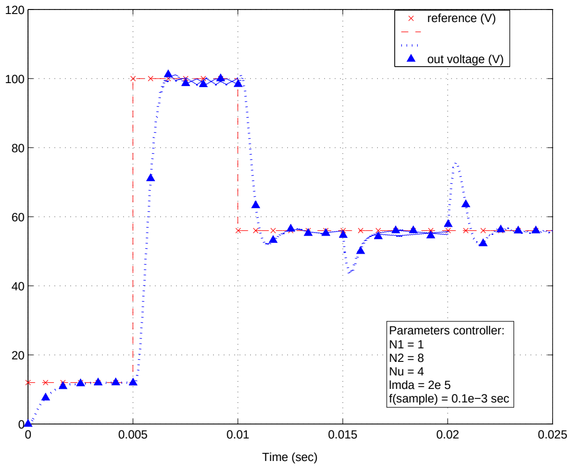

Fig. 2. Reference and output voltage response.

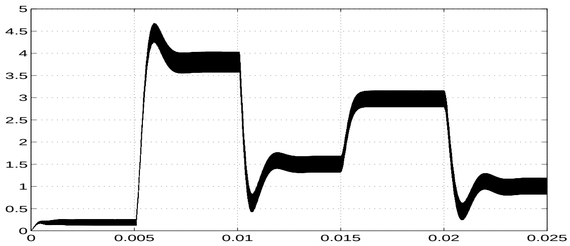

Fig. 3. Inductor current response.

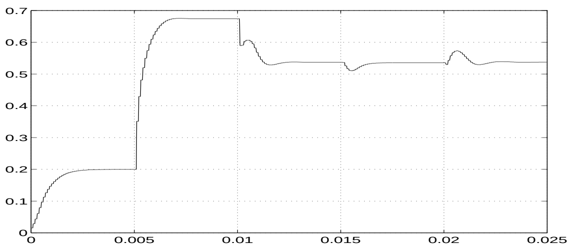

Fig. 4. Duty ratio.

major advantages of the use of a predictive approach are: (i) that the controller can be used with any BBC without modifications in the control structure, as only the parameters V_i, R, L and C must be modified in the algorithm when a new application is desired; (ii) the input and output constrains could be considered in the control law; (iii) a good compromise between robustness and performance could be obtained using an adequate tuning of the horizons and weighting factor.

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