OPTIMAL DECISIONS FOR LARGE SCALE SYSTEMS

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Abstract: This paper presents the software package SISCON, dedicated to the evaluation of optimal decisions for large scale systems. SISCON evaluates mathematical models nonlinear systems and computes the optimal decisions solving the mathematical non-linear programming problems. The LSS is a complex system structure and global approach computation can not be carried out. After decomposition of the large-scale problems, the corresponding sub-problems are solved using standard optimization techniques. SISCON is used in control and supervision strategies for complex systems, improving the support of real-time applications. An application on steel plant, focused on the combustion process optimization of pre-heating installations, was implemented.

Keywords: large scale systems, decomposition techniques, optimization methods, software package

1. INTRODUCTION

It is well known that optimization theory and mathematical nonlinear programming, have been developed by several authors among whom we mention for instance: Rosen, Fletcher, Reeves, Powell, Lasdon, Himmelblau. During the 1970's, many researchers have considered the optimization theory a good operating environment for the development of other domains such as system control or system identification. In the early 1980's, the work in this field was oriented towards the achievement of optimal decisions in control and supervision. In our days, this domain is still of special interest, as the optimization techniques are used by specialized software to compute optimal decisions for large scale systems control.

Large scale systems are supposed to be represented by subsystem collections respecting some given arrangements and some given interconnections. Each subsystem is described through a specific model. Interconnections between subsystems represent the constraints of these ones. The management of the global system is quite complicated and decomposition and organizing techniques are required. The original problem is transformed into an equivalent one in order to distribute and reduce the effort made in optimization computing (Himelbau 1974), (Ghaoui 1997).

The computed decision of the considered system is the solution of a standard linear or non-linear mathematical programming problem having the form:

$$opt\{F(x) = F(x_1, x_2, ..., x_N)\} \qquad (1)$$

$$g_i(x) = 0, \quad i = 1, ..., m$$

$$g_j(x) \leq 0, \quad j = 1, ..., s$$

where F is the criterion function, g_i, g_j are the constraints functions and $x_1, x_2, ..., x_N$ are sub-vectors.

The proposed SISCON software package reduces the computation complexity, allowing adequate calculus effort per each subsystem and can be used in many applications of supervision architectures.

SISCON determines the optimal solution x^* using numerical computation techniques selected by taking into account the characteristics of the criterion-functions and of the associated constraints (Serbanescu 1999).

2. DECOMPOSITION TECHNIQUES

The following types of decomposition techniques can be used depending on the global optimization problem characteristics (Lasdon 1975), (Roberts 2001), (Oshuga 1993):

a) decomposition for block-diagonal structure problems associated with weak coupling systems;

b) decomposition additively separable problems on criterion-functions and constraints;

c) relaxation and partitioning techniques;

- *The first category includes the linear case:*

$$\min_{x,y}(c^T x + c_0{}^T y) \tag{2}$$

with the coupling constraints:

$$Ax + D_0 y = b_0 \tag{3}$$

$$Bx + Dy = b \tag{4}$$

where: $x = [x_1 \mid x_2 \mid ... \mid x_N]$ is a set of sub-vectors x_i of dimension n_i and y is the coupling vector of the subsystems with $x_i \geq 0, i = 1,..., N, y \geq 0$; $c = [c_1 \mid c_2 \mid ... \mid c_N]$ is the set of corresponding coefficients, $A = [A_1 \mid A_2 \mid ... \mid A_N]$ is a set of $(m_0 \mathrm{x} n_i)$ dimension matrixes A_i, D_0 is a $(m_0 \mathrm{x} n_0)$ dimension matrix,

$$B = \begin{bmatrix} B_1 & 0 & 0 & 0...... & 0 \\ 0 & \mathrm{B}_2 & 0 & 0...... & 0 \\ \multicolumn{5}{c}{..................................} \\ 0 & 0 & 0 & 0....... & \mathrm{B}_\mathrm{N} \end{bmatrix}$$ is a block

diagonal matrix with B_i, $(m_i \mathrm{x} n_i)$ matrix, $D = [D_1 \mid D_2 \mid ... \mid D_N]$ where D_i are $(m_i \mathrm{x} n_0)$ matrixes and $b = [b_1 \mid b_2 \mid ... \mid b_N]$, with b_i vectors.

In the general case, the solution is given by Ritter's partitioning method and for the particular case $y = 0$, by Rosen's linear method.

The non-linear case:

$$\min_{x,y}(c^T x + f(y)) \tag{5}$$

with the coupling constraints: $Ax + D(y) \geq b$ and $x \geq 0$, where: $x = [x_1 \mid x_2 \mid ... \mid x_N]$, is a set of n_i - dimension sub-vectors x_i and y is the coupling vector of the subsystems, $x_i \geq 0, i = 1,..., N, y \geq 0$; $c = [c_1 \mid c_2 \mid ... \mid c_N]$ is the set of corresponding coefficients, $A = [A_1 \mid A_2 \mid ... \mid A_N]$ is the set of $(m_0 \mathrm{x} n_i)$ dimension matrices A_i, f is a scalar non linear function of y; $D \in R^m$ is a vector in y and S is an admissible subset in E^p, selected according to functional constraints.

- *The second category includes the following problems:*

$$\max_{(x,m)} F(x,m) \tag{6}$$

having the input-output constraints

$$z^i = G^i(m^i,\ x^i) \quad i = 1,\ 2,\ ...,\ N \tag{7}$$

and the coupling constraints,

$$x^i = H^i(z^1,\ z^2,\ ...,\ z^N) \tag{8}$$

The original problem is additively separable with respect to criterion-functions and constraints,

$$F(x,m) = \sum_{i=1}^{N} f^i(m^i, x^i) \tag{9}$$

where $f^i(m^i, x^i)$ are the criterion-functions of the subsystem S_i and each subsystem S_i is determined by the input-output $z^i = G^i(m^i,\ x^i),\ i = 1,2,...,N$ relations and by the interconnection constraints $x^i = H^i(z^1,\ z^2,\ ...,\ z^N)$.

Consequently, the associate Lagrangian-function of this problem is:

$$L = \sum_{i=1}^{N} f^i(m^i, x^i) + \sum_{i=1}^{N} (\mu^i)^T (G^i - z^i) + \sum_{i=1}^{N} (\rho^i)^T (H^i - x^i) \tag{10}$$

where μ^i, ρ^i stand for Lagrange multipliers.

The Lagrangian is additively decomposable and the global problem is transformed into N optimization sub problems as:

$$\max_{(x^i,m^i)} f^i(x^i, m^i) \tag{11}$$

$$z^i = G^i(m^i,\ x^i), \quad i = 1,2,...,\ N \tag{12}$$

$$x^i = H^i(z^1,\ z^2,\ ...,\ z^N) \tag{13}$$

keeping the previous constraints.

- *The third category includes* optimization problems with many constraints and/or many variables.

Partitioning is applied when the number of variables is important.

$$\begin{gathered} \max f(x) \\ g_i(x) \geq 0 \quad i = 1,...,m;\ x \in S \end{gathered} \tag{14}$$

f, g_i concave functions of n - dimensional x vector, S – subset of E^n; x –vector whose size is large enough to cause difficulties to occur during problem solving.

This procedure splits the variables of the problem into two subsets. At first, it acts on the variables belonging to one of the subsets and afterwards on variables in the other one.

Relaxation is used when many constraints are present. Let us consider the problem:

$$\begin{gathered} \max f(x) \\ g_i(x) \geq 0 \quad i = 1,...,m; \quad x \in S \end{gathered} \tag{15}$$

f, g_i concave functions of n-dimensional x vector, S – subset of E^n, $g_i(x) \geq 0$, constraints causing difficulties to occur during problem solving.

The relaxation procedure consists in the temporary ignoring some constraints and in solving the problem with the remaining ones. If the solution complies with relaxed constraints, then the solution is an optimal one. If not, one or several constraints are imposed and the procedure is reiterated.

Gradient optimization methods are used after the decomposition of large scale problems is carried out.

3. SEARCH DIRECTION FOR GRADIENT OPTIMIZATION TECHNIQUES

These techniques are used to solve the optimization problems in standard representation forms. The standard algorithm for all gradient optimization methods includes the following important steps:

- The choice of the input data (criterion function, starting point, stop conditions, additional initializations);
- The evaluation of the search direction d_k, in the optimization space;
- The computation of the optimal length step s_k;
- The computation of the recursive relation:
  $$x_{k+1} = x_k + s_k d_k;$$
- If the stop criterion is satisfied, the algorithm stops.

In constructing our software-tools, we tried a more efficient exploitation of the original approach concerning the computation of the search direction in gradient optimization methods (Serbanescu 1999). A new global approach for computing the search gradient direction d_k is proposed.

Let us consider $F(x)$, the criterion function to be minimized and let us define the distance between two points x_1 and x_2 in the Hilbert R^n space:

$$d(x_1, x_2)^2 = (x_1 - x_2)^T A(x_1 - x_2) \qquad (16)$$

where $A_{[n\times n]}$ is a positive definite metric-matrix, assuring $d(x_1, x_2) > 0,\ (\forall) x_1 \neq x_2$.

The geometrical locus of points $x \in R^n$ placed at a distance d from the fixed point x_k, is an ellipsoid having its center in x_k, described by the following equation:

$$d^2 = (x - x_k)^T A(x - x_k) = (\Delta x)^T A \Delta x \qquad (17)$$

where $\Delta x = x - x_k$

Now the shift from x_k to an arbitrary point of the ellipsoid, noted x, where the value of $F(x)$ is the lowest, will be determined. This point is $x = x_k + \Delta x_k$, which is in fact the following x_{k+1}, used in the standard recursive algorithm $x_{k+1} = x_k + \Delta x_k$. Then the optimization problem is solved:

$$\min_x \{F(x) \mid (\Delta x)^T A \Delta x = d^2\} \qquad (18)$$

If an approximation of $F(x)$ by a Taylor series around x_k is admitted and this series is cut down after its linear part, we obtain:

$$F(x) \cong F(x_k) + (\Delta x)^T \nabla F(x_k) \qquad (19)$$

and (18) is reduced to:

$$\min_x \{F(x_k) + (\Delta x)^T \nabla F(x_k) \mid (\Delta x)^T A \Delta x = d^2\} \qquad (20)$$

It can be observed that (20) is an optimization problem with constraints that can be solved using Lagrange multipliers.

The Lagrange function is now built:

$$L(x, \lambda) = F(x_k) + (\Delta x)^T \nabla F(x_k) + \lambda[d^2 - (\Delta x)^T A \Delta x] \qquad (21)$$

and we fix its stationary conditions:

$$\frac{\partial L}{\partial x} = \nabla F(x_k) - 2\lambda A \Delta x = 0 \qquad (22)$$

$$\frac{\partial L}{\partial \lambda} = (\Delta x)^T A \Delta x - d^2 = 0 \qquad (23)$$

From condition (22),

$$\Delta x = \frac{1}{2\lambda} A^{-1} \nabla F(x_k) \qquad (24)$$

is obtained allowing to write:

$$x_{k+1} = x_k + \frac{1}{2\lambda} A^{-1} \nabla F(x_k) \qquad (25)$$

This is the solution of the relation (18), i.e. the direction to follow in order to minimize $F(x)$.

This direction is:

$$d_k = A^{-1} \nabla F(x_k) \qquad (26)$$

and satisfies the descent condition:

$$\left\langle d_k, \nabla F(x_k) \right\rangle = \left\langle \nabla F(x_k)^T A^{-1}, \nabla F(x_k) \right\rangle < 0 \qquad (27)$$

with $A^{-1} > 0$.

To have an adequate choice of matrix A, one should take into account the already known gradient methods:

- for $A=I$, where I is the unit matrix, then $d_k = -\nabla F(x_k)$, i.e. first order gradient method (Cauchy);
- for $A^{-1} = \beta_k I$,

$$\text{with} \quad \beta_k = \frac{\nabla F(x_k)^T \nabla F(x_k)}{\nabla F(x_{k-1})^T \nabla F(x_{k-1})} \qquad (28)$$

then: $d_k = -\beta_k I \nabla F(x_k)$, i.e. conjugate gradient methods;

- for $A^{-1}=H_k$, with $H_k>0$, then $d_k = -H_k \nabla F(x_k)$, i.e. first order gradient methods with variable metrics;
- for $A^{-1} = \theta_k I$, then $d_k = -\theta_k I \nabla F(x_k)$ i.e. gradient projection methods;
- for $A^{-1} = Hess^{-1}(F(x_k))$,

then $d_k = -Hess^{-1}(F(x_k))\nabla F(x_k)$ i.e. second order gradient methods (Newton Raphson).

4. SISCON PACKAGE - PRESENTATION

The SISCON software is developed in such way as to enable algorithms to be traced step by step. The nucleus algorithms are implemented in dynamically linked libraries to allow them to be called from various modules (e.g. SIMPLEX and BOXE or gradient algorithms) and to have more rigorous control of applications. Algorithm implementation in dynamically linked libraries has also the advantage of better management of computer resources (e.g. stacks operation management in order to enable any kind of function to be taken over).

The algorithms in the library are put into classes and for each class there are procedures for initialization, control and erasing algorithms from computer storage. Since object oriented techniques are used, new algorithms can be developed on the basis of existing ones. The programs are written in C++ language.
The interface of the designed SISCON software package is flexible and allows data to be easily introduced.

For example, in case a) block-diagonal problems, Rosen's technique is applied, as it can be observed from the following figures:

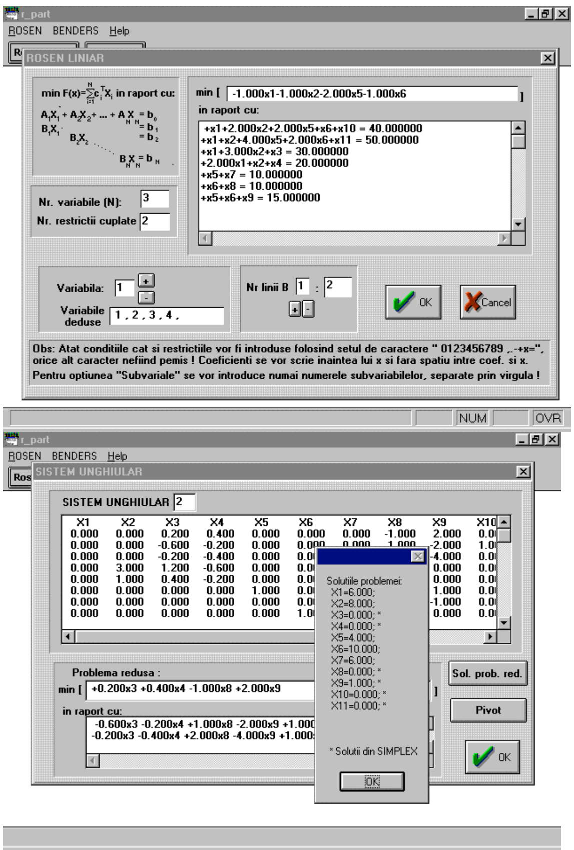

Fig.1. Example of software optimization procedure using Rosen's technique

The user defines the following objects:

- the number of blocks and the number of coupling constraints;
- the block diagonal system variables for each block;
- the number of constraints corresponding to each block.

The variables of the block diagonal structure are partitioned and then displayed. To carry out partitioning, initial basic arrays corresponding to each block are determined by solving the sub-problems min $c_i^T x_i$ with respect to $B_i x_i = b_i$.

The problem is reduced to the canonical form for the base variables in each block, by performing a set of pivot operations using these bases. Then the resulting angle system and the reduced problem are displayed.

The reduced problem is solved and the optimality test is performed. If the test fails, the initial angle system is pivoted again and a new angle system results. This new angle system as well as the new reduced problem is displayed.
The pivoting process is reiterated until the optimality test is met. Then the final solution is displayed.

For case b) the coordination by Lagrange multipliers, the number of subsystems, the criterion function for each subsystem, the maximum number of iterations and the stop condition are put in.

Also, for each subsystem the number of model constraints and of coupling constraints, the model and coupling constraints, the initial values of ρ multipliers and the number of implicit and explicit technological constraints are put in.

The same conditions are imposed for the other coordination procedures (Serbanescu 1999).

At a local level, optimum problems are solved using Box algorithm. The solutions are transmitted to the coordinating level where a new ρ vector is determined. The procedure is reiterated until the convergence criterion is met.

For case c) the use of Benders technique is suggested. The criterion function and the constraints are put in and the primal problem is transformed into a dual one which is displayed.

The primary problem is made linear by introducing the start vector y.
The dual problem is solved and the solutions are displayed.

The reduced problem is built and displayed. Then it is solved and the solutions are displayed.

The resulting solutions are tested and the procedure is reiterated in case the constraints are not met.

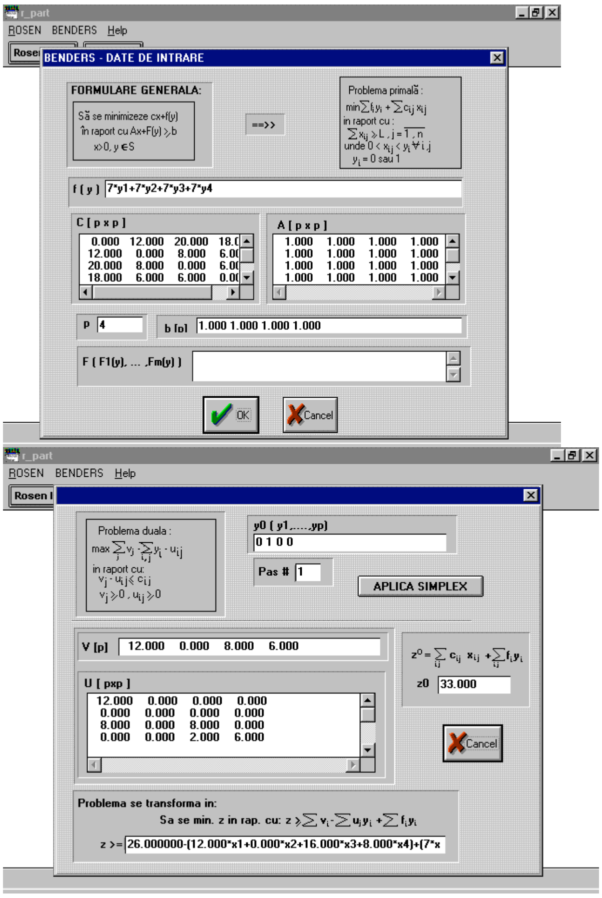

Fig.2. Example of software optimization procedure using Bender's technique

The criterion function can show any non-linearity, because the syntactic analyzer takes the string of characters and changes it into calls for C++ functions. The user interface allows the search procedure to be stored in a file and to be displayed for criterion function values as well as for the individual value of each variable in the criterion function.

Before solving the global optimization problem SISCON determines also the decision mathematical models of the systems. These may be linear or non-linear ones. Thanks to the syntactic analyzer, which reads and interprets the functions, the present program may handle almost any type of non-linearity. It gives also the possibility to select the input variables or to automatically generate combinations of input variables in order to find that combination determining the model which mostly approximates reality. The data can be introduced by text files or directly by keyboard.

The software package SISCON is dedicated to optimal decision problems working priority with gradient techniques. It can be integrated in a decision control strategy class of large scale industrial or economic systems. SISCON improves the decision support in real time applications of control and supervision.

Some recent attempts in implementing the present software in industrial processes in order to reduce manufacturing costs while increasing its performance have given satisfactory results.

5. STUDY CASE ON PRE-HEATING STEEL PLANTS

The SISCON software was successfully used for optimization of the combustion process at the pre-heating installations of steel furnace in ISPAT-SIDEX Galati, Romania.

ISPAT-SIDEX is an important steel plant in the Eastern Europe. A program of modernization was launched in order to improve the performances of feeding the plant's blast furnaces with hot air from the cowpers ensemble.
Some particularities of this heating process can be noticed.

- The large dimensions of the installation imply a plant model with large delays and distributed parameters, engaging important flow materials.
- The used fuel has many components: methane gas, coke gas and furnace gas, with different caloric powers. A convenient recipe must be calculated in order to feed the burners.
- The quality of the combustion gas and the process nonlinearities introduce important disturbances in exploitation. To evaluate the combustion process, the composition of the flue gases is analyzed; more precisely, the concentration of O_2 and CO are measured and computed.

Our major interest was to improve the cowper's efficiency using an adequate automation solution.

The work has been focused on two main directions:
- to construct a data acquisition and control system in order to maintain the installation in a nominal operating point;
- to optimize the burning process, important consumer of fuel gas.

The system was designed as a hierarchical structure, organized on two interconnected levels: data acquisition and control level and supervision level, respectively.
For the first level, the design methodology uses software resources, based on experimental identification techniques and on pole-placement methods to compute the control algorithms. To improve control systems performances, adaptive and robust mechanisms were used during the implementation phase (Popescu,1989).

The second hierarchical level evaluates the optimal decision for the combustion process, solving a parametric optimization problem and this work will be presented below.

The purpose of the decision level is to optimize the combustion process in restrictive technological conditions (Wismer, 1972; Roberts 2001).
First of all, a supervision model $z(\%O_2) = f(x_1, x_2)$ was evaluated, and after that, the constraints models: CO concentration $\hat{z}_1$, cowper cupola temperature $\hat{z}_2$ and flue gases temperature $\hat{z}_3$ depending on fuel flow x_1 and combustion air flow x_2 were calculated:

$$\begin{aligned} \hat{z}_1(\%CO) &= f_2(x_1, x_2) \\ \hat{z}_2(T_{cowper\,cupola}) &= f_3(x_1, x_2) \\ \hat{z}_3(T_{flow\,gas}) &= f_4(x_1, x_2) \end{aligned} \tag{29}$$

These models were computed using LS experimental identification method (Popescu 1989).
The procedure of data acquisition is accomplished during the first interval of cowper heating phase, on an imposed duration, with an acquisition rate of 20 seconds and a resolution of 256 observations.
For the usual data set, measured in real-time conditions, following non-linear models are estimated:

$$\begin{aligned} \hat{z} &= -9.665 + 0.229x_1 - 0.0009x_1^2 + 0.010x_2 \\ \hat{z}_1 &= 4282.875 - 21.566x_1 - 0.077x_1^2 - 21.500x_2 \\ \hat{z}_2 &= 1277.613 + 0.001x_1^2 - 0.387x_2 \\ \hat{z}_3 &= 499.161926 - 0.002147x_1^2 - 3.49945x_2 \end{aligned} \tag{30}$$

A parametric optimization problem was built:

$$\min \ \hat{z} = -9.665 + 0.229x_1 - 0.0009x_1^2 + 0.010x_2 \tag{31}$$

with the following restrictions:

$$\begin{aligned} 0 \le \hat{z}_1 &\le 450\,ppm \\ 0 \le \hat{z}_2 &\le 1300^o C \\ 0 \le \hat{z}_3 &\le 340^o C \\ 96.309 \le x_1 &\le 102.452 \\ 46.602 \le x_2 &\le 57.992 \end{aligned} \tag{32}$$

The solution obtained is the optimal operating point for the combustion process:

$x_1^* = 97469.85$ m^3/h – fuel flow

$x_2^* = 47804.16$ m^3/h –air combustion flow

for which it results a minimum value of O_2 concentration in flue gases

$$z_{\min}(\%O_2) = 4.65\% \tag{33}$$

Using this approach, the fuel gas consumption was reduced by 7.2%.
At the same time, corresponding values are obtained for

$$\begin{aligned} z_1(\%CO) &= 415.73\,ppm \\ z_2(T_{cupola}) &= 1273.25^o C \\ z_3(T_{flow\ gas}) &= 311.47^o C. \end{aligned} \tag{34}$$

The computed optimal point, meaning optimal decision (x_1^*, x_2^*), is automatically transferred as reference $(r_1^* = x_1^*, r_2^* = x_2^*)$ to the inferior control level, which has the task to bring the combustion process in this optimal exploitation point.
The decision level is implemented on the operator console of the hierarchical structure control.

6. CONCLUSIONS

SISCON is a software package dedicated to optimal decisions evaluation for large scale systems configurations. It is an efficient software package in solving the complex optimization problems, based on decomposition and partitioning techniques.

SISCON package is dedicated to control and supervision strategies for complex industrial and economic systems, improving the support for real-time control applications.

SISCON software was used optimization of combustion process at pre-heating installations of steel furnace. The system was designed as a hierarchical structure, organized on two interconnected levels, control and supervisor levels, and is implemented as a real time industrial application on the blast furnace in ISPAT – SIDEX Galati, Romania, respectively.
To improve control system performances the second hierarchical level evaluates the optimal decisions for the combustion process, solving a parametric optimization problem.

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