

TWO CONTROL LAWS FOR A SPRAY SYSTEM WITH TIME VARYING DELAY AND DEAD BAND

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Abstract: Two control laws, to control the liquid pressure of an agricultural spray boom are presented and compared. Characteristic for the process is a dead zone and a time varying delay. The first control law is rather intuitive and is based on direct dead zone compensation and a pole placement idea. The other control law originates from sliding mode control where the continuous part of the control contains some compensation dynamics, assuring stability. Both control laws are compared and tested in practice. *Copyright*© 2005 IFAC

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1. INTRODUCTION

In agriculture, spraying is one of the most important methods to distribute herbicides and liquid fertilizer to plants. Current practice is to distribute them as homogeneously as possible across the field to assure good biological efficiency. To compensate for irregularities in spray deposit due to speed variations, a control system adapts the flow rate to the speed. Recently, sensors to discriminate between weeds and plants have been developed (Vrindts *et al.*, 2002), (Tian *et al.*, 1999), (Wang *et al.*, 2001) in research labs allowing site-specific spraying *i.e.* spraying where the weeds are. This new method of spraying, puts new requirements to the spray hydraulics. Instead of spraying continuously, nozzles are opened and closed resulting into pressure variations. However, to assure

the right dose and correct droplet spectrum size, the pressure needs to be maintained as constant as possible. Very often, the dynamics of actual spray equipment may contain a variable time delay and a dead zone. The objective of this paper is to study control laws which can handle these phenomena.

Dead zones are often introduced by complex friction behaviour and deteriorate system performance. Several methods exist to handle friction in control systems. An overview is given in the work of Armstrong *et al.* (Armstrong-Hélouvy *et al.*, 1994) and Olsson *et al.* (Olsson *et al.*, 1998). They range from friction compensation based on accurate determined models, robust control methods like sliding mode e.g. (Hatipoglu and Özgüner, 1999), (Su *et al.*, 2000), (Young *et al.*, 1999) or adaptive algorithms, identifying on-line the friction e.g. (Ge and T.H. Lee, 2001), (Wang *et al.*, 2004).

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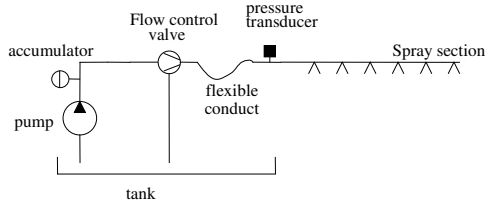


Fig. 1. Schematic representation of the pressure control system

About the study and control of systems with time delays, a good overview is given by Richard (Richard, 2003). Several methods exist to examine systems' stability with time varying delays. The Lyapunov-Razumikhin approach is known to be conservative (Cao *et al.*, 1998) but can deal with time varying delays without any restriction on the derivative of the delay. With the Lyapunov-Krasovskii method, information about the time derivative used to be known until the work of Fridman *et al.* (Fridman *et al.*, 2004) and Richard *et al.* (Richard *et al.*, 2001). As their results are promising, this approach is followed in the paper.

The organisation of the paper is as follows. First, a description of the system is provided and a model is presented. Two control laws are proposed and compared. Stability issues are discussed and finally, practical implementation results are shown.

2. SYSTEM DESCRIPTION

Figure (1) shows the lay out of the system. It is actually one section of an agricultural spray boom for application of herbicides and fertilizer to the plants. A pump, containing two pistons, operating in anti-phase, feeds the circuit. Pressure peaks, resulting from fast activation of valves or originating from the pulsating flow of the pump, are attenuated by the accumulator. The pressure at the nozzles is regulated by a flow control valve by adjusting the opening to the return. A long flexible conduct links the pressure control valve with the metal conduct, on which the nozzles are mounted. An electronic transducer measures the pressure at the entrance of the metal conduct. This is the pressure of interest which is measured and should be controlled. The system is secured by a check valve, limiting the pressure to 7bar.

In the laboratory, the total non-linear system has been modelled and validated. For controller

design, the following reduced model, which represents actually the slow dynamics, in the sense of singular perturbed systems (Khalil, 2002), is sufficient.

$$\begin{aligned} \dot{x} &= K_m f_d(u) \\ \sqrt{y} &= \frac{\alpha x(t-h) + \beta}{x(t-h) + \gamma} \quad 0 \leq h_{min} \leq h \leq h_{max} \end{aligned} \quad (1)$$

The only relevant state x is the angle of the valve, which is actuated by an electrical DC motor. A complex mechanism which originates from compliance of the motor struts, which is needed for assembly reasons, introduces the time varying delay. Therefore, the time delay changes by the change of direction of the electrical motor. Due to dry friction between the ball of the valve and the housing a dead zone $f_d(u)$ occurs.

$$\begin{aligned} f_d(u) &= u - c_0 \text{sign}(u) & \text{if} & \quad c_1 \leq |u|, \\ f_d(u) &= 0 & \text{if} & \quad c_1 > |u|, \\ & \text{and} & & \quad 0 \leq c_0 \leq c_1. \end{aligned} \quad (2)$$

The dead zone model $f_d(u)$ is clearly a simplification of the underlying friction behaviour.

3. STATE ESTIMATOR DESIGN

In order to stabilize the state equation in (1), the rotation angle x of the valve should be known. The valve doesn't contain a measurement system to determine the rotation angle, such that only the delayed state $x(t-h)$ is available from the output \sqrt{y} . Applying directly the delayed state $x(t-h)$ in a control law leads to limit cycle behaviour. Therefore, a state estimator is constructed.

$$\hat{\dot{x}}(t) = K_m f_d(u) + E(x(t-h) - \hat{x}(t-\hat{h})) \quad (3)$$

where \hat{x} is an estimate of the state x , \hat{h} an estimate of the delay h and E the Kalman gain. Actually, equation (3) can only have an interpretation of a Kalman filter if $h = \hat{h}$.

4. FIRST CONTROL LAW

Based on the prediction of the state \hat{x} , the control law is constructed:

$$u = -\frac{1}{K_m} (u_d \text{sign}(\hat{x}(t) - \hat{x}_d) + a(\hat{x}(t) - \hat{x}_d)) \quad (4)$$

Constant $\hat{x}_d = \frac{\beta - \gamma \sqrt{p_d}}{\sqrt{p_d} - \alpha}$ is the desired rotation angle, calculated back through the second part of

equation (1) by replacing y by the desired pressure p_d . Constants a and u_d are design parameters. Such a simple control law, with direct dead band compensation, is often found back in practical control systems. It is based on the idea of pole placement, in which a is the final pole location, when the dead zone is compensated perfectly and $c_1 = c_0$ equal u_d/K_m . However, in order to ensure that the dead zone is compensated, u_d/K_m should be larger than c_1 . Therefore the determination of the parameters of the observer and control law should be based on the theory of sliding modes. From equation (4) the sliding surface $s(t)$ can be derived:

$$s(t) = \hat{x}(t) - \hat{x}_d \quad (5)$$

In sliding regime, the observer (3) is in sliding mode and the dynamics of the reduced system is:

$$\dot{x} = -E(x(t-h) - \hat{x}(t-\hat{h})) \quad (6)$$

It is striking that during sliding, the dynamics of the system are governed by gain E of the predictor and not by gain a of the control law in equation (4). By the simple reasoning of perfect compensation and pole placement, a was faulty thought to determine the closed loop pole. In order to ensure a stable sliding mode, constant E should be determined such that equation (6) is stable. The state in equation (6) contains a time delay h , which changes by altering the direction of $x(t)$. In the worst case, h may change discontinuously such that proper selection of gain E is not straightforward. Next theorem provides a sufficient condition on E such that equation (6) is stable.

Theorem 1. Let $E \geq 0$, if :

$$\frac{\pi}{2h_{max}} > E, \quad (7)$$

the system in equation (6) is stable with bounded delay: $h \leq h_{max}$

Proof:

See (Anthonis *et al.*, 2004)

□

To formulate the stability condition on the design parameters of the observer and control law, two lemma's are required.

Lemma 1. Let the delay of the system and observer satisfying $h, \hat{h} \leq h_{max}$ and the size of the delay h only changes by the change of the direction of the angle x of the control valve. The sliding line $s(t)$ given by equation (5) is attractive

and reached in finite time for the control law in equation (4). However, once the system is on the sliding line, it is not ensured that it remains on it.

Proof:

See (Anthonis *et al.*, 2004)

□

Lemma 2. The system given by

$$\dot{q} = A_0q(t) + A_1q(t-h) + A_2q(t-\hat{h}) \quad (8)$$

with

$$A_0 = \begin{pmatrix} 0 & 0 \\ 0 & -a \end{pmatrix} \quad A_1 = \begin{pmatrix} -E & -E \\ E & E \end{pmatrix} \\ A_2 = \begin{pmatrix} 0 & E \\ 0 & -E \end{pmatrix}$$

with time varying delays h, \hat{h} and $h \leq h_{max}$ $\hat{h} \leq \hat{h}_{max}$ is stable if there exist 2×2 matrices $0 < P_1, P_2, P_3, Z_{1i}, Z_{2i}, Z_{3i}$ and $R_i, i = 1, 2$ that satisfy the following LMIs:

$$\Psi_1 < 0 \text{ and } \begin{pmatrix} R_i & (0 \ A_i) P \\ * & Z_i \end{pmatrix} \geq 0 \quad (9)$$

where

$$P = \begin{pmatrix} P_1 & 0 \\ P_2 & P_3 \end{pmatrix} \quad Z_i = \begin{pmatrix} Z_{1i} & Z_{2i} \\ * & Z_{3i} \end{pmatrix} \\ \Psi_1 = \Psi_0 + \sum_{i=1,2} \begin{pmatrix} 0 & 0 \\ 0 & h_{max} R_i \end{pmatrix} + h_i Z_i$$

$$h_1 = h_{max} \quad h_2 = \hat{h}_{max}$$

$$\Psi_0 = P^T \begin{pmatrix} 0 & I \\ A & -I \end{pmatrix} + \begin{pmatrix} 0 & I \\ A & -I \end{pmatrix}^T P$$

and

$$A = A_0 + A_1 + A_2$$

Proof:

The theorem is an extension of a theorem of Fridman (Fridman *et al.*, 2004).

□

Theorem 2. The control law (4) makes the surface $s(x) = 0$ attractive and reached in finite time. The equilibrium $x = \hat{x}_d$ is then globally asymptotically stable for $h \leq h_{max}$ $\hat{h} \leq \hat{h}_{max}$ provided that $E < \frac{\pi}{2h_{max}}$ and if there exist 2×2 matrices $0 < P_1, P_2, P_3, Z_{1i}, Z_{2i}, Z_{3i}$ and $R_i, i = 1, 2$ that satisfy the LMIs (9) and P_1 diagonal.

Proof:

First the system equations (1), (3) are transformed into regular form:

$$\begin{aligned}\dot{e}(t) &= -Ee(t-h) - Es(t-h) + Es(t-\hat{h}) \\ \dot{s}(t) &= Ee(t-h) + Es(t-h) - Es(t-\hat{h}) + \\ &\quad K_m f_d(u)\end{aligned}\quad (10)$$

With the control law (4) this results in:

$$\begin{aligned}\dot{q} &= A_0 q(t) + A_1 q(t-h) + A_2 q(t-\hat{h}) \\ &\quad - (u_d - K_m c_0) \text{sign}((0 \ 1) q(t))\end{aligned}\quad (11)$$

Similar to the proof of lemma 2 and (Fridman *et al.*, 2004), the system is transformed to descriptor form and the same Lyapunov-Krasovskii functional is selected to proof stability. After elaboration, this results in:

$$\dot{V}(t) = \dot{V}^*(t) - 2P_{11}(u_d - K_m c_0)|s(t)| \quad (12)$$

in which $\dot{V}^*(t)$ is the derivative of $V(t)$ in case equation (11) doesn't contain the *sign* term on the right hand side *i.e.*, the system of equation (8). P_{11} is the first row, first column element of P_1 . Provided the LMIs (9) are satisfied, $\dot{V}^*(t) < 0$ therefore $\dot{V}(t) < 0$. Moreover, the system is globally asymptotically stable, as $V^*(q_e, y_e) = 0$ (Fridman *et al.*, 2004), $V(q_e, y_e) = 0$, with q_e and y_e the equilibrium states and $V(q, y) \rightarrow \infty$ for $\|(q^T \ y^T)\| \rightarrow \infty$. Furthermore, the switching line is reached in finite time by the stability of the exponential stability of the system and theorem 1.

On the switching line $s(x) = \dot{s}(x) = 0$ such that, given the regular form equations (10) the reduced system is governed by the following equation:

$$\dot{e}(t) = -Ee(t-h) \quad (13)$$

which has been proved stable with the requirement (7) by theorem 1.

□

5. SECOND CONTROL LAW

The first control law (4) was rather intuitive. The second control law relies on the same switching line (5) but is synthesised, based on design methods, used in the theory of the sliding modes which often employ the regular form as in equation (10).

The dead zone $f_d(u)$ can be considered as the input $u(t)$ and a disturbance $d(t)$ which leads to:

$$\dot{s}(t) = Ee(t-h) + K_m u(t) + K_m d(t) \quad (14)$$

From equation (2), it is easy to see that $d(t)$ is bounded:

$$|d(t)| < c_1 \quad (15)$$

The control law is inspired on the work of Richard (Richard *et al.*, 2001), who proposed variable structure control laws for linear time invariant systems with time varying delay and bounded disturbance.

$$\begin{aligned}u(t) &= -\frac{1}{K_m} \{ Ee(t-h) + Es(t-h) - Es(t-\hat{h}) + \\ &\quad u_d \text{sign}(s(t)) + as(t) \}\end{aligned}\quad (16)$$

where constants u_d and a are defined as

$$u_d = m_1 + K_m c_1 \quad (17)$$

and constants

$$a, m_1 > 0 \quad (18)$$

Control law (16) is equivalent to:

$$\begin{aligned}u(t) &= -\frac{1}{K_m} \left\{ E \left(x(t-h) - \hat{x}(t-\hat{h}) \right) + u_d \times \right. \\ &\quad \left. \text{sign}(s(t)) + as(t) \right\}\end{aligned}\quad (19)$$

This control law is apart from the term $E(x(t-h) - \hat{x}(t-\hat{h}))$ exactly the same as the first control law (4) and can easily be implemented as $x(t-h)$ is, through \sqrt{y} , readily available from the measurement. In the first control law (4), the discontinuous term is selected such that $|u(t)| \geq c_1$, in order to assure that whenever a control action is desired, it really reaches the system and is not blocked by the dead zone. Note that this requirement is not necessarily fulfilled in the second control law (16). Nevertheless, asymptotic stability is achieved.

Theorem 3. A) The control law (16) makes the surface (5) attractive and reached in finite time for the state equations (1), (3).

B) The equilibrium $x_1 = \hat{x}_1 = \hat{x}_d$ is then globally asymptotically stable for $h_{min} \leq h \leq h_{max}$, bounded delay estimation \hat{h} and for E satisfying inequality (7).

Proof:

For part A), the following Lyapunov function is selected:

$$V(t) = s(t)s(t) \quad (20)$$

Its derivative along the solution of (1), (3) is:

$$\dot{V}(t) = s(t) \left(Ee(t-h) + Es(t-h) - Es(t-\hat{h}) + K_m u(t) + K_m d(t) \right) \quad (21)$$

Inserting control law (15) renders:

$$\dot{V} < -2m_1\sqrt{V} - 2aV < -2m_1\sqrt{V} \quad (22)$$

which proves that $s(t) = 0$ is a sliding surface, reached in finite time.

The proof of part B of the theorem relies on the attraction of the sliding line and by the fact that equation (22) is always valid such that once $s(t) = \dot{s}(t) = 0$, by the bounded delays h and \hat{h} , also $s(t-h)$ and $s(t-\hat{h})$ evolve in finite time to zero. Therefore in sliding mode, the reduced system corresponds to equation (13), of which the stability has already been proved by theorem 1.

□

The stability conditions of the second control law (16) are less restrictive than those of the first control law (4). However for both laws, the conditions are sufficient and it is not clear how conservative they are.

6. IMPLEMENTATION

The observer and the two controllers are programmed on a digital controller (ADWIN Gold, Jaeger GMBh). As a discretisation rule, the zero order hold transformation was utilized. In order to approximate the behaviour of a continuous controller, a rather high sampling frequency of 1000Hz has been selected. It ensures also that the Shannon principle is not violated. The two piston membrane pump generates harmonics of 23Hz.

Table 1. System parameters and parameters of the control law.

K_m ($^\circ/(\text{Vs})$)	α ($\sqrt{Pa}/^\circ$)	β (\sqrt{Pa})
1.71	1.59×10^3	-1.04×10^5
γ ($^\circ$)	c_0 (V)	u_d (V)
-53.9	2.72	4

All relevant parameters of the system are listed in table 1. As stated in the introduction, the delay changes by changing the direction of rotation of the valve. However, there is no fixed value of the delay for a certain direction. After some experiments, a maximum delay of 0.2s and a minimum

delay of 0.1s has been observed. The estimation of the time delay \hat{h} is selected fixed and equal to the average of the maximum and minimum delay: 0.15s. Parameter $a = 5.85$ is determined as high as possible, avoiding input saturation. Applying theorem 1 results in a maximum E , assuring stability, of 7.85. With respect to stability of the first control law, the LMIs of theorem 2 need also to be verified allowing a maximum E of about 5.6. By decreasing \hat{h} , the maximum value of E increases. In case \hat{h} is put equal to zero, E can be enlarged up to 7. However, experiments on the real set-up show a better performance and a more stable behaviour with $\hat{h} = 0.15$ instead of $\hat{h} = 0$. With $\hat{h} = 0.15$, gain E could be increased more before obtaining unstable behaviour. Simulations in Matlab confirm this. The simulations are based on the system model (1) and a delay h which can only take two distinct values (0.1s and 0.2s). This seeming contradiction between theory and practice can be explained by the fact that theorem 2 just considers the maximum delay values of h and \hat{h} . In practice, there is an important relation between these two *i.e.* the one is the prediction of the other. Therefore the class of systems considered by theorem 2 is much larger than required such that conservative results are obtained. For the second controller (16), decreasing the delay estimate \hat{h} to zero gives an increase in performance, without affecting stability. A possible explanation for this is that the stability of this control law is independent of the size of \hat{h} . Only a bounded \hat{h} is required. The delay estimation \hat{h} affects the term $s(t-\hat{h})$ in the control law, reflecting the distance to the sliding line in the past. Therefore, it is clear that for performance \hat{h} is the best as small as possible.

The two control laws are compared in figures 2 and 3 for $E = 7$ and $\hat{h} = 0.15$ respectively $\hat{h} = 0$. The oscillations on the figure, are due to the piston pump. Based on the figures, both controllers show the same performance. In practice, the first controller (4) seems to react more accurate than the second (16). For some set-point pressures, it shows some off-set and on small set-point jumps ($< 0.2\text{bar}$) it doesn't react.

To investigate stability, gain E is increased until unstable behaviour is obtained. In case of the first controller (4), taking E larger than 8 showed sometimes strange behaviour and after 11, the pressure went to its maximum value. For the second controller (16), even after increasing E to 30, apart from decaying oscillatory behaviour, no instability was observed. This can only be explained by the fact that the error $e(t-h)$ needs to be zero before reaching the switching line.

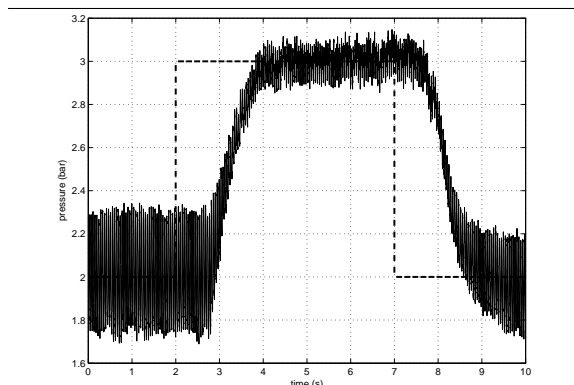


Fig. 2. Measured (full line) and desired (dashed line) pressure for the first controller (4)

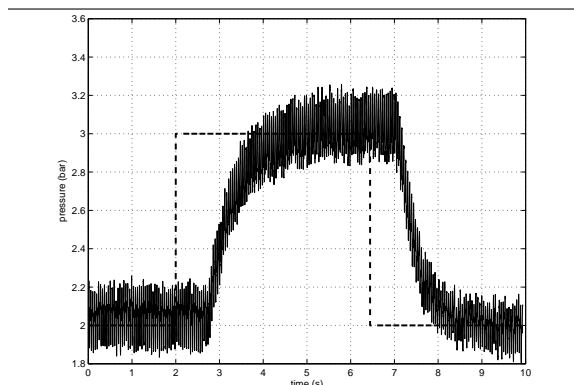


Fig. 3. Measured (full line) and desired (dashed line) pressure for the first controller (16)

7. CONCLUSIONS

Two control laws, that can cope with dead zones and time varying delays, to stabilize the pressure in a hydraulic circuit for spraying pesticides have been proposed. The first control law ensures that the control signal is always larger than the dead zone. It is intuitive but following this intuition may lead to bad design. Despite its intuitive character, it is more difficult to prove the stability. The second control law doesn't guarantee a control output larger than the dead zone but by its construction, asymptotic stability is acquired. In practice, the first control law shows a slightly better performance but the second control law is more stable. With respect to the delay estimation, requires the first control law a good estimate to guarantee a good performance and stable behaviour. In case of the second control law, the delay estimate should be just considered as a parameter and put to zero.

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