PHYSICAL SWITCHING SYSTEMS: HYBRID INCIDENCE MATRICES FOR STRUCTURED MODELLING AND ANALYSIS

C. Valentin, M. Magos, B. Maschke

LAGEP, UMR CNRS 5007, Université Claude Bernard Lyon 1, bat. 308 G, ESCPE, 43, Bd du 11 Novembre 1918, 69622 Villeurbanne cedex, France.
valentin@lagep.univ-lyon1.fr, Tel: (33)4 72 43 18 66, Fax: (33)4 72 43 18 99

Abstract: This paper extends a systematic method to analyse the set of admissible configurations for a wide class of physical switching systems (PSS). This method is based on a mathematical representation of their dynamic network graph and of its dual graph, using the hybrid incidence matrix. The descriptor system associated with the set of PSS' configurations, parameterized by the discrete state of the switches, is deduced. By analysing also sets of constrained configurations, a further step to a generic control synthesis for physical switching systems is achieved.

Keywords: hybrid dynamic systems, modelling, analysis, dynamic network graph, hybrid parameterized incidence matrix, admissible configurations, constrained configurations.

1. INTRODUCTION

In a physical switching system (PSS) the topology may change instantaneously depending on some discrete parameters (Van der Schaft, and Schumacher, 2000; Zaytoon, *et al.*, 2001). The goal of this paper is to extend a systematic structured method to model and analyse the admissible and constrained configurations of a wide class of physical switching systems (PSS) from a model based on mathematical representations of network graphs. The algebro-differential equations associated with the set of PSS' configurations are deduced from the parameterized incidence matrices and they are written as a non minimal parameterized implicit port Hamiltonian formulation using a kernel representation. The modeling method does not include the conditions of switching. It leads to a family of hybrid (parameterized) incidence matrices representing a primal dynamic network graph and its dual graph, associated with the PSS. This paper is the continuation of (Magos, *et al.*, 2004-b) and (Valentin, *et al.*, 2004) which presented how to obtain these matrices. (Magos, *et al.*, 2004-a) presented how to express the flow constraint representation and to carry out the analysis of the admissible configurations related to effort sources. After some brief recalls, this paper presents how to perform the analysis of the admissible configurations related to flow sources and the constrained configurations.

This approach, based on an energetic view, is related to other works on linear switched systems (Gerritsen, *et al.*, 2002), hybrid Hamiltonian systems for electrical circuits (Jeltsema, *et al.*, 2001), mechanical systems (Haddad, *et al.*, 2003) or different power converters (Escobar, *et al.*, 1999), and hybrid models based on bond graphs where the switches are modeled by effort or flow sources (Buisson, 1993; Cormerais, *et al.*, 2002) or by nonlinear resistors (Dauphin-Tanguy, *et al.*, 1989).

The method presented in this paper is illustrated on electrical power converters. One may notice that they could be seen as the equivalent physical systems from a different field by energetic analogy. Indeed, it is important to point out that some mechanical systems or hydraulical systems have an equivalent network representation. Therefore, they may be represented as circuits or more generally modelled by bond graphs (Paynter, 1961; Karnopp, *et al.*, 1990).

Paragraph 2 recalls the definitions of a dynamic network graph and of a hybrid parameterized incidence matrix. Paragraph 3 defines the non-admissible configurations of a physical switching system (PSS) and paragraph 4 the PSS' constrained configurations. Paragraph 5 deals with the application on a power converter with a flow source. The last paragraph presents a non minimal hybrid parameterized implicit port Hamiltonian representation.

2. DYNAMIC NETWORK GRAPHS MATHEMATICAL REPRESENTATION

Network graphs (Recski, 1989) have been used to model physical switching systems in different domains (DeMarco, 2001; Frigioni, and Italiano, 2000). From these references, we got the idea to see the switches like ideal elements whose function is to change the interconnection of the functional elements. Then, switching is considered as a graph transformation in a dynamic network graph context (Magos, *et al.*, 2004-a, 2004-b; Valentin, *et al.*, 2004). In a PSS, when a switch is closed, the edges connected to its starting vertex are disconnected and reconnected to its ending vertex. Then, its starting vertex is isolated.

Definition 1: *A dynamic network graph $G_w = (V, E, E_w)$ consists of an oriented graph where:*
** V is a nonempty finite set of n_v vertices ($v_x \in V$),*
** E is a nonempty finite set of n_e pairs of elements of V called edges ($e_i \in E$ / $e_i = (v_x , v_y)$, v_x being the starting vertex and v_y being the ending vertex). The port of a functional element is associated with every of the n_e oriented edges of this graph.*
** E_w is a nonempty finite set of n_s pairs of elements of V called virtual edges ($e_{wi} \in E_w$ / $e_{wi} = (v_x , v_y)$, $(v_x,v_y) \in V^2$). The port of a switching element is associated with every of the n_s oriented virtual edges of this graph.*

Each edge is associated with a dipole which is linear or not and then, with an effort variable and a flow variable. Edges orientation is a convention: usually it is flow orientation. If $v_y = v_x$, the edge is a self-loop. For electrical circuits the functional elements might be inductors, resistors, capacitors for mechanical systems, masses, frictions, springs, ...

The most suitable mathematical representation of the dynamic network graph for a systematic modelling and analysis of the system structure, especially if it is varying, is the incidence matrix. A physical switching system is a multiconfiguration system which is mathematically represented by a family of models. The hybrid (parameterized) incidence matrix defined in (Magos, *et al.*, 2004-a) gives the geometric interconnection structures of all these configurations in a single matrix parameterized by the discrete state of the switches. For each switch Sw_k, a discrete variable $w_k \in \{0, 1\}$ is defined, such that: $w_k=1$ if the switch is closed and $w_k=0$ if the switch is open. Thus the discrete state of the model is given by: $W = [w_1, w_2, ..., w_{ns}]^T$.

Let recall here the definition of a hybrid incidence matrix, following (Magos, *et al.*, 2004-a, 2004-b; Valentin, *et al.*, 2004).

Definition 2: *Let us consider a dynamic oriented network graph G_w.*
** The term $\left(I_{n_v} + M_{Tk}(G_w)(w_k)\right)$ represents the transformations of the geometric interconnections between the functional elements produced in closing the switch Sw_k. $M_{Tk}(G_w)(w_k)$ is defined by:*

$$M_{Tk}(G_w)(w_k)_{i,j} = \begin{cases} w_k & \text{if} \quad e_{wk} = (v_j, v_i) \\ -w_k & \text{if} \quad i = j \text{ and } e_{wk} = (v_j, .) \\ 0 & \text{Otherwise} \end{cases}$$

$$\text{for } (i, j) \in \{1,..,n_v\}^2.$$

** The transformation matrix, $M_T(G_w)(W)$, depends on W and represents the graph transformation from the reference configuration G_r (network graph without the virtual edges i.e. with all switches open) to another configuration given by the discrete state of the n_s switches, W. $M_T(G_w)(W)$ is the following ordered product (from $k=1$ to $k=n_s$):*

$$M_T(G_w)(W) = \prod_{k=1}^{n_s} \left(I_{n_v} + M_{Tk}(G_w)(w_k)\right) \qquad (1)$$

$$= \left(I_{nv} + M_{T1}(G_w)(w_1)\right) \ \dots \ \left(I_{nv} + M_{Tns}(G_w)(w_{ns})\right)$$

** The hybrid parameterized incidence matrix $IM(G_w)(W)$ of the 2^{ns} PSS' configurations, is given by $IM(G_w)(W) = M_T(G_w)(W) \cdot IM(G_r)$, $W \in \{0, 1\}^{ns}$ (2)*

Each row of the hybrid incidence matrix gives the edges connected to the corresponding vertex and each column gives the two vertices connected to an edge associated with a functional element.

Proposition 1: *The hybrid parameterized incidence matrix is correct as calculated in definition 2 if the oriented dynamic network graph associated with the PSS satisfies the following two assumptions:*

Assumption 1. The outdegree of each vertex of the subgraph composed of all virtual edges with their incident vertices is below or equal to 1.

Assumption 2. All virtual edges are indexed such that an oriented sequence of virtual edges is decreasing.

Proposition 1 defines the class of physical switching systems (PSS) studied in this paper. The hybrid incidence matrices of a primal dynamic network graph and its dual graph point out several interesting features of the corresponding PSS: functional elements' short-circuits and/or open-circuits and devices connected in parallel or in series.

It is also interesting to notice that the edges' orientation in the network graphs is not unique and predefined by the structure or the logic of the system as in Petri nets based models for example, and that a PSS may be modelled by a family of hybrid incidence matrices depending on the edges' orientation of its network graphs.

To illustrate these definitions, let now give the hybrid incidence matrices of the dynamic network graph and its dynamic dual graph (figure 2) associated with the simplified Buck converter schemed figure 1.

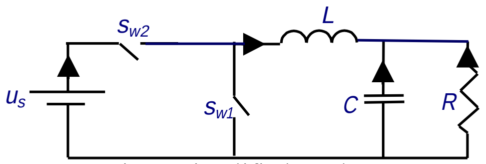

Fig. 1. simplified Buck converter

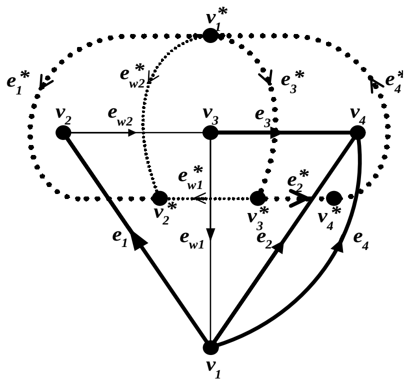

Fig. 2. the primal and dual dynamic network graphs of the Buck converter: G_{wa} and G_{wa}^*

The primal network graph G_{wa} is represented with full lines and virtual edges with thin lines, and the dual network graph G_{wa}^* is represented with dotted lines and dual virtual edges with thin dotted lines. Their orientation is chosen in accordance with proposition 1. It is not unique.

$$IM(G_{wa})(W) = \begin{bmatrix} 1-w_1w_2 & 1 & w_1 & 1 \\ -1+w_2 & 0 & 0 & 0 \\ w_2(w_1-1) & 0 & 1-w_1 & 0 \\ 0 & -1 & -1 & -1 \end{bmatrix} \quad (3)$$

$$IM(G_{wa}^*)(W) = \begin{bmatrix} w_2 & 0 & w_2 & -w_2 \\ -w_2 & 1-w_1 & w_1-w_2 & -(1-w_2) \\ 0 & w_1 & -w_1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad (4)$$

with $(w_1, w_2) \in \{0, 1\}^2$.

3. ADMISSIBLE CONFIGURATIONS ANALYSIS

Some states of the switches, may lead to non-admissible configurations of the physical system with variable topology. Indeed, a problem arises if there exist conflicts between generalized Kirchhoff's laws and independent non zero effort or flow sources. Thus, it is of prime importance to remove them for the control synthesis procedure.

Proposition 2*: a non-admissible configuration corresponds to:*

i) An effort source in short-circuit or several independent effort sources connected in parallel.

ii) A flow source connected in an open-circuit or several independent flow sources connected in series.

A main advantage of the incidence matrix $IM(G_w)(W)$ is that its direct analysis gives all the admissible configurations dealing with effort sources, because it leads to the generalized Kirchhoff's flow laws. Non-admissible configurations dealing with flow sources may be deduced from the dual dynamic network graph and then the dual hybrid incidence matrix defined in (Magos, *et al.*, 2004-b; Valentin, *et al.*, 2004) which lead to the generalized Kirchhoff's effort laws. If a flow source is connected in an open-circuit, its dual edge shows a short-circuit. If several independent flow sources are connected in series, their dual edges are connected in parallel.

Then, to allow a systematic analysis of the hybrid incidence matrices, the edges in the reference network graph G_r are indexed in gathering the elements of each type together. The following edges indexation is arbitrarily chosen: effort sources, capacitors, inductors, dissipative elements and flow sources. Thus, the characterization of non-admissible configurations is:

Definition 3: *If the physical switching system Σ_w, the geometric structure of which is modelled by hybrid incidence matrices $IM(G_w)(W)$ and $IM(G_w^*)(W)$, includes n_{es} effort sources and n_{fs} flow sources, a non-admissible configuration defined by a vector $W \in \{0, 1\}^{ns}$ fulfills one of the four following conditions:*

1) If $n_{es} \neq 0$, $\exists\ j \in \{1, \ldots, n_{es}\}$ / $IM_{\bullet j}(G_w)(W) = 0$.

2) If $n_{es} > 1$, $\exists\ (i, j) \in \{1, \ldots, n_{es}\}^2$ with $i \neq j$ /

$$|IM_{\bullet j}(G_w)(W)| = |IM_{\bullet i}(G_w)(W)|.$$

3) If $n_{fs} \neq 0$, $\exists\ j \in \{n_e - n_{fs}+1, \ldots, n_e\}$ /

$$IM_{\bullet j}(G_w^*)(W) = 0.$$

4) If $n_{fs} > 1$, $\exists\ (i, j) \in \{n_e - n_{fs}+1, \ldots, n_e\}^2$ with $i \neq j$ /

$$|IM_{\bullet j}(G_w^*)(W)| = |IM_{\bullet i}(G_w^*)(W)|.$$

$|V|$ represents the vector where each component is the absolute value of each component of V. Condition 1) detects effort sources in short-circuit, condition 2) effort sources connected in parallel, condition 3) flow sources connected in open circuit and condition 4) flow sources connected in series in the configuration characterized by W. The set of admissible configurations (according to proposition 2) of the physical switching system Σ_w is denoted $A(\Sigma_w)$. Note that: $A(\Sigma_w) \subset \{0, 1\}^{ns}$.

Let now consider the simplified Buck converter presented figure 1. Its primal and dual dynamic network graphs G_{wa} and G_{wa}^*, given figure 2, respect the index order of the functional elements proposed in this section and there is only one effort source. This source is represented by the first column in $IM(G_{wa})(W)$ (eq. (3)). And:

$$IM(G_{wa})(W)_{\bullet 1} = 0 \Leftrightarrow \begin{bmatrix} 1-w_1 w_2 \\ -1+w_2 \\ w_2(w_1-1) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Then, the only non-admissible configuration among the four possible is: $W_a=[1\ 1]^T$. Therefore, as there are no flow sources in the circuit:

$$A(\Sigma_{wa}) = \{0, 1\}^2 - \{[1\ 1]^T\}.$$

4. CONSTRAINED CONFIGURATIONS ANALYSIS

One may notice that some configurations of a physical switching system lead to implicit state space representations with dynamic modes and algebraic modes which introduce constraints. As a consequence, state variables' jumps may happen when entering into these configurations at a time when the constraint is not satisfied. Then, it is interesting to be aware of these constrained configurations for the control synthesis procedure.

Proposition 3: *a constrained configuration corresponds to inductors connected in series or in open-circuit or capacitors connected in parallel or in short-circuit.*

Definition 4: *If the physical switching system, Σ_w, includes n_{es} effort sources, n_c capacitors and n_L inductors, a constrained configuration is defined by vector $W \in \{0, 1\}^{ns}$ if one of the four following conditions is satisfied:*

1) *If $n_c \neq 0, \exists\ j \in \{n_{es}+1, \ldots, n_{es}+n_c\}$ /*

$$IM_{\bullet j}(G_w)(W) = 0\ .$$

2) *If $n_c>1$, $\exists\ (i, j) \in \{n_{es}+1, \ldots, n_{es}+n_c\}^2$ with $i \neq j$ /*

$$|IM_{\bullet j}(G_w)(W)| = |IM_{\bullet i}(G_w)(W)|.$$

3) *If $n_L \neq 0, \exists\ j \in \{n_{es}+n_c+1, \ldots, n_{es}+n_c+n_L\}$ /*

$$IM_{\bullet j}(G_w^*)(W) = 0.$$

4) *If $n_L>1$, $\exists\ (i, j) \in \{n_{es}+n_c+1, \ldots, n_{es}+n_c+n_L\}^2$ with $i \neq j$ / $|IM_{\bullet j}(G_w^*)(W)| = |IM_{\bullet i}(G_w^*)(W)|$.*

Condition 1) detects capacitors in short-circuit, condition 2) capacitors connected in parallel condition 3) inductors connected in open-circuit and condition 4) inductors connected in series in the configuration characterized by W. The set of constrained configurations of the physical switching system Σ_w is denoted $C(\Sigma_w)$. A constrained configuration is admissible, then $C(\Sigma_w) \subset A(\Sigma_w)$.

Let for example study all the cases of potential jumps in the simplified Buck converter. If the only inductor is in open-circuit, the third column in $IM(G_{wa}^*)(W)$ (eq. (4)) satisfies the following equations:

$$IM(G_{wa}^*)(W)_{\bullet 3} = 0 \Leftrightarrow \begin{bmatrix} w_2 \\ w_1-w_2 \\ -w_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

If the only capacitor is in short-circuit, the second column in $IM(G_{wa})(W)$ satisfies the following equations:

$$IM(G_{wa})(W)_{\bullet 2} = 0 \Leftrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow no\ solution$$

Then, only one configuration among the three admissible is constrained. Therefore:

$$C(\Sigma_{wa}) = A(\Sigma_{wa}) - \{[0\ 0]^T\}.$$

5. APPLICATION TO ANOTHER CONVERTER

The electrical power converter outlined figure 3 controls the power provided to the load by the current source through the control of the switches Sw_i.

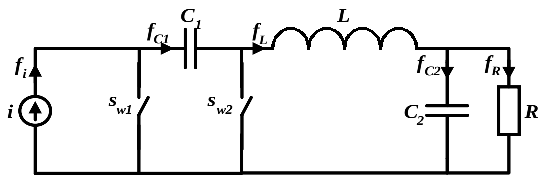

Fig. 3. simplified power converter

The hybrid incidence matrices of this simplified power converter are:

$$IM(G_{wb})(W) = \begin{bmatrix} w_1-w_2 & -1 & w_2 & -1 & 1-w_1 \\ 1-w_1 & 0 & 0 & 0 & -1+w_1 \\ -1+w_2 & 0 & 1-w_2 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{bmatrix} \quad (5)$$

$$IM(G_{wb}^*)(W) = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ -1+w_1 & -(1-w_1)(1-w_2) & -(1-w_1)(1-w_2) & 0 & -1 \\ -w_1 & -w_1(1-w_2) & -w_1(1-w_2) & 0 & 0 \\ 0 & -w_2 & -w_2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \quad (6)$$

with $(w_1, w_2) \in \{0, 1\}^2$.

The only flow source is represented by $IM(G_{wb}^*)(W)$ last column (eq. (6)). And $IM_{\bullet 5}(G_{wb}^*)(W) \neq 0$, for all W, then the four possible configurations are admissible in this power converter (system Σ_{wb}). Therefore, $A(\Sigma_{wb}) = \{0, 1\}^2$.

Let study all the constrained configurations of this power converter. If the only inductor is in open-circuit, the third column in $IM(G_{wb}^*)(W)$ (eq. (6)) satisfies the following equations:

$$IM(G_{wb}^*)(W)_{\bullet 3} = 0 \Leftrightarrow$$
$$\begin{bmatrix} 1 \\ -(1-w_1)(1-w_2) \\ -w_1(1-w_2) \\ -w_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \text{no solution}$$

If one of the two capacitors is in short-circuit or if they are in parallel, the first or second columns in $IM(G_{wb})(W)$ satisfy the three equations' sets:

$$IM(G_{wb})(W)_{\bullet 1} = 0 \Leftrightarrow \begin{bmatrix} w_1-w_2 & 1-w_1 & -1+w_2 & 0 \end{bmatrix}^T = 0_{4\times 1}$$
$$\Rightarrow \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$$

$$IM(G_{wb})(W)_{\bullet 2} = 0 \Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix}^T = 0_{4\times 1} \Rightarrow \text{no solution}$$

$$\left|IM(G_{wb})(W)_{\bullet 1}\right| = \left|IM(G_{wb})(W)_{\bullet 2}\right| \Leftrightarrow$$
$$\begin{bmatrix} w_1-w_2 & 1-w_1 & -1+w_2 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix}^T$$
$$\Rightarrow \text{no solution}$$

Then, only one configuration among the four admissible is constrained. Therefore:

$$C(\Sigma_{wb}) = A(\Sigma_{wb}) - \{[1\ 1]^T\} = \{0, 1\}^2 - \{[1\ 1]^T\}.$$

6. NON MINIMAL PARAMETERIZED PORT HAMILTONIAN FORMULATION

A non minimal implicit parameterized port Hamiltonian formulation of a PSS can be deduced from the hybrid incidence matrices of their primal and dual dynamic network graphs (Magos, *et al.*, 2004-b). The so-called kernel representation (Dalsmo and Van der Schaft, 1998) has been extended to admissible configurations of a physical switching system Σ_w as follows:

$$\begin{bmatrix} IM(G_w)(W) \\ 0 \end{bmatrix} \underline{pf} + \begin{bmatrix} 0 \\ IM(G_w^*)(W) \end{bmatrix} \underline{pe} = 0$$

with: $W \in A(\Sigma_w)$, $H = \sum_{i=1}^{nc} \frac{1}{2}\frac{q_i^2}{C_i} + \sum_{j=1}^{nl} \frac{1}{2}\frac{\phi_j^2}{L_j}$,

$$\underline{pf} = \begin{bmatrix} i_S & \dot{q} & \frac{\partial H}{\partial \phi} & i_R \end{bmatrix}^T, \quad \underline{pe} = \begin{bmatrix} u_S & \frac{\partial H}{\partial q} & \dot{\phi} & u_R \end{bmatrix}^T$$

Note that flow and effort vectors are composed of both derivative states variables and state variables.

For the simplified Buck converter we get:

$$\begin{bmatrix} 1-w_1w_2 & 1 & w_1 & 1 \\ -1+w_2 & 0 & 0 & 0 \\ w_2(w_1-1) & 0 & 1-w_1 & 0 \\ 0 & -1 & -1 & -1 \\ & 0_{4x4} & & \end{bmatrix} \begin{bmatrix} i_s \\ \dot{q} \\ \frac{\partial H}{\partial \phi} \\ i_R \end{bmatrix} +$$
$$\begin{bmatrix} & 0_{4x4} & & \\ w_2 & 0 & w_2 & -w_2 \\ -w_2 & 1-w_1 & w_1-w_2 & -(1-w_2) \\ 0 & w_1 & -w_1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_s \\ \frac{\partial H}{\partial q} \\ \dot{\phi} \\ u_R \end{bmatrix} = 0$$

with $H = \frac{q^2}{2C} + \frac{\phi^2}{2L}$ and $W \in \{[0\ 0]^T, [0\ 1]^T, [1\ 0]^T\}$.

This non minimal kernel representation of the parameterized implicit port Hamiltonian formulation of a PSS may be rewritten, after reduction of variables related to resistors, to express a non minimal set of equations, of descriptor type. For the simplified Buck converter we get:

$$\begin{bmatrix} w_2 & \frac{1}{R}w_2 \\ (1-w_2) & \frac{1}{R}(w_1-w_2) \\ 0 & w_1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{\phi} \end{bmatrix} =$$
$$\begin{bmatrix} 0 & -w_2 \\ \frac{1}{R}(w_1-1) & \frac{1}{R}(w_1-w_2) \\ w_1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial \phi} \end{bmatrix} + \begin{bmatrix} -\frac{1}{R}w_2 \\ \frac{1}{R}w_2 \\ 0 \\ 0 \end{bmatrix} u_s$$
$$\begin{bmatrix} 1-w_1w_2 \\ w_2-1 \\ w_2(w_1-1) \end{bmatrix} i_s = \begin{bmatrix} 0 & 1-w_1 \\ 0 & 0 \\ 0 & w_1-1 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial \phi} \end{bmatrix}$$

with $W \in \{[0\ 0]^T, [0\ 1]^T, [1\ 0]^T\}$

A minimal implicit port Hamiltonian formulation of each PSS's configuration is obtained from this non minimal single representation after instantiation of W. Notice that the discrete dynamics of W will be determined by the conditions of autonomous switching (diodes, ...) and the control (transistors, valves, ...) which synthesize the sequence of configurations of the system depending on time (Manon, *et al.*, 2002; Fibrianto, and Dochain, 2003).

7. CONCLUSION AND PERSPECTIVES

A systematic method to analyse all the admissible and the non-constrained configurations of a large class of dissipative physical systems with sources and switching topology has been proposed. It is based on hybrid incidence matrices associated with a primal dynamic network graph of the physical switching system (PSS) and its dual graph. Indeed, it is of prime importance to remove the non-admissible configurations from the control synthesis procedure and to be aware of constrained configurations which may lead to state jumps in the trajectory. This method has been illustrated on the examples of the simplified Buck converter and of another power converter with a flow source.

The approach presented here is particularly well-suited to the context of a modular analysis of complex non-regular systems. Indeed, the hybrid (parameterized) incidence matrices can be calculated for all the subsystems (regular or not) and then connected through the ports.

A continuation of this work can be to extend control synthesis methods based on continuous Hamiltonian systems such as Interconnection Damping Assignment Passivity Based Control (Ortega, *et al.*, 2001; Maschke, *et al.*, 1999) and continuous control synthesis method for hybrid port-controlled Hamiltonian systems with autonomous switching as impacts (Haddad, *et al.*, 2003) to dissipative physical switching system with sources and controlled switches.

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