SYNTHESIS OF OPTIMAL SIGNALS FOR DYNAMICAL SYSTEMS BY SPECIAL TYPES OF BANG-BANG ACTUATORS

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Abstract: We describe a new constructive method for solving a linear indirect optimal control problem with allowance for the features of special types of bang- bang actuators. The method makes possible to obtain necessary and sufficient optimality conditions to the control problems with bang-bang actuators, bang-bang actuators with stagnation zones and bang-bang actuators with delay in feedback loop. On the base of this approach the algorithms of optimal open- loop and on-line optimization are justified for systems with constraints on control function and terminal states. All results are illustrated by examples.

Keywords: Real time, linear indirect optimal control problem, bang-bang actuators with stagnation zones, bang-bang actuators with delay in feedback loop.

1. INTRODUCTION

The paper concerns the problem of on-line optimization of linear dynamic systems by different types of bang- bang actuators (regulators) (A.AQ. Feldbaum, 1963). It is supposed that the structures of the actuators are given. At forming control signal they can take into account them to be relay (bang- bang), bang- bang actuator with delays in feedback loop, with stagnation zone.

From mathematical point of view, the peculiarity of control in consideration consists in choosing control signal and functions from known special classes but not in very wide traditional classes such as piecewise- continuous, measurable and generalized ones. In doing so, the approach suggested in (Gabasov *et al.*, 1995; Balashevich *et al.*, 2001) are developed to construct realizations of optimal feedbacks in real time.

In the paper along with the criteria of optimality, algorithm of optimal open- loop and online optimization are justified to control systems with constraints on control function and terminal states. Some of the actuators leads to investigations of the optimal control problems under state constraints what means considerable complication in the study. The paper develops the mention results of the authors and based on the adaptive method (Gabasov *et al.*, 1995) and fast algorithms of optimal control.

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2. BANG— BANG ACTUATORS

Dynamic system control by bang-bang actuator. Consider the dynamic system

$$\dot{x} = Ax + bu, x(0) = x_0, \tag{1}$$

which is controlled by two-positioned bang-bang actuator on the time interval $T = [0, t^*]$. Here: t^* is fixed, $x = x(t) \in R^n$ is a state vector of the dynamical system, $u = u(t) \in R$ is a value of control function (input signal of the control system), A is a given $n \times n$ matrix of control system dynamics, b is a given n-vector of parameters of input device.

System (1) is controlled in the following way. At each moment $t \in T_{h_0} = \{0, h_0, \ldots, t^* - h_0\}, h_0 = t^*/N_0, N_0 \in N$ is given, a current system state $x(t)$ is measured. This information is transmitted to the control device which on that basis forms a control signal and then gives the signal to the input of the bang-bang actuator. The bang-bang actuator produces the control function. The latter can take only two values ± 1 and change them only at the switching points $0 < t_1 < t_2 < \ldots < t_p < t^*$ (Fig.1). It is clear that on-off bang- bang control can be uniquely defined via the initial value of the control function $u(0) = u_0 = \pm 1$ and vector of control parameters $\tau = (t_1, \ldots, t_p)$. Denote by U_p^1 a set of control functions $u(t), t \in T$, produced by the bang-bang actuator.

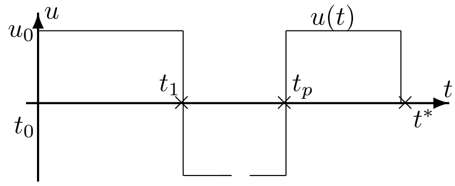

Fig. 1.

Linear optimal control problem. Optimality condition. Consider a terminal optimal control problem:

$$J(u) = c'x(t^*) \longrightarrow \max_u, \dot{x} = Ax + bu, x(0) = x_0,$$

$$Hx(t^*) = g, u(\cdot) \in U_p^1, p = 1, 2, \ldots. \qquad (2)$$

Here: g - is a given m- vector; H is a given $m \times n$ matrix, $I = \{1, 2, \ldots, m\}$, $J = \{1, 2, \ldots, n\}$ are sets of indexes of rows and columns of matrix H, $\text{rank}H = m < n$, correspondingly.

A bang-bang control function $u(\cdot) = (u(t), t \in T)$ is called admissible if corresponding trajectory $x(t), t \in T$, of linear system (1) satisfies the constraint: $x(t^*) \in X^* = \{x \in R^n : Hx = g\}$. An admissible control function $u^0(\cdot)$ is said to be optimal if the optimal trajectory $x^0(t),\ t \in T$, satisfies the equality $c'x^0(t^*) = \max c'x(t^*),\ u(\cdot) \in U_p^1$.

Let us consider an admissible control function $u(\cdot)$ with the vector of control parameters τ and u_0. Any another control function $\bar{u}(\cdot)$ can be obtained from $u(\cdot)$ if: a) to shift points t_0, t_{p+1} into the interval T; b) at every interval $[t_i, t_{i+1}[$ insert $2k_i$ additional switching points $\zeta_1^i \pm \Delta\zeta_1^i, \zeta_2^i \pm \Delta\zeta_2^i, \ldots, \zeta_{k_i}^i \pm \Delta\zeta_{k_i}^i$, $(t_i < \zeta_j^i \pm \Delta\zeta_j^i < t_{i+1}), j = \overline{1, k_i}$); c) move points $t_i, i = \overline{1, p}$, preserving their order. Note, that these actions describe the class of used control functions. The vector of the control parameters $\bar{\theta}$ of a new control function $\bar{u}(\cdot)$ is given by $\bar{\theta} = (t_0 + \Delta t_0, \ldots, t_i + \Delta t_i, \zeta_1^i - \Delta\zeta_1^i, \zeta_1^i + \Delta\zeta_1^i, \ldots, \zeta_{k_i}^i + \Delta\zeta_{k_i}^i, t_{i+1} + \Delta t_{i+1}, \ldots, t_{p+1} + \Delta t_{p+1})$, where $\Delta t_0 \geq 0; \Delta t_k, k = \overline{1, p}; \Delta\zeta_j^i \geq 0, j = \overline{0, k_i}, i = \overline{0, p}; \Delta t_{p+1} \leq 0$ are arbitrary small by modulo values.

As a result of some special perturbations of (2), using new notation, one can get the linearized problem in the increments

$$l'\Delta\theta \longrightarrow \max_{\Delta\theta},\ D\Delta\theta = g^0, d_* \leq \Delta\theta \leq d^*. \qquad (3)$$

To get an optimality criterion of (3) we use the next statement:

Lemma 1. If $u^0(\cdot)$ is an open-loop control of (2) and $\text{rank}D = m$, then the vector $\Delta\theta = 0$ is the optimal feasible solution of (3).

Problem (3) is a special case of canonical linear programming problem. Introduce the function $\psi'(t) = (c' - \nu' H)F(t^*)F^{-1}(t),\ t \in T$, which is the solution of the adjoint system $\dot{\psi} = -A'\psi,\ \psi(t^*) = c - H'\nu$. The function $\psi(t),\ t \in T$, is said to be a co-trajectory, $\Delta(t) = \psi'(t)b,\ t \in T$, be a co-control. To formulate the optimality criterion of the vector $\Delta\theta = 0$ of problem (3) we use the proper results of the adaptive method (Gabasov *et al.*, 1995) and lemma 1. The statement is true.

Theorem 2. (Maximum principle). An admissible control $u(t),\ t \in T$, of problem (2) is optimal iff it is given by

$$u(t) = sign\Delta(t),\ t \in T. \qquad (4)$$

Construction of optimal open-loop controls. ***Step 1.*** Set a number $N = lN_0 \geq m, l \geq 1, N_0 \in N$. In the class of discrete controls $u_h(t) = u(kh),\ t \in [kh, (k+1)h[,\ k = \overline{0, N-1}$; with the quantization step $h = t^*/N$, solve the problem

$$c'x(t^*) \longrightarrow \max_u,\ \dot{x} = Ax + bu, x(0) = x_0,$$

$$Hx(t^*) = g,\ |u(t)| \leq 1, t \in T, \qquad (5)$$

using method (Gabasov *et al.*, 1995). According to (Gabasov *et al.*, 2001) the open-loop solution $u_h^0(t),\ t \in T$, of problem (5) has the form $u_h^0(t) = u^0(kh) = sign\Delta_h^0(kh),\ t \in [kh, (k+1)h[,\ k = \overline{0, N-1}$; where $\Delta_h^0(kh) = \int_{kh}^{(k+1)h} \Delta^*(t)dt/h,\ \Delta^*(t) = \psi'(t)b,\ \dot{\psi} = -A'\psi,\ \psi(t^*) = c - H'\nu,\ \nu$ is an optimal vector of potentials.

Let the solution to problem (5) identified the structure of the optimal control function of (2), i.e.

the number of zeros [2] of the functions $\Delta^0(t),\ t \in T,\ \Delta_h^0(t),\ t \in T_h$, are equal and $sign\Delta_h^0(t_0) = sign\Delta(t_0)$ (excluding special cases when $\Delta_h^0(t_0) \approx 0,\ \Delta^0(t_0) \approx 0$). If the structure of the optimal control function is identified, then proceed to step 2. Otherwise, increase N and solve problem (5) by the dual method taking the previous optimal base as an initial one.

Step 2. Construct the bang- bang control $u^1(t), t \in T$, using the optimal discrete open-loop control $u_h^0(t),\ t \in T$, of (5). For that every zero t_k^h of co-control $\Delta_h^0(t),\ t \in T_h$, with $|u_h^0(t-h)| \neq 1$ should be replaced by the point $t_k = t_k^h + (u_h^0(t_k^h - h) - 1)h/2$. New points t_k and the remaining zeros of the co-control $\Delta_h^0(t),\ t \in T_h$ are used as the initial approximation τ^1 of the vector of control parameters for the finishing procedure (Gabasov *et al.*, 1995). Another way to construct τ^1 is to compose it from the zeros of the function $\Delta^*(t), t \in T$.

The finishing procedure bases on the Newton method and consists in solving system (6) with respect to unknown $\nu,\ t_k, k = \overline{1,p}$.

$$\Delta(t_k) = 0,\ k = \overline{1,p};\ f(\tau, u_0) = \tilde{g} \tag{6}$$

If $\dot{\Delta}(t_k) \neq 0,\ k = \overline{1,p}$, then the Jacobian of system (6) is nonsingular and (6) can be solved by the Newton method.

Fulfill 3-5 iterations starting from the approximation τ^1 . If there is no quadratic convergence of the method or the points $t_k, k = \overline{1,p}$ are gluing, then go to step 1, increasing N, or to step 3. Otherwise the solution τ^0 of problem (2) can be constructed with a required accuracy.

Step 3. Solve problem (3) on $u^1(\cdot)$ with τ^1 and the discrepency $w^1 = \tilde{g} - f(\theta^1, u_0)$:

$$l'\Delta\theta \longrightarrow \max_{\Delta\theta},\ D\Delta\theta = w,\ d_* \leq \Delta\theta \leq d^*. \tag{7}$$

Using the solution $\Delta\theta^1$ one can construct a new approximation $\theta^2 = \theta^1 + \delta\Delta\theta^1, \delta > 0$. If the inequality $f_0(\theta^2, u_0) - (\nu^1)' f(\theta^2, u_0) > f_0(\theta^1, u_0) - (\nu^1)' f(\theta^1, u_0)$, holds true for θ^1 and ν^1, then (7) should be solved with θ^2 and w^2.

When s approximations have been fulfilled (s is defined by necessary exactness), one can go to the finishing procedure (step 2) with the initial approximation θ^s. Otherwise N is increased to go back to the step 1. More details are given in (N. Kavalionak, 2004).

For constructing the optimal feedbacks of (3) by the bang- bang actuators the special scheme is proposed, which represent a procedure of correcting the optimal open- loop control in real time.

[2] The moment $t_k^h = kh$ is called a zero of co-control $\Delta_h^0(t),\ t \in T_h$, if $\Delta_h^0(t_k^h - h)\Delta_h^0(t_k^h) \leq 0, \Delta_h^0(t_k^h) \neq 0$.

3. BANG-BANG ACTUATORS WITH DELAY IN FEEDBACK LOOP

In section 1 it was assumed that the processing of information on states of the control system takes $s < h_0$ time units, and therefore was neglected. Let before starting the control process we know that if the last measurement of the current state of control system (1) was made at the moment t, then the actuator gives the corresponding control signal to the input of the control system only at the moment $t + \alpha$, i.e with delay $\alpha > 0$. Denote by U_p^2 a set of control signals generated by this actuator. As the delay in feedback loop affects only the control process, it is sufficient to describe the algorithm of functioning the optimal controller under new conditions.

Before the control process starts the optimal controller calculates the optimal open- loop control $u^0(t|0, x(0)), t \in T$, of (2) for the initial instant $(0, x_0)$ by the method described for the bang- bang actuators. Using this information the actuator gives the control function $u^{**}(t) = u^0(t|0, x_0)$ on $[0, \alpha + h_0[$ to the input of the control system. Let the delay in feedback loop is only caused by the time that needed to process the measurement of the current state $x(t)$.

At the moment $t = h_0$ the optimal controller gets the measurement $x^*(h_0)$. Using $u^0(t|0, x_0), t \in [0, \alpha + h_0]$ it calculates the state $x^0(h_0 + \alpha)$ of the system $\dot{x} = Ax + bu^0, x(h_0) = x^*(h_0)$, for the instant $(h_0 + \alpha, x^0(h_0 + \alpha))$ by the method for the bang- bang actuators. Then find the open- loop solution $u^0(t|h_0 + \alpha, x_0(h_0 + \alpha)), t \in [h_0 + \alpha, t^*]$, of problem (2) to feed to the system on $[h_0 + \alpha, 2h_0 + \alpha[$: $u^{**}(t) = u^0(t - \alpha|h_0 + \alpha, x^0(h_0 + \alpha)), t \in [h_0 + \alpha, 2h_0 + \alpha[$.

Hereinafter the operations of the optimal controller depend on the quantity of used microprocessors. If there are not less then $k = [\alpha/h_0]$ microprocessors, the actuator gives the control function $u^{**}(t), t \in [h_0 + \alpha, 2h_0 + \alpha[$ to the input of the control system. To process the next measurement $x^*(2h_0)$ the optimal controller switch on the second microprocessor that will calculate the control function for $u^{**}(t), t \in [\alpha + 2h_0, \alpha + 3h_0[$ and so on. By the moment $\alpha + h_0$ the first microprocessor has finished its work and the control process is repeated.

From the described algorithm one can see that the optimal controller with several microprocessors can calculate given measurements with the same rate as it gets them. The moments of states measurements and the moments when the control system gets corresponding signals differ from the value α. In this case we say that the optimal controller realizes optimal feedback in real time with the delay α. If $\alpha < h_0$ one can say about the real time (with no delay).

Suppose that there is only one microprocessor that needs the α time units to process a current measurement. In this case at the moment h_0 the optimal controller gets the measurement $x^*(h_0)$, then calculates $u^0(t|h_0, x^*(h_0)), t \in T$, that can be used only on $[\alpha + h_0, 2\alpha + h_0[$: $u^{**}(t) = u^0(t - \alpha|h_0, x^*(h_0)), t \in [\alpha + h_0, 2\alpha + h_0[$. The optimal controller can not process the measurements $x^*(2h_0), \ldots, x^*(\alpha)$, so we do not need them to be fulfilled. The next measurement that can be processed by the optimal controller is $x^*(\alpha + h_0)$.

4. BANG-BANG ACTUATORS WITH STAGNATION ZONE

Control by bang-bang actuators with stagnation zones. Necessary and sufficient optimality conditions. The above described actuators were able to switch on the control function from a limit value to another one instantly. Consider the bang-bang actuator with stagnation zones. Before changing a current limit value to another one it switch off from the control object for a time 2β (Fig.2). During this time the control object gets a fixed limited signal $u(t) \equiv u_\beta$. Its value defined by characteristics of the control device. Let $u_\beta = 0$.

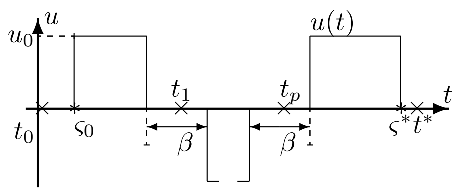

Fig. 2.

In the absence of the optimal feedback an open-loop control for this kind of actuators will be as follows. Before the beginning of the control process the optimal controller calculates the initial value of the control signal $u_0 = \pm 1$, switching on ς_*, switching off ς^* moments and the switching moments $t_i, i = \overline{1,p}$: $\varsigma_* < t_1 < t_2 < \ldots < t_p < \varsigma^*$, $(t_{k+1} - t_k \geq 2\beta, k = \overline{0,p}; \varsigma_* \geq 0, t^* - \varsigma^* \geq 0)$.

According to these signals the actuator sets the value u_0 (in advance, before the beginning of the control process), starts to give it to the control object at the moment ς_* and then switches the control signal (taking into account stagnation zones) till the moment ς^*. Typical parts of the control signal $u(\cdot) \in U_p^3$ are given:1. $a)\varsigma_* = 0, u(t) = u_0, t \in [0, t_1 - \beta[$; $b)$ $0 \leq \varsigma_* \leq 2\beta, u(t) = 0, t \in [0, \varsigma_*[; u(t) = u_0; t \in [\varsigma_*, t_1 - \beta[$; $c)$ $\varsigma_* = 2\beta, u(t) = 0, t \in [0, \varsigma_*[; u(t) = u_0, t \in [\varsigma_*, t_1 - \beta[$; 2. $u(t) = (-1)^i u_0, t \in [t_i + \beta, t_{i+1} - \beta[; u(t) = 0, t \in [t_i - \beta, t_i + \beta[, i = \overline{1, p-1}$; 3. $a)\varsigma^* = t^*, u(t) = (-1)^p u_0, t \in [t_p + \beta, \varsigma^*]$; $b)$ $t^* - 2\beta \leq \varsigma^* \leq t^*, u(t) = (-1)^p u_0, t \in [t_p + \beta, \varsigma^*]$, $u(t) = 0, t \in [\varsigma^*, t^*]$; $c)$ $\varsigma^* = t^* - 2\beta$, $u(t) = (-1)^p u_0, t \in [t_p + \beta, \varsigma^*]$, $u(t) = 0, t \in [\varsigma^*, t^*]$. The case when the control function has the same value before and after the stagnation zone are supposed to be singular and does not investigated.

Optimal control problem (2) is simplified to linear increment problem by the scheme implemented in the previous part of the paper.

Necessary and sufficient optimality condition can be proved which lead to the local optimality of the control:

Theorem 3. (Maximum principle). An admissible control $u(t)$, $t \in T$, of problem (2) is local optimal iff

$$u(t) = \mathrm{sign}\Delta(t), t \in [0, t_1 - \beta[\cup[t_k + \beta,$$

$$t_{k+1} - \beta[\cup[t_p + \beta, t^*], k = \overline{1, p-1};$$

$$u(t) = 0, t \in [t_k - \beta, t_k + \beta[, k = \overline{1,p}. \quad (8)$$

Scheme of functioning of the optimal controller. We construct an optimal open-loop control function of problem (2) in the class of discrete controls (Gabasov *et al.*, 2001), find optimal discrete control $u_h^0(\cdot)$ with no stagnation zones and with switching points $t_k^h, k = \overline{1,N}$, and then operate under one of the following ways:

A. In the neighborhood of the discrete switching points t_k^h the optimal discrete control function is changed by considering the bang-bang control function with stagnation zones $[t_k - \beta, t_k + \beta[$. Then with the help of the iterative method and the linearized problem in increments we find admissible control function $u^1(\cdot) \in U_p^3$. Refine the initial approximation to such degree that a special finishing procedure based on the Newton methods convergences. Its equations are given by

$$\Delta(t_k - \beta) + \Delta(t_k + \beta) = 0, k = \overline{1,p};$$

$$f(\tau, u_0) = \tilde{g}. \quad (9)$$

The Jacobian of system (9) is nonsingular if $\dot{\Delta}(t_k - \beta) + \dot{\Delta}(t_k + \beta) \neq 0$, $k = \overline{1,p}$. On the base of this special procedure find the optimal solution $u^0(\cdot) \in U_p^3$ of problem (2) with the required accuracy.

B. Choose a sufficiently small number $\beta^1 < \beta$, and construct an initial bang-bang control function $u(\cdot) \in U_p^3$ with $2\beta^1$. Using the special finishing procedure, find the solution of problem (2) with the stagnation zone $2\beta^1$. Increasing the stagnation zone up to $2\beta^2, \beta^2 > \beta^1$, and using the previous solution as the initial one construct the open-loop solution $u^2(\cdot) \in U_p^3$ with the stagnation zone $2\beta^2$ by the finishing procedure. Through a finite number of steps one can get $\beta^k = \beta$ and the open-loop solution $u^0(\cdot) \in U_p^3$ of problem (2).

The algorithm of optimal controller work. Before the beginning of the control process the optimal controller calculates the open- loop solution $u^0(\cdot)$ of problem (2) with the switching points $t_k, k = \overline{1,p}$. Let $\varsigma_* = 0, \varsigma^* = t^*$. At the moment $t = 0$ the actuator begins to give the control function $u^{**}(t) = u^0(t), t \in [0, h_0[$ to the input of the control system. At the moment h_0 the optimal controller gets the measurement $x^*(h_0)$, on which it calculates $u^0(t|h_0, x^*(h_0)), t \in T(h_0)$, with the switching points $t_k(h_0), k = \overline{1,p}$.

If $t_1(h_0) - \beta \geq 2h_0$, then the actuator keeps control function $u^0(t) = u_0$ on the interval $[h_0, 2h_0]$. The situation does not change till the moment $\tau_1 \in T(\tau_1)$, when the inequalities $\tau_1 < t_1(\tau_1) - \beta \geq \tau_1 + h_0$ are hold at first time. On the interval $[t_1(\tau_1) - \beta, t_1(\tau_1) + \beta[$ the actuator switch off from the control system: $u^{**}(t) \equiv 0$. But the optimal controller continue to calculate the optimal solutions and the corresponding switching moments $t_i(\tau), i = \overline{2,p}$ on the base of the measurements $x^*(\tau), \tau \in [t_1(\tau_1) - \beta, t_1(\tau_1) + \beta[$. At the same time it takes into account that $u^{**}(t) \equiv 0, t \in [t_1(\tau_1) - \beta, t_1(\tau_1) + \beta[$. Let $t_2(\tau_1) - \beta > t_1(\tau_1) + \beta$, except special cases. At the moment $t_1(\tau_1) + \beta$ the actuator connected to the control object and produces the control function $u^{**}(t) = -u_0, t \in [t_1(\tau_1) + \beta, \tau_1 + 2\beta + h_0[$. Up to the end of the control process the optimal controller and actuator repeat the scheme.

5. EXAMPLE

The following problem was used in computer experiments:

$$J(u) = \int_0^{t^*} u(t)dt \longrightarrow \min_u, , u(t) \in U,$$

$$\ddot{y} = -y(t) + u(t), y(0) = y_0, \dot{y}(0) = 0,$$

$$y(t^*) = 0, \dot{y}(t^*) = 0, t \in T = [0, t^*]. \qquad (10)$$

The bang-bang actuator. Initial values $U = \{0, 1\}, t^* = 12; y_0 = 3$; the optimal open-loop solution of (10) was obtained with the optimal vector of switching points: $\tau^0\{0.737, 2.375, 7.269, 8.544\}$, and $u_0 = 0$. If dynamical system (10) is under unknown disturbances $w(t) = 0.1 sint, t \in [0, 7.4]; w(t) = 0, t \in]7.4, t^*]$; the optimal on-line solution $\tau^0 = (0.7384, 2.3604, 7.3151, 8.5748)$, $u_0 = 0$ of (10) was obtained.

Bang-bang actuator with delay in feedback loop. Think the initial value be the same as for bang-bang actuator. If dynamical system (10) is under unknown disturbances $w(t) = 0.1 sint, t \in [0, 7.4]; w(t) = 0, t \in]7.4, t^*]$; and delay $\alpha = 0.1$ then the next optimal on- line solution $\tau^0 = (0.6384, 2.2604, 7.2151, 8.4748)$, $u_0 = 0$ of (10) could be obtained.

The bang- bang actuator with stagnation zone. Let $u(t) \in U = \{0, 1\} \cup \{u_\beta\}; w(t) = 0.13 sint, t \in [0, 5.8]; w(t) = 0, t \in]5.8, t^* = 12]$. For $\beta = 0.4, u_\beta = 0.5$ the vector of the optimal switching points is $\tau^0 = (0.620327, 2.4761407, 7.2427714, 8.5592467), u_0 = 0, J(u) = 3.172$; For $\beta = 0.1, u_\beta = 0.5$— $\tau^0 = (0.727246, 2.3779, 7.28556, 8.50384)$; $u_0 = 0$, $J(u) = 2.868934$.

6. CONCLUSION

The finite algorithms of on-line optimization for linear dynamic systems with constraints on terminal states and control functions are justified. To organize the control the actuators of different kind are used such as the bang-bang actuators, bang- bang actuators with delay in feedback loop, bang- bang actuators with stagnation zone. Numerical computation conducted demonstrate the efficiency of the algorithms of optimal on-line control.

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