ROBUST GENERIC MODEL CONTROL FOR PARAMETER INTERVAL SYSTEMS

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Abstract: A multivariable control technique is proposed for a type of nonlinear system with parameter intervals. The control incorporates the feedback linearization scheme called Generic Model Control, and alters the control calculation by utilizing parameter intervals, employing an adaptive step, averaging control predictions, and applying an interval problem solution. The proposed approach is applied in controlling a nonlinear arc welding system.

Keywords: Nonlinear control systems, Intervals, Parameters, Feedback linearization, Control applications

1. INTRODUCTION

There is now a greater need for nonlinear control to meet greater performance and cost demands in industry. Also, the capability to apply nonlinear control techniques has also risen due to the computational power increases and cost reductions of the controller hardware. Nonlinear systems and their models are prevalent in industry; however, the true parameters of the model are often uncertain within a reasonable range and may change throughout time. Thus, a control algorithm that is easily implemented to control nonlinear systems with parametric intervals is not only an interesting topic of research but also could have extensive applications in the industrial setting.

The proposed control algorithm was developed for multi-input multi-output (MIMO), nonlinear, state-feedback systems with parameter intervals. Generic model control (GMC) is a feedback linearization scheme and has been selected to improve for the following reasons: the control law has the ability to incorporate a nonlinear model, the controller design parameters are intuitive and have a definable affect on the closed loop output, and the controller is already suitable for MIMO systems. The modifications to GMC include the ability to handle parameter interval systems, the use of averaging control predictions to induce control smoothness, and the calculation of a control stability interval that restricts the control signal to satisfy a specified output interval constraint. Results will be presented for the application of the developed algorithm in controlling a nonlinear arc-welding process.

2. BACKGROUND OF CONTROL ALGORITHM

In this section, six areas of the proposed control algorithm will be briefly overviewed. First, a review of interval mathematics will be given, which is followed by the reduction of the nonlinear model to an interval problem. Then GMC is reviewed, which is followed by the introduction of an adaptive step and a control smoothing improvement. Finally, some calculation steps necessary in a MIMO implementation are presented.

2.1 Interval Mathematics Review.

A real interval $[Y] \in \mathbb{R}$ is a closed, connected, and bounded subset of $\mathbb{R}$, such that

$$[Y] = \left[Y^- \; Y^+\right] = \left\{Y \mid Y^- \le Y \le Y^+\right\} \quad (1)$$

Interval arithmetic is generalized for addition, subtraction, multiplication, and division, see (Hansen and Walster, 2004).

Given the interval mathematics problem below:

$$\left[Y^- \; Y^+\right] = \left[H^- \; H^+\right] + \left[B^- \; B^+\right] \times \left[U^- \; U^+\right] \quad (2)$$

where the intervals [B], [U], and [H] are known, the interval [Y] is found using:

$$\begin{aligned} Y^- &= H^- + \min\left(B^-U^-, B^-U^+, B^+U^-, B^+U^+\right) \\ Y^+ &= H^+ + \max\left(B^-U^-, B^-U^+, B^+U^-, B^+U^+\right) \end{aligned} \quad (3)$$

If given an inverse interval problem where the goal is to find the [U] that satisfies a particular [Y], then two simplifying assumptions will be made. For it is known that, in general, the solution to this linear inverse interval problem can only be solved through nonlinear programming methods, see (Hansen and Walster, 2004). First, it is assumed that $[B] \geq 0$, and secondly it is assumed that the [Y] is feasible, defined by the width of [H] being smaller or equal to the width of [Y]. It can be shown (Istre, 2004), that the inverse problem can be solved whereby for a given feasible [Y], the necessary [U] can be calculated using:

$$\begin{aligned} U^- &= \max\left(\frac{Y^- - H^-}{B^-}, \frac{Y^- - H^-}{B^+}\right) \\ U^+ &= \min\left(\frac{Y^+ - H^+}{B^-}, \frac{Y^+ - H^+}{B^+}\right) \end{aligned} \quad (4)$$

Thus, the inverse interval problem can be used to calculate an acceptable range of U, called $[U]_S$, that satisfies a stability criteria for $[Y]_S$, where the "S" subscript signifies that the interval is used for defining a "stability" interval. Then for the output criteria $[Y]_S = [Y^- \; Y^+]_S$, the acceptable $[U]_S$ would be found using the following equation:

$$[U]_S = \left[\max\left(\frac{Y^- - H^-}{B^-}, \frac{Y^- - H^-}{B^+}\right), \min\left(\frac{Y^+ - H^+}{B^-}, \frac{Y^+ - H^+}{B^+}\right)\right] \quad (5)$$

2.2 Nonlinear Model to Interval Problem.

Consider the nonlinear scalar model:

$$Y_k = \left(\sum_{i=1}^{n} \varphi_i \cdot f_i(x_{i_k})\right) + \beta \cdot g(x_{0_k}) \cdot u_{k-1} \quad (6)$$

where x_i and x_0 the are states of the nonlinear system, the f_i*'s* and g are nonlinear functions, and φ_i's and β are the parameters of the model. The parameters φ_i and β are known to fall within intervals $\varphi_i^- \le \varphi_i \le \varphi_i^+$ and $\beta_i^- \le \beta_i \le \beta_i^+$, where the intervals are found by system identification. Finding these parameter intervals through system identification is easily accomplished and is a sensible method for characterizing model uncertainty and system disturbances. Note that this model structure given in Equation 6 is capable of being controlled by the feedback linearization control technique, called GMC. Moreover, linear systems with state feedback fall within this structure.

Assume all of the states, x, of Equation 6 are fully known at each time k. Then each of the values of the nonlinear functions $f_1(x_{1,k})$, $f_2(x_{2,k})$,…., $f_n(x_{n,k})$, $g(x_{0,k})$ are also known. Therefore, the nonlinear model can be simplified to an interval problem, for $[B_k]>0$, given by:

$$\left[Y_k^- \; Y_k^+\right] = \left[H_k^- \; H_k^+\right] + \left[B_k^- \; B_k^+\right] * \left[U_k^- \; U_k^+\right]$$

where,

$$\begin{aligned} H_k^- &= \sum_{i=1}^{n} \min\left(\varphi_i^- \cdot f_i(x_{i_k}), \varphi_i^+ \cdot f_i(x_{i_k})\right) \\ H_k^+ &= \sum_{i=1}^{n} \max\left(\varphi_i^- \cdot f_i(x_{i_k}), \varphi_i^+ \cdot f_i(x_{i_k})\right) \\ B_k^- &= \beta^- \cdot g(x_{0_k}) \\ B_k^+ &= \beta^+ \cdot g(x_{0_k}) \end{aligned} \quad (7)$$

The utility of Equation 7 is that the nonlinear model is reduced to an interval equation of the form given in Equation 2. If $[B_k] \geq 0$, then a closed form solution for the inverse interval problem, Equation 5, can be used to find a $[U]_S$ that satisfies a given $[Y]_S$ criteria.

2.3 Generic Model Control.

Generic Model Control (GMC), see (Lee and Sullivan, 1988), which was not developed for parameter interval systems, solves the feedback linearization problem, for the system of Equation 6 and its formulation given in Equation 7, according to the following equations:

$$U_k = \frac{Y_{ref,k} - H_{avg,k}}{B_{avg,k}}$$

where,

$$\begin{aligned} Y_{ref,k} &= T_S \begin{Bmatrix} K_1\left(Y_{set_{k-1}} - Y_{k-1}\right) + \\ K_2 \sum_{i=1}^{k-1} T_S \cdot \left(Y_{set_i} - Y_i\right) \end{Bmatrix} + Y_{k-1} \\ H_{avg,k} &= \left(H_k^- + H_k^+\right)/2 \\ B_{avg,k} &= \left(B_k^- + B_k^+\right)/2 \end{aligned} \quad (8)$$

Essentially, GMC uses the inverse nonlinear model and calculates the necessary control to achieve the desired reference trajectory, Y_{ref}. In Equation 8, the signal Y_{set} is the desired setpoint of the system output, the signal Y_{ref} is the linear reference trajectory specified by GMC parameters K_1 and K_2, and the values H_{avg} and B_{avg}, are the average of the maximum and minimum of the simplified model intervals.

2.4 GMC with Adaptive Factor.

Instead of using the average of the maximum and minimum parameters for each of the nonlinear functions, as necessary for use by GMC, an adaptive factor based upon the system's previous history could be used to calculate a more suitable control. Two models, a maximum and minimum model, could be defined by using only the maximum or minimum parameters. These two models then bound the behavior of the actual system. Thus, based upon the actual system's behavior within those bounding models, an estimate could be made of the system's future behavior within the future maximum and minimum bounds.

Using the maximum and minimum parameters for the model in Equation 6, the maximum and minimum models are defined as:

$$\begin{aligned} Y_k^{\min} &= \left(\sum_{i=1}^{n} \varphi_i^- \cdot f_i(x_{i_k}) \right) + \beta^- \cdot g(x_{0_k}) \cdot u_k \\ Y_k^{\max} &= \left(\sum_{i=1}^{n} \varphi_i^+ \cdot f_i(x_{i_k}) \right) + \beta^+ \cdot g(x_{0_k}) \cdot u_k \end{aligned} \tag{9}$$

Defining the actual system's behavior within the maximum and minimum models by a parameter F_k and assuming that F_k remains constant for the next sampling time, a more suitable model is described for use in the GMC calculation of the control as is shown in the following equations:

$$F_{k-1} = \frac{Y_{k-1} - Y_{k-1}^{\min}}{Y_{k-1}^{\max} - Y_{k-1}^{\min}} \tag{10}$$

$$U_k = \frac{Y_{ref,k} - H_{F,k}}{B_{F,k}}$$

where

$$\begin{aligned} H_{F,k} &= H_k^- + F_{k-1} \cdot \left(H_k^+ - H_k^- \right) \\ B_{F,k} &= B_k^- + F_{k-1} \cdot \left(B_k^+ - B_k^- \right) \end{aligned} \tag{11}$$

Thus, as shown in Equation 10, the adaptive GMC uses, from the previous sampling time, the systems actual response, Y_{k-1}, and the maximum and minimum calculated responses, Y_{k-1}^{max} and Y_{k-1}^{min} , to determine the adaptive factor, F_{k-1}. Then assuming that F_{k-1} does not change, or remains approximately constant to the present sampling time, a more "system aware" U_k can be calculated using Equation 11 by adjusting the model values H_k and B_k according to their intervals by the adaptive factor.

Note that in implementation, the adaptive factor, F_{k-1}, as defined in Equation 10 can vary substantially between sampling times, counter to the constant assumption, due to both the width of the parameter intervals and the actual variation of the system's parameters. This variation can subsequently result in a highly varying control signal. In order to counteract this variation of the control signal, a solution is to reasonably filter this change in the adaptive factor to more appropriately match the likely system movement throughout its interval.

However, at this point in the control calculation, output stability has not been guaranteed. The interval relations developed in section 2.1, however, can be used to guarantee stability. Simply, to ensure that at the next sampling time the system's output is within the user-specified stable output interval, $[Y_k]_S$, the control, U_k, found using Equation 11, must be constrained by $[U_k]_S$, found using Equation 5. Thus, at each sampling time, constraining the control within $[U_k]_S$, as shown in the following equation, always guarantees the output stability as defined by $[Y_k]_S$.

$$\begin{aligned} U_k \in [U_k]_S \Big| \quad U_k &= \min\left(U_k^+, \max\left(U_k^-, U_k \right) \right) \\ &\Rightarrow Y_k \in [Y_k]_S \end{aligned} \tag{12}$$

2.5 Reducing Controller Oscillation through Predictive Control.

An additional step in the proposed control algorithm is to predict future controls and then actually implement a weighted average of them that is still constrained by $[U_k]_S$. For a one step prediction, after calculating $U_{P,k}$ and constraining it by $[U_k]_S$, as in the previous section, the algorithm uses the system model to predict the next output of the system, $Y_{P,k}$. Then using $Y_{P,k}$ as real feedback would predict a next control, $U_{P,k+1}$, and constrain it by $[U_{k+1}]_S$. Then the actual implemented control would be a weighted average of $U_{P,k}$ and $U_{P,k+1}$ that is constrained by $[U_k]_S$. More generally, if p control predictions are made, then the actual implemented control is given in the following equation, where the W's are the weights of the average.

$$\begin{aligned} U_k &= \sum_{i=0}^{p-1} W_i \cdot U_{P_{k+i}} \\ U_k \in [U_k]_S \Big| \quad U_k &= \min\left(U_k^+, \max\left(U_k^-, U_k \right) \right) \end{aligned} \tag{13}$$

The benefit gained in averaging control predictions is a smoother control signal. Feedback linearization techniques, such as GMC, solve the model inverse for the instantaneous control that will produce the desired process output. The control found by solving the nonlinear model inverse can, especially for systems with varying parameters, produce overly intense actuation and may result in a highly oscillating control signal that is unrealizable in hardware. Thus, by predicting a number of control calculations ahead and averaging them, the result may not *exactly* produce the desired output but it will achieve a more smooth control signal. Moreover, depending upon the weights, W_i, the deviation from the desired output response can be minimal. Furthermore, as long as the implemented U_k is constrained within $[U_k]_S$, then the output will be stable.

2.6 Concerning MIMO Implementation.

All the calculations discussed so far are expandable to MIMO systems except for the stabilizing control interval $[U]_S$, given in Equation 5, which can be used only for scalar systems. For multivariable systems, additional steps will be necessary to determine the multivariable $[\mathbf{U}]_S$; however, it is important to note that no matrix inversion will be required. The following equations are for the most general case where the only assumption is that all elements of $[\mathbf{B}_k]$ are greater than or equal to zero. If further assumptions can be made for either $\mathbf{Y}_k^+$, $\mathbf{Y}_k^-$, $\mathbf{U}_k^+$, or $\mathbf{U}_k^-$, then the calculations can be simplified.

The first step shown in Equation 14 is to find the difference between the output stability criteria and the worst case outputs that would occur if the desired $\mathbf{U}_k$, found from adaptive GMC, were implemented.

$$\begin{aligned} \mathbf{V}_k^+ &= \left(\mathbf{Y}_k^+ - \mathbf{H}_k^+ - \mathbf{B}_k^+ \mathbf{U}_k\right) \\ \mathbf{V}_k^- &= \left(\mathbf{Y}_k^- - \mathbf{H}_k^- - \mathbf{B}_k^+ \mathbf{U}_k\right) \end{aligned} \tag{14}$$

The next shown step in Equation 15 is to find the multiple by which all the inputs should be increased or decreased in order to satisfy all of the output stability criteria. Note the notation $|\mathbf{U}_k|$ denotes the absolute value of the input vector, and the notation ". / " denotes element-wise division of the two vectors. Lastly, the resulting argument of the min and max functions will be a vector; therefore, the values L_k^+ and L_k^- are scalars.

$$\begin{aligned} L_k^+ &= \min\left\{\left(\mathbf{V}_k^+\right) ./ \left(\mathbf{B}_k^+ \left|\mathbf{U}_k\right|\right)\right\} \\ L_k^- &= \max\left\{\left(\mathbf{V}_k^-\right) ./ \left(\mathbf{B}_k^+ \left|\mathbf{U}_k\right|\right)\right\} \end{aligned} \tag{15}$$

The third step calculates the actual input interval constraint, $[\mathbf{U}_k]_S$ according to Equation 16.

$$\begin{aligned} \mathbf{U}_k^+ &= \mathbf{U}_k + \left|\mathbf{U}_k\right| L_k^+ \\ \mathbf{U}_k^- &= \mathbf{U}_k + \left|\mathbf{U}_k\right| L_k^- \end{aligned} \tag{16}$$

The final step, shown in Equation 17, is to constrain $\mathbf{U}_k$ according to the calculated input interval found from the previous equation. Note that the max and min functions in Equation 17, assuming column vectors for $\mathbf{U}_k$, $\mathbf{U}_k^+$, and $\mathbf{U}_k^-$, are applied across the rows of the three vectors.

$$\mathbf{U}_k = \min\left(\mathbf{U}_k^+, \max\left(\mathbf{U}_k^-, \mathbf{U}_k\right)\right) \tag{17}$$

3. SUMMARY OF CONTROL ALGORITHM

An outline is created below of the proposed control algorithm's steps, by assembling all of the algorithm pieces from Section 2 into a cohesive presentation.

1. Calculate F_{k-1} from the output sample Y_{k-1} and the maximum and minimum output predictions, Y_{k-1}^{max} and Y_{k-1}^{min}, from the previous iteration as in Equation 10.
2. Determine all model states x_i's and x_0, and then reduce the nonlinear model to the interval problem as in Equation 7.
3. If using GMC, calculate the next sample of the desired reference trajectory, $Y_{ref,k}$, as in Equation 8.
4. Calculate the adaptive control, by using the model intervals and the adaptive factor as is done in Equation 11. $U_{P,k} = (Y_{ref,k} - H_{F,k})/B_{F,k}$.
5. Find $[U_k]_S$, for scalar systems use Equation 5, for MIMO systems use the process outlined in section 2.6.
6. Constrain $U_{P,k}$ by $[U_k]_S$.
7. Simulate the system by using the maximum and minimum models and the adaptive factor to predict the next output $Y_{P,k}$ for the input $U_{P,k}$.
8. Use $Y_{P,k}$ as if it were real feedback and repeat steps 1 through 8 a p number of times.
9. Calculate a weighted average of the predicted controls as in Equation 13. $U_k = \Sigma(W_i \cdot U_{P,k+i})$
10. Finally, again constrain the weighted average, U_k, by $[U_k]_S$, ($U_k \in [U_k]_S$), and then implement this control.
11. Predict the next Y_{k-1}^{max} and Y_{k-1}^{min} for the U_k from step 10. These predictions will to be used in step 1 for the next sampling time.

Note that in step 7, during the simulation of the process, an adaptive factor must be assumed. Moreover, artificially adjusting the adaptive factor about its previously known state, in order to represent the actual process, will prevent the predicted controls from possibly settling prematurely to some erroneous value.

The control algorithm outlined above has some distinct advantages. It provides a great deal of controller design flexibility to match the controller to the application. The designer selects the GMC parameters K_1 and K_2, chooses the model parameter intervals, can create an adaptive factor filter, may smooth the signal via control predictions, and most importantly selects the output stability criteria. Thus, specific attributes can be emphasized, whether stability, smoothness, or number of computations.

4. ARC WELDING IMPLEMENTATION OF CONTROL ALGORITHM

4.1 Plasma Arc Welding System.

The proposed control algorithm was implemented for a SISO plasma keyhole welding process. The nonlinear model describing the process is given by the following equation.

$$T_{p,k} = a_0 + a_1 I_{p,k-1} + a_2 I_{p,k-2} T_{p,k-1} + a_3 I_{p,k-3} T_{p,k-2} \quad (18)$$

The model was developed by consideration of the energy input into the system. The plasma keyhole process is illustrated in Figure 1.

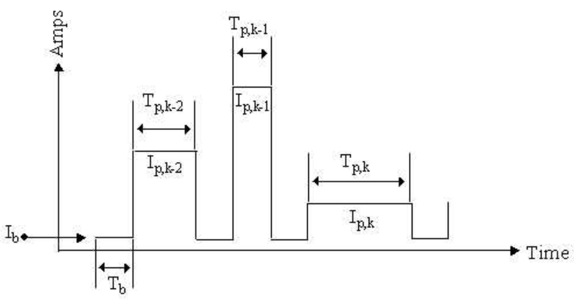

Fig. 1. Keyhole-Plasma Welding Process.

The process input oscillates between a base current, I_b, and a peak current, I_p. Once the peak current has been applied long enough, full weld penetration will be sensed and the process input is switched to the base current for a specified amount of time. The goal of the process is to naturally achieve full penetration at a certain time period. Thus, the input to the system is the amplitude of the peak current, and the output of the system is the measured time period to achieve full penetration.

For the model given in Equation 18, system identification was completed using random inputs within the likely process operating range, and a least squares algorithm was employed for determining the process parameters. This was completed four times to enable the construction of the probable max and min of each of the four model parameters. The range of each parameter is given in the following table:

Table 1 Parameter Intervals for Arc Welding System

	a_0	a_1	a_2	a_3
max	734.66	−4.53	$0.7E{-3}$	$-14E{-4}$
min	707.15	−4.96	$-1.3E{-3}$	$-4E{-4}$

After system identification, closed loop control could be attempted. The controller parameters selected were as follows. First, the desired reference trajectory, $Y_{ref,k}$, was selected to be the GMC reference trajectory with reasonable choices for K_1 and K_2 based upon the system's known response time. For control smoothing, the controller averaged two prediction controls (i.e. p=2) with the weights W_0=2/3 and W_1=1/3. Lastly, it was found that the controller improved with a filter restricting the change in F_k between sampling times as discussed in section 2.4. The filter used was a moving average of the newly calculated F_k with its four previous iterations.

Typical results of the welding experiment are shown in the following figures.

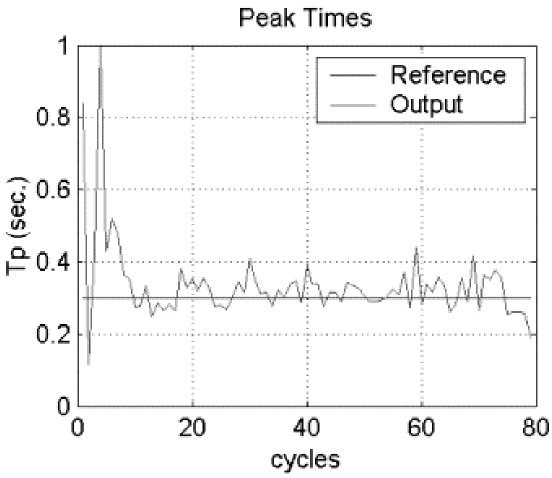

Fig. 2. Weld Penetration Times (System Output)

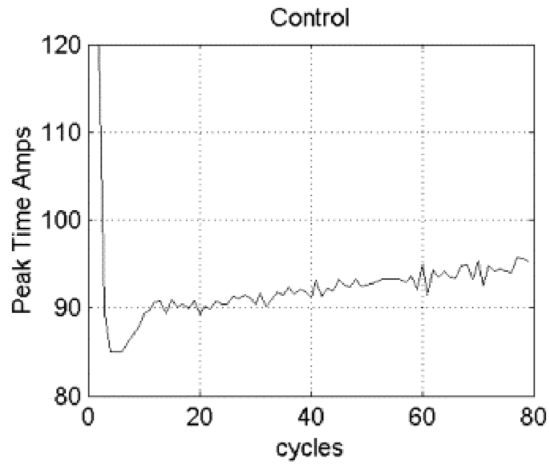

Fig. 3. Peak Current Amperage (System Input)

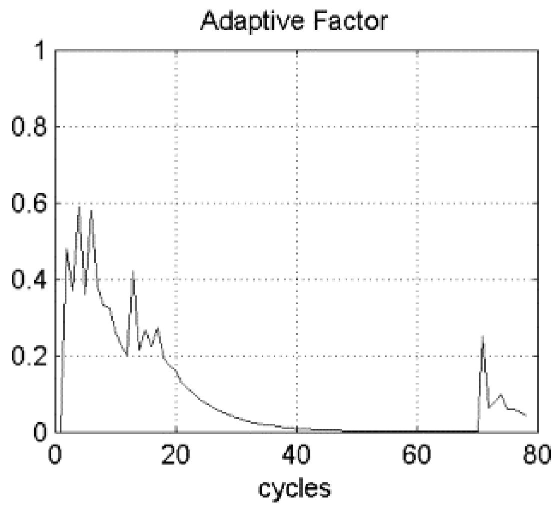

Fig. 4. History of Adaptive Factor, F_k.

The peak time (system output) for each weld cycle is shown in Figure 2 along with the desired setpoint of 300 ms. After approximately 10 cycles, the output has reached the setpoint. However, note that the control signal in Figure 3 does not reach steady state, which suggests that the controller is still adapting the system input despite the output reaching a quasi-steady state at the setpoint. This demonstrates one of the nonlinear characteristics of this system.

In Figure 4, the adaptive factor F_k is plotted for every cycle. Note the correlation of the change in the adaptive factor with the change in the control in the first 20 cycles. This demonstrates the interrelatedness of the control calculation with the system's behavior throughout its maximum and minimum models. Also, though the parameter interval factor has varied widely throughout the entire process, variation between cycles is relatively small, which justifies using the adaptive factor from the previous sampling time in the control calculation, see Equation 11. It is important that the adaptive factor filter must approximately match the process real movement. This will also greatly contribute to the control smoothness without the use of control predictions.

An additional item to observe is that during the period between 45 and 70 cycles, the adaptive factor saturates to the minimum model. This suggests that the minimum model is not truly the minimum. In this situation without the actual minimum model, the minimum stability criterion is not absolutely guaranteed. However, for this process the minimum criterion was not a particular concern, and an approximate minimum model with greater relevancy in the normal operating range was of greater use. This demonstrates the utility of using parameter interval systems and the flexibility of this control algorithm to match the process needs. Images of the weld are shown in Figure 5.

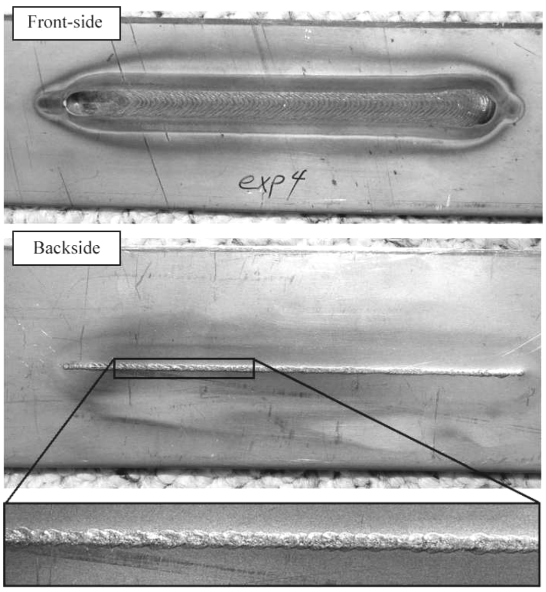

Fig. 5. Weld Images (Front View, Backside View, and Zoomed View of Backside)

The goal of the keyhole plasma process is to obtain slightly overlapping keyhole penetration spots to minimize the energy input and achieve a high quality weld. This can be particularly seen in the zoomed backside view of the weld in the images given in Figure 5, where the penetration spots look like overlapping circular patterns.

5. CONCLUSION

A control algorithm was proposed for processes described by nonlinear models with parameter intervals. The control algorithm is based on a feedback linearization technique, called GMC, but also incorporates the following:
1. The use of parameter intervals.
2. An adaptive factor to characterize the process behavior throughout its maximum and minimum models.
3. The use of controller predictions to smooth the control signal.
4. The use of a closed form solution to an inverse interval problem to find a stabilizing control interval.

The algorithm greatly improves the robustness of GMC by enhancing its ability to track a desired reference trajectory for processes with significant uncertainties. Also, the algorithm is applicable to MIMO systems without the need for computationally intensive matrix inversions.

The algorithm is applied to a nonlinear arc-welding process and the results are discussed.

An area for future work is finding a closed form solution to the interval problem for MIMO systems where $[\mathbf{B}_k]$ has elements with different signs.

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