

# TRAFFIC-RESPONSIVE SIGNALLING CONTROL THROUGH TIMED PETRI NETS

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Abstract: This paper addresses the optimization of the green light durations in a signalized urban area, by means of a traffic-responsive control strategy. The adopted model, mainly consisting of signalized intersections and roads, is microscopically represented by means of deterministic-timed Petri nets (DTPNs), which allow a compact representation of the traffic behaviour. The proposed optimization algorithm includes the solution to a mathematical programming problem constrained by the DTPN state equations. *Copyright © 2005 IFAC*

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## 1. INTRODUCTION

The application of *Intelligent Transportation Systems* (ITS) makes the use of information technology for the control of any transportation system workable. In urban traffic networks, *traffic signalling control* is the major measure applied to regulate vehicle flows. The relevant control strategies typically consist in changing the intersection stage specification, the relative green duration of each stage, the intersection cycle time, and/or the offset between cycles for successive intersections, according to the time-varying behaviour of the incoming traffic (Papageorgiou *et al.*, 2003). The traffic signalling control strategies are split into *fixed-time strategies* and *traffic-responsive strategies*. Fixed-time strategies consider a given time of a day and determine the optimal splits (i.e., the optimal green durations), the optimal cycle time, or even the optimal staging, based on historical values of traffic demand in the considered area (e.g., TRANSYT (Robertson, 1969)).

Traffic-responsive strategies make use of real-time measurements to compute the suitable signal settings (e.g., SCOOT (Hunt *et al.*, 1982), and OPAC (Gartner, 1983)).

This paper describes a control strategy for the optimization of the relative green duration of each stage, assuming fixed, and a-priori known, the stage specification. In doing so, a model of a signalized urban traffic area, as composed of signalized intersections and roads, is used, which falls in the class of *microscopic* traffic representations, where each vehicle is distinctly modelled in the traffic stream. As a matter of fact, a network of traffic lights can be viewed as a complex discrete event system, and then represented via timed Petri nets (PN) (Murata, 1989; Di Febbraro and Giglio, 2004). The above mentioned optimization is accomplished by solving an algorithm which contains a mathematical programming problem, whose cost function minimizes the overall number of tokens in the timed PN representing the

signalized urban area, and whose constraints are derived from the PN state equations. The considered decision variables are the *area stages*, that is, the time intervals during which a given combination of green and red lights does not change in a signalized area with several traffic signals.

The paper is organized as follows. The adopted model of a signalized urban area is briefly introduced in section 2, whereas its representation via *deterministic-timed* PN is reported in section 3. The control algorithm and the optimization problem are described in details in section 4. Conclusions are reported in section 5.

## 2. THE MODEL OF THE SIGNALIZED URBAN AREA

The proposed model of an urban area consists of  $n_I$  *signalized intersections*  $I_i$ ,  $i = 1, \dots, n_I$ , and  $n_R$  *roads*  $R_j$ ,  $j = 1, \dots, n_R$ . Each road is characterized by its *number of lanes*  $s_j$  and its *capacity*  $c_j$ , that is, the maximum number of vehicles which can stay at once in the road itself. Eastbound and westbound directions, as well as southbound and northbound, are separately considered, that is, each direction is modelled by a road. Also road lanes are separately considered. Let  $R_{j(k)}$ ,  $k = 1, \dots, s_j$ , denote the  $k$ -th *lane* in road  $R_j$ .

Let  $IN(I_i)$  and  $OUT(I_i)$  be the sets of *incoming roads* and *outgoing roads*, respectively, of intersection  $I_i$ . A *turning rate* value  $\alpha_{j(k),l(h)}$ , with  $0 \leq \alpha_{j(k),l(h)} \leq 1$ , is associated with each ordered pair  $(R_{j(k)}, R_{l(h)})$ , with  $R_j \in IN(I_i)$ ,  $k = 1, \dots, s_j$ , and  $R_l \in OUT(I_i)$ ,  $h = 1, \dots, s_l$ . Such a value expresses the percentage of vehicles coming from the  $k$ -th lane in road  $R_j$ , that is  $R_{j(k)}$ , and going to the  $h$ -th lane in road  $R_l$ , that is  $R_{l(h)}$ . A value  $\alpha_{j(k),l(h)} = 0$  means that direction  $R_{l(h)}$  is forbidden for vehicles coming from  $R_{j(k)}$ , whereas  $\alpha_{j(k),l(h)} = 1$  represents a mandatory direction. To guarantee the flow conservation, the condition

$$\sum_{l:R_l \in OUT(I_i)} \sum_{h=1}^{s_l} \alpha_{j(k),l(h)} = 1 \quad (1)$$

$\forall R_j \in IN(I_i)$ ,  $\forall k = 1, \dots, s_j$ , must always be verified for any signalized intersection  $I_i$ .

In multi-lane roads, vehicles are allowed to change lane according to the relevant traffic rules. Then, it is necessary to know the percentages of vehicles  $\beta_{j(k)}$ , with  $0 \leq \beta_{j(k)} \leq 1$ , which exit from road  $R_j$  using lane  $R_{j(k)}$ ,  $k = 1, \dots, s_j$ . Obviously, it must be

$$\sum_{k=1}^{s_j} \beta_{j(k)} = 1 \quad (2)$$

$\forall R_j : s_j > 1$ . Note that single-lane roads may be viewed as multi-lane roads with  $s_j = 1$ , and then all the previous considerations hold for single-lane roads, too. Examples of signalized intersections and of roads are shown in Fig. 1.

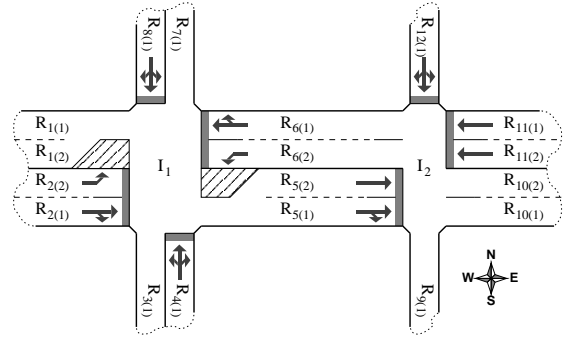


Fig. 1. An urban traffic area consisting of two intersections and twelve roads.

In order to model the behaviour of vehicles when crossing an intersection, the whole intersection area is divided into a finite number of parts (*crossing sections*), so as to take into account the physical space that a vehicle crossing the intersection occupies (Di Febbraro *et al.*, 2002a). Details of such a modelling aspect are here omitted for the sake of brevity, but it is worth noting that this choice will allow to easily and efficiently manage macroscopic entities (turning rates  $\alpha_{j(k),l(h)}$  and percentages  $\beta_{j(k)}$ ) within a microscopic representation tool (the deterministic-timed PN). In any case, the reader can refer to (Di Febbraro and Giglio, 2004) for a detailed description of the model and of the crossing sections.

A multi-stage *traffic signal*  $TS_i$  is always associated with a signalized intersection. The number of *stages* in its *cycle* depends on both the structure of the intersection and the allowed flow directions. In the following, the notation  $\phi_{i,p}$  will be used to indicate the  $p$ -th stage of the traffic signal  $TS_i$ .

All traffic signals in the considered urban traffic area are here managed at once. In this connection, it is possible to determine the stage specification for the whole area. Let it consist of  $F$  “*area*” *stages*, namely  $\psi_f$ ,  $f = 1, \dots, F$ . As an example, consider again the urban traffic area represented in Fig. 1. It consists of two intersections and twelve roads.  $I_1$  is controlled by a three-stage traffic signal (in stage  $\phi_{1,1}$ , vehicles in road lanes  $R_{2(1)}$  and  $R_{6(1)}$  find a green light; in  $\phi_{1,2}$ , vehicles in road lanes  $R_{2(2)}$  and  $R_{6(2)}$  find a green light; in  $\phi_{1,3}$ , vehicles in road lanes  $R_{4(1)}$  and  $R_{8(1)}$  find a green light), whereas  $I_2$  is controlled by a two-stage traffic signal (in stage  $\phi_{2,1}$ , vehicles in road lanes  $R_{5(1)}$ ,  $R_{5(2)}$ ,  $R_{11(1)}$ , and  $R_{11(2)}$  find a green light; in  $\phi_{2,2}$ , vehicles in road lane  $R_{12(1)}$  find a green light). A compatible whole area stage specification, consisting of five area stages, is represented in Fig. 2.

A very important aspect of the proposed model is that the duration of each area stage is not fixed, but constrained between a lower and an upper bound. Then, it can change in order to differently split the cycle, according to the traffic conditions.

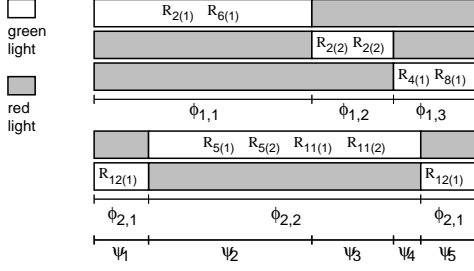


Fig. 2. A compatible stage specification for the urban traffic area represented in Fig. 1.

On the contrary, the *cycle time* is assumed to be constant and equal to  $C$ ; then, the following constraint must hold for any possible area stage specification:

$$\sum_{f=1}^F \psi_f = C \quad (3)$$

### 3. DETERMINISTIC-TIMED PETRI NETS AND URBAN AREA REPRESENTATION

The described traffic model is microscopically represented by means of timed Petri nets (TPNs), with the purpose of providing a suitable modelling tool for traffic management and control. In the adopted TPN model, timings are associated with transitions. The formal definition of the adopted TPN model is given in the following.

*Definition 1.* A *deterministic-timed Petri net* (DTPN) is a five-tuple  $\{P, T, Pre, Post, D\}$ , where  $P$  is a finite non-empty set of  $n = \text{card}(P)$  places, namely  $p_1, \dots, p_n$ ;  $T$  is a finite non-empty set of  $m = \text{card}(T)$  transitions, namely  $t_1, \dots, t_m$ ;  $Pre$  is a  $[n \times m]$  matrix (the pre-incidence matrix) whose element  $Pre_{i,j}$  is equal to  $w$  if an arc with weight  $w$  joining  $p_i$  and  $t_j$  exists, and 0 otherwise;  $Post$  is a  $[n \times m]$  matrix (the post-incidence matrix) whose element  $Post_{i,j}$  is equal to 1 if an arc with weight  $w$  joining  $t_j$  and  $p_i$  exists, and 0 otherwise;  $D : T \rightarrow (\mathbb{R}^+)^2$  is a function which associates a pair of non-negative real numbers, namely  $d_j^{\min}$  and  $d_j^{\max}$ ,  $d_j^{\max} \geq d_j^{\min}$ , with each transition in the net.

*Definition 2.* The *marking*  $M_i(\tau_k) \geq 0$  is the number of *tokens* which are within place  $p_i \in P$  at time instant  $\tau_k$ ,  $k = 1, 2, \dots$ ;  $M(\tau_k) = [M_i(\tau_k), i = 1, \dots, n]^T$  is the marking vector of the DTPN;  $M(\tau_0)$  is the initial marking vector.

In a DTPN, a pair of deterministic *firing times* is always associated with a transition of the net.  $d_j^{\min}$  represents the lower bound of the firing time of  $t_j$ , whereas  $d_j^{\max}$  is the upper bound. Transitions  $t_j$  such that  $d_j^{\max} = d_j^{\min} = 0$  will be denoted in the following as *immediate transitions* (graphically represented by thin bars), whereas transitions having  $d_j^{\min} > 0$  will be named *timed transitions* (represented by boxes). Moreover, the firing time

of a transition may be fixed and a-priori specified (in this case,  $d_j^{\max} = d_j^{\min}$ ), or not ( $d_j^{\max} > d_j^{\min}$ ). The DTPN representation of a given urban traffic system can be obtained by applying a suitable *synthesis* procedure which analyzes an instance of the urban traffic network model, and provides the net representation fulfilling the network properties. Details of such a procedure are not reported here, since the main objective of the paper is to define a traffic-responsive control procedure.

The resulting DTPN can be viewed as the merging of three distinct nets:

- the DTPN<sup>a</sup>, namely  $\{P^a, T^a, Pre^a, Post^a, D^a\}$ , representing signalized intersections and roads (Fig. 4). Tokens within such a net model either vehicles in the urban area or the availability of intersection/road sections. Such a net only includes immediate transitions or timed transitions whose firing time is a-priori specified (that is,  $d_j^{\max} = d_j^{\min} \forall t_j \in T^a$ ), but presents several conflicts necessary to model all the allowed vehicle drivers' behaviours in the area;  $M^a(\tau_k)$  denotes the marking vector of the DTPN<sup>a</sup>;
- the DTPN<sup>s</sup>, namely  $\{P^s, T^s, Pre^s, Post^s, D^s\}$ , representing the whole area staging (Fig. 3). Such a net models the a-priori defined stage specification of the urban area. It includes both immediate and timed transitions, and all the timed transitions have  $d_j^{\max} > d_j^{\min}$ , that is, the firing time  $d_j$  of each timed transition  $t_j \in T^s$  ranges in the interval  $[d_j^{\min}, d_j^{\max}]$ , and thus has to be specified through a suitable optimization algorithm, as described in the following section;  $M^s(\tau_k)$  denotes the marking vector of the DTPN<sup>s</sup>;

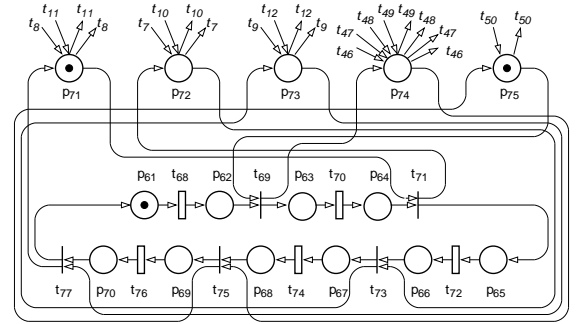


Fig. 3. The DTPN<sup>s</sup> representing traffic signal phases.

- the ordinary PN<sup>c</sup>, namely  $\{P^c, T^c, Pre^c, Post^c\}$ , representing macroscopic entities management. In particular, such a net solves conflicts which are in the DTPN<sup>a</sup>, according with percentages  $\alpha$  and  $\beta$  previously defined. This is accomplished by connecting certain places of the PN<sup>c</sup> with certain conflicting transitions of the DTPN<sup>a</sup>, thus preventing the firing of one between two conflicting transitions. The part of such a PN, only including immediate transi-

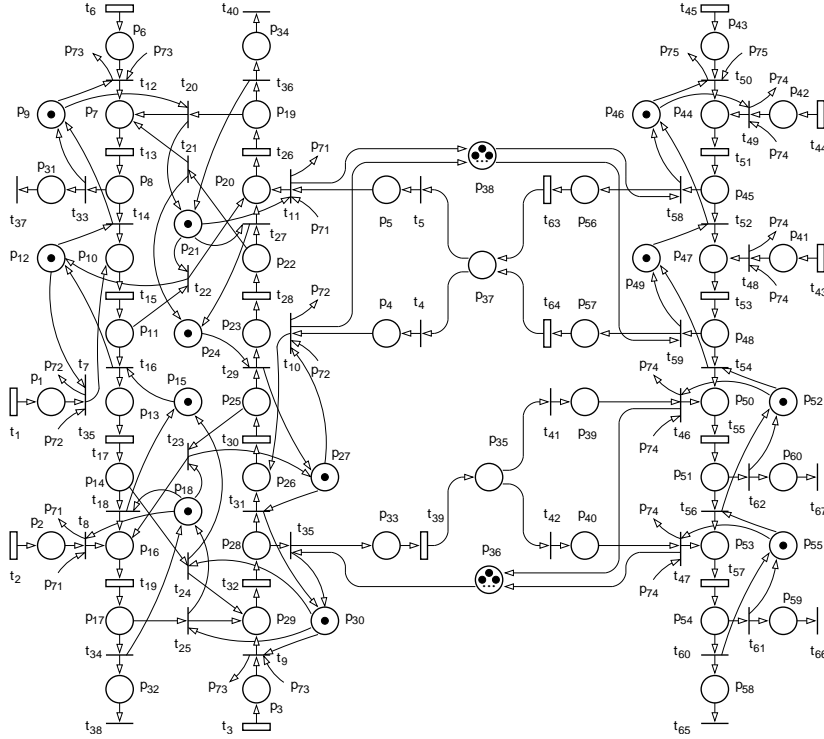


Fig. 4. The DTPN<sup>a</sup> representing signalized intersections and roads of the urban traffic area of Fig. 1.

tions, which solves the conflict between  $t_{14}$  and  $t_{33}$  in Fig. 4 is represented in Fig. 5.  $M^c(\tau_k)$  denotes the marking vector of the DTPN<sup>c</sup>.

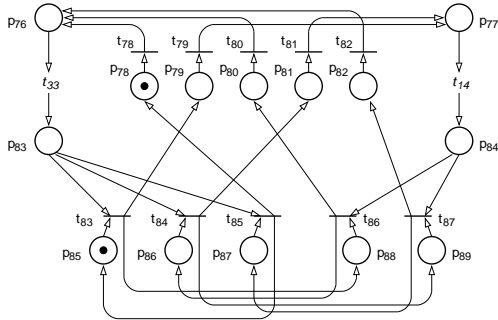


Fig. 5. Structural solution of the conflict between  $t_{14}$  and  $t_{33}$  of the DTPN<sup>a</sup> in Fig. 4.

The dynamics of the three DTPNs is expressed by their state equations. As some immediate transitions are shared between the DTPN<sup>a</sup> and the DTPN<sup>s</sup>, and between the DTPN<sup>a</sup> and the PN<sup>c</sup>, it is necessary to merge the three distinct nets, and then writing the state equations for the newly obtained overall deterministic-timed Petri net, namely  $DTPN^o = \{P^o, T^o, Pre^o, Post^o, D^o\}$ . The procedure which merges the three nets is quite simple (details of such a procedure are here omitted for the sake of brevity).

The DTPN state equations can be written assuming that the firing times of all timed transitions in the net are integer numbers with reference to a particular time unit  $\delta$  (*sampling interval*). Such an assumption introduces a certain degree of approximation in the representation of a real traffic system. However, traffic signal plans are usually

defined adopting a time unit equal to one second, and, moreover, the slow dynamics of urban traffic systems does not seem to require a shorter time unit. Then, it is reasonable to adopt a sampling interval equal to one second ( $\delta = 1s$ ). The assumption of integrity allows the analytic representation of the DTPN system state evolution. The DTPN system state, at a given (integer) time instant, is represented by the joint information consisting of both the marking of the net and the residual firing times of firing timed transitions.

*Definition 3.* The *residual firing time*  $\Theta_j(\tau_k) \geq 0$  is a vector containing the numbers of *time intervals* which have to elapse until the end of the firings of transition  $t_j \in T$ , at time instant  $\tau_k$ ,  $k = 1, 2, \dots$ ; the generic element of  $\Theta_j(\tau_k)$  is  $\Theta_{j,h}(\tau_k)$ , with  $h = 1, \dots, b_i$ , being  $b_i$  the upper bound (*capacity*) of input place  $p_i$  of  $t_j$ , that is,  $i : p_i \bullet \equiv t_j$ .

The above definition makes the firing of a timed transition possible even if a firing is already occurring in that transition. This is necessary in the adopted DTPN representation of roads. It is worth noting that, in the DTPN model, each timed transition has one and only one input place, and it is possible to prove that the input place of each timed transition has finite capacity, that is, the number of tokens which can be in that place, whichever firing sequence occurs, is upper bounded. Moreover, it is assumed  $\Theta_{j,h}(\tau_0) = 0$ ,  $\forall j \in \{1, \dots, m\}$ ,  $\forall h \in \{1, \dots, b_i\}$ , that is, no timed transition is firing at the initial time instant. According with the previous definitions, the state of a DTPN is given by

$$\underline{x}(\tau_k) = [M_i(\tau_k), i = 1, \dots, n, \Theta_{j,h}(\tau_k), \\ j = 1, \dots, m, h = 1, \dots, b_i, \\ i : p_i^\bullet \equiv t_j] \quad (4)$$

The state evolution takes place by considering alternately *zero-length* time intervals, in which only immediate transitions fire, and *one-second-length* time intervals, in which only timed transitions fire or “burn” one time unit of their firing (Di Febraro *et al.*, 2002b). In particular, between time instants  $\tau_k$  and  $\tau_k + \delta$  the state evolves according to the following two-step algorithm (in the following, it will be assumed  $\delta = 1$ s and  $\tau_k$  expressed in seconds as well).

*Algorithm 1.*

**1. Zero-length phase.** At  $\tau_k$ , all the enabled immediate transitions, if any, fire simultaneously (this is possible since the DTPN<sup>o</sup> is *conflict-free*, as all the conflicts which are in the DTPN<sup>a</sup> are structurally solved by the PN<sup>c</sup>). The firing of an immediate transition may cause the enabling of further transitions, which were initially not enabled at  $\tau_k$ ; then, the “zero-length” time interval consists of some subsequent firings of immediate transitions, all occurring at  $\tau_k$ , in the logical step  $r = 0, 1, 2, \dots$ . This is ruled by the following two equations:

$$M_i^o(\tau_k, r+1) = M_i^o(\tau_k, r) + \\ + \underline{\rho}_i^T \cdot C^o \cdot \underline{\sigma}_j(\tau_k, r) \quad (5)$$

$\forall i : p_i \in P^o$ , where  $\underline{\rho}_i$  is a vector which selects the  $i$ -th place of the net, and  $\underline{\sigma}_j(\tau_k, r)$  is a vector whose  $j$ -th element is equal to 1 if immediate transition  $t_j \in T^o$  is enabled, at  $\tau_k$ , in logical step  $r$ , and 0 otherwise, and whose  $p$ -th element,  $p \neq j$ , is 0;

$$\Theta_j^o(\tau_k, r+1) = \Theta_j^o(\tau_k, r) \quad (6)$$

for any  $j : t_j \in T^o$ . Equations (5) and (6) are initialized,  $\forall i : p_i \in P^o$  and  $\forall j : t_j \in T^o$ , by

$$M_i^o(\tau_k, 0) = M_i^o(\tau_k) \quad (7)$$

$$\Theta_j^o(\tau_k, 0) = \Theta_j^o(\tau_k) \quad (8)$$

However, it is very important to observe that, whatever firing occurs, the system reaches a marking in which no immediate transition is enabled. This results from the absence in the DTPN<sup>o</sup> of *closed paths* (“cycles”) which only contain immediate transitions.

**2. One-second-length phase.** At  $\tau_k$ , all the enabled timed transitions, if any, fire simultaneously, and all the timed transitions that are already firing, if any, “burn” 1 second of their firing times.

In the first case, the state of the net evolves as follows:

$$M_i^o(\tau_k + 1) = M_i^o(\tau_k, R) \quad (9)$$

$$\forall i : p_i \in P^o;$$

$$\Theta_{j,h}^o(\tau_k + 1) = d_j - 1 \quad (10)$$

for any  $j$  such that timed transition  $t_j \in T^o$  is enabled in marking  $M^o(\tau_k, R)$  (the enabling conditions of both immediate and timed transitions are here omitted), and  $h$  such that  $\Theta_{j,h}^o(\tau_k, R) = 0$  and  $\Theta_{j,h'}^o(\tau_k, R) > 0$ ,  $\forall h' = 1, \dots, h - 1$ .

In the second case, being  $t_j \in T^o$  the transition which is already firing, if there exists  $h$  such that  $\Theta_{j,h}^o(\tau_k, R) = 1$ , then

$$M_i^o(\tau_k + 1) = M_i^o(\tau_k, R) + \\ + \underline{\rho}_i^T \cdot C^o \cdot \underline{\sigma}_j(\tau_k, R) \quad (11)$$

$\forall i : p_i \in P^o$ , else if  $\Theta_{j,h}^o(\tau_k, R) > 1$  or  $\Theta_{j,h}^o(\tau_k, R) = 0$ ,  $\forall h = 1, \dots, b_i$ ,  $i : p_i^\bullet \equiv t_j$ , then

$$M_i^o(\tau_k + 1) = M_i^o(\tau_k, R) \quad (12)$$

$$\forall i : p_i \in P^o;$$

$$\Theta_{j,h}^o(\tau_k + 1) = \Theta_{j,h}^o(\tau_k, R) - 1 \quad (13)$$

$\forall h \in \{1, \dots, b_i\}$ ,  $i : p_i^\bullet \equiv t_j$ , such that  $\Theta_{j,h}^o(\tau_k, R) > 0$ .

Note that the condition  $\Theta_j^o(\tau_k, R) = 1$  for which (11) is defined means that the firing transition will end its firing at the end of the considered sampling period. For this reason, tokens are removed from the input places of the firing transition and inserted into the output ones.

For any transition  $t_j \in T^o$  which is neither enabled or already firing simply holds

$$\Theta_j^o(\tau_k + 1) = 0 \quad (14)$$

#### 4. OPTIMIZATION OF AREA STAGES

In the proposed model of an urban area, the duration of area stages, as defined in section 2, may vary according with traffic conditions. This is modelled in the DTPN representation (and, in particular, in the DTPN<sup>s</sup>) by deterministic-timed transitions whose firing times may range within a given interval. Then, the main object of the paper is to provide an optimization algorithm which determines the optimal stages (through the determination of the optimal firing times in the DTPN<sup>s</sup>), as described in the following.

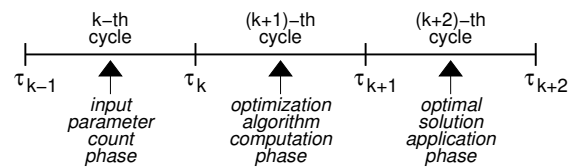


Fig. 6. Schematization of the optimization of area stages

Assume that the number of vehicles which enter the considered area, in the time interval  $[\tau_{k-1}, \tau_k)$

( $k$ -th cycle), can be counted (through, for instance, suitable counter devices such as electromagnetic induction loops and/or visual recognizers). This represents the *input parameter count phase* (Fig. 6). Moreover, assume that the traffic system dynamics is “slow” (such an hypothesis is usually fulfilled by urban traffic systems), then such a number can be considered to be an estimate of the number of the vehicles which will enter the considered area in the subsequent time interval, that is, in  $[\tau_k, \tau_{k+1}]$  ( $(k+1)$ -th cycle). Then, at  $\tau_k$ , it is possible to set the firing time of “input” (or “source”) transitions (in the proposed DTPN representation, an input transition is a timed transition having no incoming arcs, as  $t_1, t_2, t_6, t_{43}, t_{44}$ , and  $t_{45}$  in Fig. 4), so that the number of tokens generated by their firings during the time interval  $[\tau_k, \tau_{k+1}]$  is equal to the number of counted vehicles during the interval  $[\tau_{k-1}, \tau_k]$ .

The optimal area stages  $\psi_{k,f}^\circ$ ,  $f = 1, \dots, F$ , are determined in the time interval  $[\tau_k, \tau_{k+1}]$  through the solution of an optimization problem whose objective function minimizes the number of tokens within the net (note that, as the numbers of tokens within the DTPN<sup>s</sup> and the PN<sup>c</sup> are constant, it is sufficient to minimize the number of tokens within the DTPN<sup>a</sup>), by optimizing the firing times of the timed transitions of the DTPN<sup>s</sup>. This corresponds to minimize the number of vehicles which are in the area at the end of the considered time interval, that is, at  $\tau_{k+1}$ , by suitably setting the length of stages. The objective function is subject to the cycle length constraint (3) and to the DTPNs dynamic equations provided by (5)÷(14), and initialized by the marking vectors  $M^a(\tau_k)$ ,  $M^s(\tau_k)$ , and  $M^c(\tau_k)$ . This represents the *optimization algorithm computation phase*. Then, the computed optimal area stages are applied to the real system from time  $\tau_{k+1}$ , that is, within the  $(k+2)$ -th semaphoric cycle, which is the *optimal solution application phase*. The whole procedure can be summarized as follows.

*Algorithm 2.*

1. Within the time interval  $[\tau_{k-1}, \tau_k]$ , count the number of vehicles entering the area from road lane  $R_{p(q)}$ , for any  $p$  such that  $R_p \in IN(I_s)$ ,  $s = 1, \dots, n_I$ , and  $R_p \notin OUT(I_t)$ ,  $t = 1, \dots, n_I$ ,  $t \neq s$ , and for any  $q = 1, \dots, s_p$ . Let  $\sigma_{p(q)}$  be the number of counted vehicles.
2. At time  $\tau_k$ , set  $d_j^{\max} = d_j^{\min} = \lfloor \frac{C}{\sigma_{p(q)}} \rfloor$ , being  $j$  such that  $t_j \in T^a$  represents road lane  $R_{p(q)}$ .
3. Within the time interval  $[\tau_k, \tau_{k+1}]$ , compute  $\psi_{k,f}^\circ$ ,  $f = 1, \dots, F$ , by solving

$$\min_{d_j} \sum_{i \in P^a} M_i^a(\tau_{k+1}) \quad (15)$$

$$j : t_j \in T^s, d_j^{\min} > 0$$

subject to

$$\sum_{j : t_j \in T^s, d_j^{\min} > 0} d_j = C \quad (16)$$

$$M_i^a(\tau_{k+1}) = \mathcal{G} \left( \begin{array}{l} \text{DTPN}^a, M^a(\tau_k), \\ \text{DTPN}^s, M^s(\tau_k), \\ \text{PN}^c, M^c(\tau_k) \end{array} \right) \quad (17)$$

where  $\mathcal{G}$  represents a structured procedure which carries out the dynamic evolution of the net, i.e., executes the “token game”. In particular, the equations implemented in  $\mathcal{G}$  are those described by (5)÷(14) in Algorithm 1.

4. At time  $\tau_{k+1}$ , set  $\psi_{k+1,f} = \psi_{k,f}^\circ$ ,  $f = 1, \dots, F$ .

## 5. CONCLUSIONS

In this paper, an algorithm for the optimization of traffic signal stages, within urban areas, has been introduced and discussed. It is worth noting that the adopted PN representation can be viewed as a set of interconnected “smaller” nets, each of them representing a particular intersection or a particular road. Such a modularity is very appreciated when building large nets representing wide urban areas. The proposed model as well as the optimization of the area stages are being tested in a real traffic area in the city of Genova, North-West of Italy.

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