LONGITUDINAL CONTROL FOR A LABORATORY HELICOPTER VIA CONSTRUCTIVE APPROXIMATE BACKSTEPPING

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Abstract: This article presents a controller design based on Lyapunov functions, which can be applied to a class of underactuated systems that cannot be feedback linearized in a specific set point. Particularly, this design will be applied in order to control the longitudinal dynamics of a laboratory helicopter. The longitudinal system is an electro-mechanical, non-linear and underactuated system. The proposed methodology consists of modifying the model of the system using an overparametrization in such a way that the backstepping theory can be applied to the approximate system. The methodology has been tested by simulated and experimental results and it has been proved for the closed-loop system that the origin is LES.

Keywords: Non-linear Control, Lyapunov Functions, Feedback Linearization, Helicopters, Double Rotor.

1. INTRODUCTION

This article presents a control design using Lyapunov functions. The design methodology can be used for a class of underactuated systems in which the backstepping technique cannot be directly applied, because this class of systems cannot be feedback linearized in the set point chosen.

This article does not impose as an essential condition that the system is controllable in all the workspace, as mentioned in Lai et al. (1994), but it does impose that an approximate model can be obtained in such a way that the system can be considered controllable in all the workspace.

The controller design will be tested on a system which represents the longitudinal motion of a laboratory helicopter.

The remainder of the paper is structured as follows:

Section 2, provides an overview of the considered system and the model chosen. Some considerations on the system controllability will be taken into account in *Section 3*. *Section 4* presents the controller design and proves the closed-loop stability. *Section 5* shows simulated results and finally *Section 6* provides a set of remarks and conclusions to the paper.

2. SYSTEM DESCRIPTION AND MODEL

The laboratory helicopter consists of a 2 DOF mechanism thrusted by two rotors resembling a

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helicopter. The degrees of freedom are the yaw and the pitch angles. This equipment has the following characteristics: It is multivariable, underactuated, nonlinear, strongly coupled and with non-minimum phase behaviour.

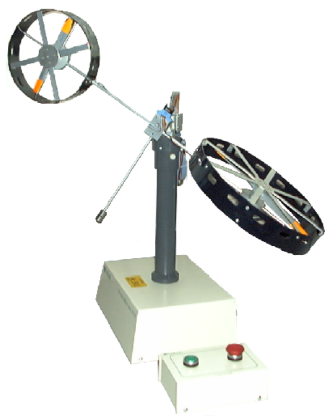

Fig. 1. Double Rotor Laboratory Helicopter

In this analysis, the yaw angle is fixed ($\theta = const$), and the angular velocity of the tail rotor is null ($\omega_t = const = 0$). The longitudinal motion will be controlled by the main rotor.

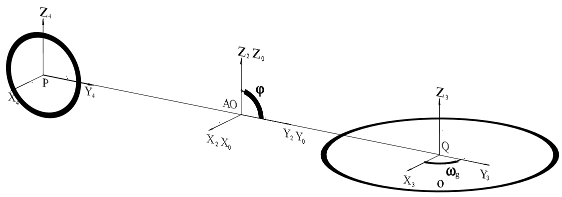

Fig. 2. Longitudinal subsystem

The equations of the longitudinal subsystem are as follows

$$I_\varphi \ddot{\varphi} + GS(\varphi - \varphi_{eq}) + K_\varphi \cdot \dot{\varphi} = L_g |\omega_g| \omega_g \quad (1)$$

$$I_g \dot{\omega}_g = u - (B_g + D_g |\omega_g|)\omega_g \quad (2)$$

where:

- φ: Longitudinal Angle measured from the horizontal plane.
- I_φ: Inertia of the longitudinal system with respect to its rotation axis.
- ω_g: Angular Velocity of the main rotor.
- I_g: Inertia of the propeller with respect to its rotation axis.
- $\hat{L}_g \omega_g$: Torque due to the aerodynamic force of propulsion in main rotor.
- $K_\varphi \cdot \dot{\varphi}$: Friction Torque.
- $GS(.)$: Gravity Torque. ($S(.) = sin(\varphi - \varphi_{eq})$)
- u: Engine Torque.
- B_g: Friction constant of the engine.
- $\hat{D}_g$: Drag Constant of the propeller.

It can be seen that there is only one engine (u) and 2 DOF, the pitch angle (φ) and the angular velocity of the rotor (ω_g). Therefore it is an underactuated system in the sense that it has less control inputs than degrees of freedom (see Fantoni and Lozano (2002) for details).

3. CONTROLLABILITY ANALYSIS

In this section, the controllability of the proposed system will be analyzed before posing a control structure. In this way the equations will be expressed as follows:

$$\dot{X} = f_{(X)} + g_{(X)} \cdot u$$

where the state vector has been defined as:

$$X = \begin{bmatrix} \varphi - \varphi_{eq} \\ \dot{\varphi} \\ \omega_g \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (3)$$

resulting

$$f_{(X)} = \begin{bmatrix} x_2 \\ \dfrac{-GS(x_1) - K_\varphi \cdot x_2 + L_g |x_3| x_3}{I_\varphi} \\ \dfrac{-(B_g + D_g |x_3|) x_3}{I_g} \end{bmatrix}$$

$$g_{(X)} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{I_g} \end{bmatrix}$$

The system will be controllable if the vector fields $\{g, ad_f(g), ad_f^2(g)\}$ are linearly independent, (see R.Su (1981), Spong and Vidyasagar (1989)), where $ad_f(g)$ is defined as $ad_f(g) = [f, g] = \nabla g \cdot f - \nabla f \cdot g$.

In order to simplify the exposition, the values of the velocity of the rotor will be considered positive, that is to say that $x_3 > 0$. Taking this fact into account, the following vector fields are obtained.

$$ad_f^0(g) = g$$

$$ad_f(g) = \begin{bmatrix} 0 \\ -\dfrac{L_g}{I_g I_\varphi} 2x_3 \\ \dfrac{1}{I_g^2}(B_g + 2x_3 D_g) \end{bmatrix}$$

and

$$ad_f^2(g) = \begin{bmatrix} \dfrac{2L_g}{I_g I_\varphi} x_3 \\ -\dfrac{2K_\varphi L_g}{I_g I_\varphi^2} x_3 - \dfrac{2L_g D_g}{I_g^2 I_\varphi} x_3^2 \\ -\dfrac{2D_g}{I_g^3}(B_g + D_g x_3) x_3 + \dfrac{1}{I_g^3}(B_g + D_g 2x_3)^2 \end{bmatrix}$$

It can be noticed that the vector fields $\{g, ad_f(g), ad_f^2(g)\}$ are linearly independent if and only if $x_3 \neq 0$. That means that the system is controllable if $x_3 \neq 0$, that is to say that the velocity of the rotor is not equal to zero.

4. CONTROLLER DESIGN

The backstepping technique will be used in order to design the controller using Lyapunov functions. This technique can be found, for example, in Khalil (1996), Krstic et al. (1995) and Sepulchre et al. (1997).

When the system can be expressed as a strict-feedback system

$$\dot{x} = f(x) + g(x)\xi \tag{4}$$

$$\dot{\xi} = f_1(x, \xi) + g_1(x, \xi)u \tag{5}$$

the theory developed so far only copes with systems whose first equation (4) has a linear control variable, that is to say, expressed in terms of u.

In the proposed system, the equation of the system are given by (1) and (2) where (1) has a quadratic control variable, where ω_g has been considered as the control variable of the subsystem.

In order to express the equations of the system in such a way that the first equation has a linear control variable, the following change of variables is defined:

$$\xi = L_g|\omega_g|\omega_g$$

where

$$\dot{\xi} = \frac{\partial \xi}{\partial \omega_g}\dot{\omega_g} = 2L_g|\omega_g|\dot{\omega_g}$$

ξ represents the torque applied to the axis of the system and now it is the new control signal.

The equations of the system are changed, when applying the definition of the state vector, into the following form:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \frac{-GS(x_1) - Kx_2}{I_\varphi} + \frac{1}{I_\varphi}\xi\\ \dot{\xi} &= f_1(x_3) + g_1(x_3)u \doteq \hat{u}\end{aligned}$$

where

$$\begin{aligned}f_1(x_3) &= \frac{\partial \xi}{\partial x_3}\frac{-(B_g + D_g|x_3|)x_3}{I_g}\\ g_1(x_3) &= \frac{1}{I_g}\frac{\partial \xi}{\partial x_3}\end{aligned}$$

It can be noticed that now the first equation block has a linear control variable. However, the change of variables has introduced a term, $g_1(x_3)$, which is zero when $\frac{\partial \xi}{\partial x_3} = 2L_g|x_3| = 0$. And this happens when $x_3 = 0$, that is to say, when the velocity of the rotor is zero, $\omega_g = 0$.

If the problem were solved using $\hat{u}$, the system would have integrator-backstepping structure. In this way, the change of variables would have to be undone, in order to obtain the value of u given by:

$$u = \frac{\hat{u}}{g_1(x_3)} - \frac{f_1(x_3)}{g_1(x_3)}$$

It can be noticed that this law is not valid when the velocity of the rotor is zero, which represents the same condition of non-controllability of the system.

Therefore, this law is quasi-global, in the sense that it is not valid only when $x_3 = 0$. In practice, for a small value of x_3, the control signal applied to the engine would be too high and would make it saturate.

4.1 Constructive approximate backstepping

The idea of a Constructive Approximate Backstepping consists of modifying g_1, in such a way that the control law is well-defined in the whole workspace. Modifying g_1 implies modifying $\frac{\partial \xi}{\partial x_3}$ and, in its turn, ξ.

Therefore, the method consists of studying another way of modelling the propulsion forces of the system, in such a way that the system is controllable and consequently the new g_1 never is zero. This will be done augmenting the model of the propulsion forces.

Starting from $g_1(x_3) = \frac{1}{I_g}\frac{\partial \xi}{\partial x_3}$, a new function $\hat{g}_1(x_3) \neq 0\ \forall x_3$ is searched. The simplest method consists of adding a constant value $\boldsymbol{\epsilon}$ to the function $\frac{\partial \xi}{\partial \omega_g}$. In this way, the result is given by $\frac{\partial \hat{\xi}}{\partial x_3} = 2L_g|x_3| + \boldsymbol{\epsilon} \neq 0\ \forall x_3$. Next, a new $\hat{\xi} = L_g|x_3|x_3 + \boldsymbol{\epsilon} x_3$ is chosen which hereinafter will be taken as the new model of propulsion.

Studying the controllability of the system for $x_3 = 0$, the following results are obtained:

$$\left(g, ad_f g, ad_f^2 g\right) = \begin{bmatrix} 0 & 0 & \frac{\boldsymbol{\epsilon}}{I_\varphi I_g} \\ 0 & -\frac{\boldsymbol{\epsilon}}{I_\varphi I_g} & -\boldsymbol{\epsilon}(\frac{K_\varphi}{I_\varphi^2 I_g} + \frac{B_g}{I_\varphi I_g^2}) \\ \frac{1}{I_g} & \frac{B_g}{I_g^2} & \frac{B_g^2}{I_g^3} \end{bmatrix}$$

The vector fields are linearly independent, and the determinant is equal to $-\frac{\boldsymbol{\epsilon}^2}{I_\varphi^2 I_g^3} \neq 0$. Therefore, the new model of the system is controllable for any values of the state vector.

4.2 Lyapunov function for the first subsystem

The dynamics of the first subsystem is given by the equation:

$$I_\varphi \ddot{\varphi} + GS(\hat{\varphi}) + K_\varphi \dot{\varphi} = \hat{\xi}$$

This equation corresponds to the dynamics of a simple pendulum but with a different static equilibrium point φ_{eq}, where a new variable has been defined: $\hat{\varphi} = \varphi - \varphi_{eq}$.

For the sake of simplicity, the Lyapunov function candidate will be chosen as the energy of the pendulum, which is given by:

$$W = E_c + E_p = G(1 - C(\hat{\varphi})) + \frac{1}{2} I_\varphi \dot{\varphi}^2$$

and is locally positive definite. $\forall(\hat{\varphi}, \dot{\varphi}) \neq 0$

On the other hand,

$$\begin{aligned} \dot{W} &= \frac{\partial W}{\partial \hat{\varphi}} \dot{\varphi} + \frac{\partial W}{\partial \dot{\varphi}} \ddot{\varphi} = GS(\hat{\varphi})\dot{\varphi} + I_\varphi \dot{\varphi}\ddot{\varphi} \\ &= GS(\hat{\varphi})\dot{\varphi} + \dot{\varphi}(-GS(\hat{\varphi}) - K_\varphi \dot{\varphi} + \hat{\xi}) \\ &= \dot{\varphi}(-K_\varphi \dot{\varphi} + \hat{\xi}) \end{aligned}$$

In order to verify $\dot{W} < 0$ the following inequality will be imposed $\hat{\xi} < K_\varphi \dot{\varphi}$. Taking $\hat{\xi} = \bar{K}_\varphi \dot{\varphi}$ and $\bar{K}_\varphi < K_\varphi$ it yields

$$\dot{W} = -[K_\varphi - \bar{K}_\varphi]\dot{\varphi}^2 = -\tilde{K}_\varphi \dot{\varphi}^2 < 0$$

Therefore, the resultant closed-loop dynamics will be given by:

$$I_\varphi \ddot{\varphi} + GS(\hat{\varphi}) + \tilde{K}_\varphi \dot{\varphi} = 0$$

It can be noticed that adding viscous friction the behaviour of the system can be changed, making the system less oscillatory.

That would imply making $\tilde{K}_\varphi = K_\varphi - \bar{K}_\varphi$ increase, so $\bar{K}_\varphi < 0$. Therefore a new positive constant is defined in such a way that $\hat{K}_\varphi = -\bar{K}_\varphi > 0$

4.3 Lyapunov function for the complete system

In order to obtain a controller for the complete system, the Lyapunov function designed to control the first subsystem will be used. The closed-loop dynamics of the first subsystem will be considered as the desired dynamics for the first subsystem of the complete system.

In the same way, the control signal in the first subsystem will be considered as the desired control signal.

Taking this into account it results:

$$\begin{aligned} \hat{\xi}_{des} &= -\hat{K}_\varphi \dot{\varphi} \\ \dot{\hat{\xi}}_{des} &= -\hat{K}_\varphi \ddot{\varphi} \\ \tilde{\xi} &= \hat{\xi} - \hat{\xi}_{des} = \hat{\xi} + \hat{K}_\varphi \dot{\varphi} \\ \dot{\tilde{\xi}} &= \dot{\hat{\xi}} - \dot{\hat{\xi}}_{des} = \hat{u} + \hat{K}_\varphi \ddot{\varphi} \\ \ddot{\varphi} &= \frac{1}{I_\varphi}(-GS(\hat{\varphi}) - K_\varphi \dot{\varphi} + \hat{\xi}) \\ &= \frac{1}{I_\varphi}(-GS(\hat{\varphi}) - K_\varphi \dot{\varphi} + \tilde{\xi} + \hat{\xi}_{des}) \\ &= \frac{1}{I_\varphi}(-GS(\hat{\varphi}) - K_\varphi \dot{\varphi} + \tilde{\xi} - \hat{K}_\varphi \dot{\varphi}) \\ &= \frac{1}{I_\varphi}(-GS(\hat{\varphi}) - \tilde{K}_\varphi \dot{\varphi} + \tilde{\xi}) \\ \dot{\tilde{\xi}} &= \hat{u} + \frac{\hat{K}}{I_\varphi}(-GS(\hat{\varphi}) - \tilde{K}_\varphi \dot{\varphi} + \tilde{\xi}) \end{aligned}$$

Next, the following Lyapunov function candidate is proposed:

$$\begin{aligned} V &= W + \frac{1}{2}\tilde{\xi}^2 \\ &= G(1 - C(\hat{\varphi})) + \frac{1}{2} I_\varphi \dot{\varphi}^2 + \frac{1}{2}\tilde{\xi}^2 \end{aligned}$$

Its derivative yields

$$\begin{aligned} \dot{V} &= \dot{W} + \tilde{\xi}\dot{\tilde{\xi}} \\ &= \frac{\partial W}{\partial \hat{\varphi}} \dot{\varphi} + \frac{\partial W}{\partial \dot{\varphi}} \ddot{\varphi} + \tilde{\xi}\dot{\tilde{\xi}} \\ &= GS(\hat{\varphi})\dot{\varphi} + I_\varphi \dot{\varphi}\ddot{\varphi} + \tilde{\xi}\dot{\tilde{\xi}} \\ &= GS(\hat{\varphi})\dot{\varphi} + \dot{\varphi}(-GS(\hat{\varphi}) - \tilde{K}_\varphi \dot{\varphi} + \tilde{\xi}) + \tilde{\xi}\dot{\tilde{\xi}} \\ &= \dot{\varphi}(-\tilde{K}_\varphi \dot{\varphi} + \tilde{\xi}) + \tilde{\xi}\dot{\tilde{\xi}} \\ &= -\tilde{K}_\varphi \dot{\varphi}^2 + \tilde{\xi}(\dot{\varphi} + \dot{\tilde{\xi}}) \end{aligned}$$

and substituting for $\dot{\tilde{\xi}}$

$$\dot{V} = -\tilde{K}_\varphi \dot{\varphi}^2 + \tilde{\xi}\left(\dot{\varphi} + \hat{u} + \frac{\hat{K}}{I_\varphi}(-GS(\hat{\varphi}) - \tilde{K}_\varphi \dot{\varphi} + \tilde{\xi})\right)$$

The control signal is obtained making $\dot{V} < 0$. In this way the parenthesis of the second term could be equaled to $-p\tilde{\xi}$, yielding

$$\begin{aligned} \dot{V} &= -\tilde{K}_\varphi \dot{\varphi}^2 - p\tilde{\xi}^2 \\ -p\tilde{\xi} &= \dot{\varphi} + \hat{u} + \frac{\hat{K}}{I_\varphi}(-GS(\hat{\varphi}) - \tilde{K}_\varphi \dot{\varphi} + \tilde{\xi}) \end{aligned}$$

$$\begin{aligned} \hat{u} &= \frac{\hat{K}}{I_\varphi}(GS(\hat{\varphi}) + \tilde{K}_\varphi \dot{\varphi}) - \frac{\hat{K}}{I_\varphi}\tilde{\xi} - \dot{\varphi} - p\tilde{\xi} \\ &= \frac{\hat{K}}{I_\varphi}GS(\hat{\varphi}) + \left(\frac{\hat{K}}{I_\varphi}\tilde{K}_\varphi - 1\right)\dot{\varphi} - \left(\frac{\hat{K}}{I_\varphi} + p\right)\tilde{\xi} \\ &= \frac{\hat{K}}{I_\varphi}GS(\hat{\varphi}) + \left(\frac{\hat{K}}{I_\varphi}\tilde{K}_\varphi - 1\right)\dot{\varphi} - \left(\frac{\hat{K}}{I_\varphi} + p\right)(\hat{\xi} + \hat{K}_\varphi \dot{\varphi}) \\ &= \frac{\hat{K}}{I_\varphi}GS(\hat{\varphi}) + \left(\frac{\hat{K}}{I_\varphi}K_\varphi - 1 - p\hat{K}_\varphi\right)\dot{\varphi} - \left(\frac{\hat{K}}{I_\varphi} + p\right)\hat{\xi} \\ &= k_1 S(\hat{\varphi}) - k_2 \dot{\varphi} - k_3 \hat{\xi} \end{aligned}$$

where applying the constructive approximation of the model of propulsion, it yields

$$\hat{\xi} = L_g|x_3|x_3 + \boldsymbol{\epsilon} x_3$$

On the other hand, the control signal which is applied to the engine is given by the following expression:

$$u = \frac{\hat{u}}{g_1(x_3)} - \frac{f_1(x_3)}{g_1(x_3)}$$

where

$$\begin{aligned} \frac{f_1(x_3)}{g_1(x_3)} &= -(B_g + D_g|x_3|)x_3 \\ g_1(x_3) &= \frac{1}{I_g}\frac{\partial \xi}{\partial x_3} \\ \frac{\partial \hat{\xi}}{\partial x_3} &= 2L_g|x_3| + \boldsymbol{\epsilon} \end{aligned}$$

Therefore

$$u = \frac{\hat{u}}{\frac{2L_g|x_3|+\boldsymbol{\epsilon}}{I_g}} + (B_g + D_g|x_3|)x_3$$

It can be noticed that this law is well-defined for all values of the velocity of the rotor $x_3 = \omega_g$

4.4 Closed-loop stability analysis

Compiling the equations that give rise to the designed controller,

$$\begin{aligned} \hat{\xi} &= L_g|x_3|x_3 + \boldsymbol{\epsilon} x_3 \\ \hat{u} &= k_1 S(\hat{\varphi}) - k_2\dot{\varphi} - k_3\hat{\xi} \\ u &= \frac{\hat{u}}{\frac{2L_g|x_3|+\boldsymbol{\epsilon}}{I_g}} + (B_g + D_g|x_3|)x_3 \end{aligned}$$

and substituting for $\boldsymbol{u}$ it yields:

$$\begin{aligned} u = &\frac{k_1 S(\hat{\varphi}) - k_2\dot{\varphi} - k_3(L_g|x_3|x_3 + \boldsymbol{\epsilon} x_3)}{\frac{2L_g|x_3|+\boldsymbol{\epsilon}}{I_g}} + \\ &+ (B_g + D_g|x_3|)x_3 \end{aligned}$$

On the other hand, the equations of the system are given by:

$$\dot{X} = f_{(X)} + g_{(X)} \cdot \boldsymbol{u}$$

$$\dot{X} = \begin{bmatrix} x_2 \\ \dfrac{-GS(x_1) - K_\varphi \cdot x_2 + \hat{L}_g|x_3|x_3}{I_\varphi} \\ \dfrac{-(B_g + D_g|x_3|)x_3}{I_g} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{I_g} \end{bmatrix} \cdot \boldsymbol{u}$$

and substituting for the value of $\boldsymbol{u}$, the closed loop equations are obtained:

$$\dot{X} = \begin{bmatrix} x_2 \\ \dfrac{-GS(x_1) - K_\varphi \cdot x_2 + \hat{L}_g|x_3|x_3}{I_\varphi} \\ \dfrac{k_1 S(x_1) - k_2 x_2 - k_3 x_3(L_g|x_3| + \boldsymbol{\epsilon})}{2L_g|x_3| + \boldsymbol{\epsilon}} \end{bmatrix}$$

Lyapunov's linearization method

Firstly, Lyapunov's linearization method will be applied in order to study the stability of the equilibrium point $X = (0, 0, 0)$.

Defining the matrix A as the Jacobian of f respect to X for $X = (0, 0, 0)$, then $\dot{X} = AX$ is the linearization of the system at this equilibrium point.

Computing the Jacobian of the system, it yields:

$$\begin{bmatrix} 0 & 1 & 0 \\ \dfrac{-G}{I_\varphi}C(x_1) & \dfrac{-K_\varphi}{I_\varphi} & \dfrac{2L_g}{I_\varphi}|x_3| \\ \dfrac{K_1 C(x_1)}{2L_g|x_3| + \boldsymbol{\epsilon}} & \dfrac{-K_2}{2L_g|x_3| + \boldsymbol{\epsilon}} & K_3\left(-1 + \dfrac{2L_g|x_3|(L_g|x_3| + \boldsymbol{\epsilon})}{(2L_g|x_3| + \boldsymbol{\epsilon})^2}\right) \end{bmatrix}$$

and substituting for $X = (x_1, x_2, x_3) = (0, 0, 0)$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{-G}{I_\varphi} & \dfrac{-K_\varphi}{I_\varphi} & 0 \\ \dfrac{K_1}{\boldsymbol{\epsilon}} & \dfrac{-K_2}{\boldsymbol{\epsilon}} & -K_3 \end{bmatrix}$$

Computing the eigenvalues of A,

$$|SI - A| = \begin{vmatrix} s & -1 & 0 \\ \dfrac{G}{I_\varphi} & s + \dfrac{K_\varphi}{I_\varphi} & 0 \\ \dfrac{-K_1}{\boldsymbol{\epsilon}} & \dfrac{K_2}{\boldsymbol{\epsilon}} & s + K_3 \end{vmatrix} = 0$$

it yields

$$\begin{aligned} |SI - A| &= (s + K_3)\left(s \cdot \left(s + \frac{K_\varphi}{I_\varphi}\right) + \frac{G}{I_\varphi}\right) = 0 \\ |SI - A| &= (s + K_3)\left(s^2 + s\frac{K_\varphi}{I_\varphi} + \frac{G}{I_\varphi}\right) = 0 \end{aligned}$$

and applying the Routh-Hurwitz Criterion, knowing that all the constants are positive, it can be noticed that A has all its eigenvalues at the left complex half-plane. Since A is Hurwitz, it can be stated that the equilibrium point is asymptotically and exponentially stable (**LES**) for the complete nonlinear system.

5. SIMULATION RESULTS

The controller is given by the following equations:

$$
\begin{aligned}
\hat{\xi} &= L_g|x_3|x_3 + \boldsymbol{\epsilon} x_3 \\
\hat{u} &= k_1 S(\hat{\varphi}) - k_2\dot{\varphi} - k_3\hat{\xi} \\
u &= \frac{\hat{u}}{\frac{2L_g|x_3|+\epsilon}{I_g}} + (B_g + D_g|x_3|)x_3
\end{aligned}
$$

where the value $\boldsymbol{\epsilon}$ is unknown and has to be experimentally determined, by comparing the propulsion torques $\hat{\xi}$ versus the velocity of the rotor $x_3 = \omega_g$. Figure 3 shows an adjustment of the parameter, using least squares identification with the experimental data and using two models of propulsion. One of them is augmented with the parameter. It can be noticed that qualitatively the curves are very similar representing quite well the experimental data. However, the augmented one is going to let us solve the control problem avoiding that the control signal goes to infinity when the velocity of the rotor is zero.

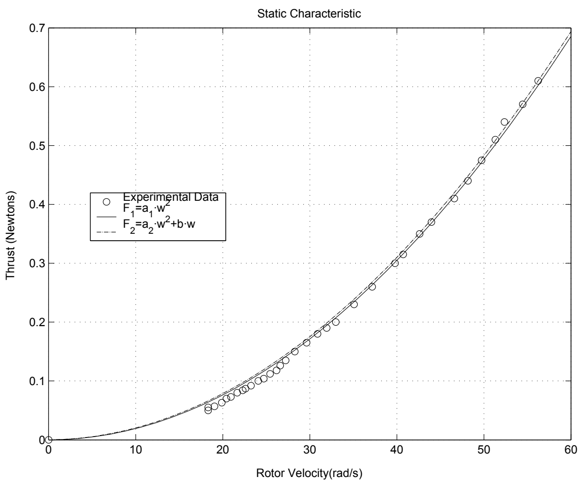

Fig. 3. Static characteristic of the propulsion forces

Figures 4 and 5 shows respectively the simulated and experimental time response of the system using the proposed controller in a regulation problem.

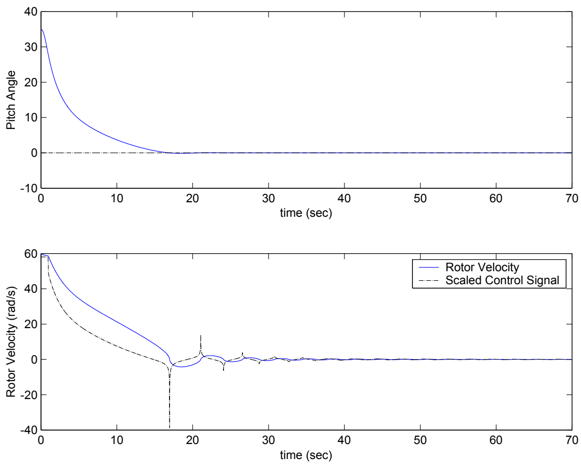

Fig. 4. Simulated response of the system

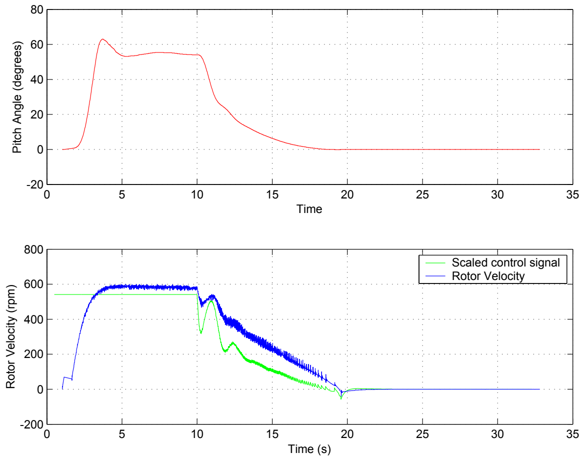

Fig. 5. Experimental response of the system

6. CONCLUSIONS

In this work, the idea of constructive design has been applied as it is meant in Sepulchre et al. (1997). Particularly, it has been applied in the design of a controller based on the backstepping technique. Applying this methodology, it has been possible to design a controller whose control signal has no peaks and has finite values when the velocity of the rotor tends to zero. The control law has been tested by simulation considering that the propulsion of the real system corresponds to a quadratic model of propulsion, and the obtained results have been successful. Finally, it has been proved that, for the closed-loop system, the origin is LES.

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