A HYBRID MODEL FOR OPTIMAL CONTROL OF SINGLE NODES IN SUPPLY CHAINS

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Abstract: Supply chains integrate production centers with raw materials and part suppliers and with logistics service providers, in an integrated environment in which co-ordination aspects as well as competitive issues may take place. In this paper, a hybrid model for the representation of a generic production center of a supply chain is presented and discussed. The model is characterized by inventory levels and by arrival and departure processes of raw materials and final products. The capacity of the considered resource is an upper bounded continuous variable. In the paper, the optimization of the dynamic behaviour of the production center is dealt with.

Keywords: Production systems, Hybrid model, Optimal control, Dynamic programming.

1. INTRODUCTION

In the last years, a growing interest of different groups of researchers has been attracted by issues concerning design, analysis, optimization, and management of complex production networks, characterized by the presence of a multiplicity of sites, where production and manufacturing operations take place. Actually, a new concept has emerged, that is, the supply chain model. A supply chain model is mainly characterized by the presence of several production centers (usually distributed over the territory), which interact with raw material and part suppliers and with logistics service providers, in an integrated environment in which co-ordination aspects as well as competitive issues may take place. Production centers, raw material and part suppliers, and service providers represent the “nodes” of the supply chain network.

The literature concerning the modelling and the management of supply chains may be classified according to the objectives of the research, and to the technique used. A first set of contributions is related to the design issues at the network level (Goetschalckx *et al.*, 2002), (Lakhal *et al.*, 2001); in this connection, the main problems addressed are those concerning the design of the supply chain as regards the number of warehouses, the sizing and the location of plants, the design of the decision structure (e.g., centralized versus decentralized), the design of the information flow structure, etc. A second research stream is focused on modelling and analysis aspects, through the use of agent-based models (Garcia-Flores *et al.*, 2000), (Wu, 2001); in this case, attention is specially paid to the design of possible coordination mechanisms among the various agents, with the purpose of achieving a satisfactory performance of the overall system. Perhaps the most effective approaches to model and to optimize the performances of supply chains are those based on the formalization and the solution of mathematical programming problems (Erenguc *et al.*, 1999), (Escudero *et al.*, 1999). However, the dimension of real systems, and the need of considering optimization horizons of significant length make the application of such approaches questionable for real applications. For what strictly concerns the modelling of the single node of a supply chain, it is worth observing that the complexity of distributed production systems inevitably leads to the choice of a simplified model. In this connection, hybrids models have been proven their ability in modelling and controlling systems in a large variety of application areas, including manufacturing systems (various authors, 2000).

The single node considered in the paper coincides with the generic production site distributed over the territory. However, the proposed model may be also applied to raw material and part suppliers and to logistics service providers, as they can be considered as specific instances of a production center. Such a model is mainly characterized by inventory levels (both those relevant to raw materials and those relevant to final products) which represent the system state variables; such variables are assumed to be continuous. The work-capacity of the production center is an upper bounded continuous variable. This allows to dedicate a fraction of such a capacity to the production of final products.

A significant optimization problem is proposed in this paper. The decision variables are mainly those relevant to the quantities of raw materials provided at each arrival, to the time instant at which raw materials enter the system, to the fractions of resource capacity dedicated to each production activity, to the time instants at which finite products exit the systems and to the amount of finite products realized. The cost functions to be minimized take into account the order costs, the inventory costs, the quadratic tardiness, and the cost due to the non-fulfilment of the external demand.

This paper is organized as follows. In section 2, the hybrid model for single node representation in supply chains is proposed. Three optimization problems are defined and discussed in section 3, and in section 4 the solution of a simplified version of the second optimization problem is provided. Some concluding remarks (section 5) end the paper.

2. THE HYBRID SINGLE NODE MODEL

The single node of a supply chain consists of a production center where parts arriving from either part suppliers or upstream production centers are carried out. One part entering the single node is processed by a single operation in order to be transformed into one product (that is, no assembly operation is present in the considered model). In the following, for the sake of clarity, only one class of parts/products will be considered.

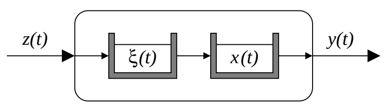

Fig. 1. Schematization of the single node model.

The model of the single node is mainly characterized by the presence of inventories for parts and products. Parts arriving from part suppliers/upstream production centers are inserted into the part inventories; then, after the processing, the resulting products are inserted into the product inventories. Such a model can be schematized as in Fig. 1, where $z(t)$ represents the flow of raw materials entering the single node, $\xi(t)$ represents the inventory level for raw materials, $x(t)$ represents the inventory level for final products, and $y(t)$ represents the flow of products exiting from the single node.

Moreover, let K be the overall work-capacity of the production center, $k(t)$ be the portion of K which is assigned to the production of products, and q be the number of finite products that the site can realize (transforming raw materials) in a time unit.

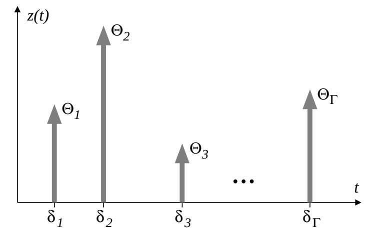

Fig. 2. The arrival process.

The flow $z(t)$ of parts arriving from part suppliers/upstream production centers is modelled as a finite (and discrete in time) sequence of arrivals (Fig. 2). An arrival has to be intended as the transportation from the external to the production center of a finite amount of parts. In the arrival process $z(t)$, Γ is the considered number of raw materials arrivals, within the considered time horizon, δ_i, $i = 1, \dots, \Gamma$ is the time instant at which the i-th arrival takes place, and Θ_i, $i = 1, \dots, \Gamma$ is the amount of raw materials entering the node at time instant δ_i (that is, the i-th ordered quantity).

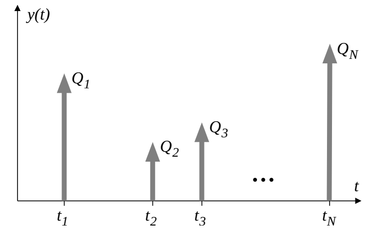

Fig. 3. The departure process of finite products.

In an analogous way, the flow $y(t)$ of products delivered to the clients of the supply chain is represented as a finite sequence of departures (Fig. 3). A departure has to be intended as the transportation of a finite amount of products from the production center to the external. Such process is characterized by the following variables: N is the considered number of finite products requests, within the considered time horizon, t_i, $i = 1, \dots, N$ is the time instant at which the i-th departure of finite products occurs, Q_i, $i = 1, \dots, N$ is the amount of finite products leaving the system at time instant t_i.

Moreover, let the external demand be characterized by due-date of the i-th departure of finite products, namely $t_i^\star$, $i = 1, \dots, N$, and by the amount of finite products required at $t_i^\star$, namely $Q_i^\star$, $i = 1, \dots, N$.

Then, the system state variables are $\xi(t)$ and $x(t)$, whereas the decision variables are

- δ_i, $i = 1, \dots, \Gamma$, Θ_i, $i = 1, \dots, \Gamma$,
- $k(t), 0 \le t \le T = \max\{\delta_\Gamma, t_N\}$,
- t_i, $i = 1, \dots, N$, Q_i, $i = 1, \dots, N$.

Note that T is the considered time horizon. On the basis of the above introduced variables, it is possible to define the state equations of the proposed supply chain's single node model. In particular, the state equation of the raw materials inventory is

$$\xi(\delta_{i+1}) = \xi(\delta_i) - q\int_{\delta_i}^{\delta_{i+1}} k(t)\,\mathrm{d}t + \Theta_{i+1} \tag{1}$$

$i = 0,\ldots,\Gamma-1$, whereas the state equation of the finite products inventory is

$$x(t_{i+1}) = x(t_i) + q\int_{t_i}^{t_{i+1}} k(t)\,\mathrm{d}t - Q_{i+1} \tag{2}$$

$i = 0,\ldots,N-1$, where $\delta_0 = 0$, $t_0 = 0$, and $\xi(0)$ and $x(0)$ are given initial inventory levels.

3. OPTIMIZATION PROBLEMS

The optimization of the dynamic behaviour of the production site subject to the external demand still need the definition of a suitable cost function. An overall optimization problem can, thus, be defined taking into account costs due to the acquisition of parts from part suppliers/upstream production centers, costs relevant to the inventory occupancy, and costs related to the non-fulfillment of external demand requisites (in terms of tardiness and difference between the actual amount of delivered products and the required one).

The cost due to the acquisition of parts from part suppliers/upstream production centers can be stated as:

$$\mathcal{C}^A = \sum_{i=1}^{\Gamma}(c_f\mu_i + c_v\Theta_i) \tag{3}$$

where c_f and c_v are the fixed and variable unitary order cost, respectively; and variables μ_i, $i = 1\ldots,\Gamma$ are binary variables taking on value 1 if some raw materials are ordered in δ_i and 0 otherwise.

Note that the binary variable μ_i is necessary in order to avoid paying unnecessarily fixed cost when the optimal value of the decision variable Θ_i is 0, that is, when no transportation of parts from part suppliers/upstream production centers occurs.

The cost due to the inventory occupancy is:

$$\mathcal{C}^I = H\int_0^{\delta_\Gamma}\xi(t)\,\mathrm{d}t + P\int_0^{t_N}x(t)\,\mathrm{d}t \tag{4}$$

where H and HP are the unitary inventory costs for raw materials and finite products, respectively.

Finally the cost term relevant to the deviations from the due-dates and from the required finite products quantities are:

$$\mathcal{C}^T = \alpha\sum_{i=1}^{N}(t_i - t_i^\star)^2 + \beta\sum_{i=1}^{N}(Q_i - Q_i^\star)^2 \tag{5}$$

being α and β suitable weighting coefficients.

The overall optimization problem can then be stated as follows.

Problem 1. Given the initial conditions $\delta_0 = 0$, $t_0 = 0$, $\xi(0) \geq 0$, and $x(0) \geq 0$, find

$$\min_{\substack{\delta_i,\Theta_i,\mu_i,\, i=1,\ldots,\Gamma\\ t_i,Q_i,\, i=1,\ldots,N\\ k(t),0\leq t\leq T}} \mathcal{C}_1 = \mathcal{C}^A + \mathcal{C}^I + \mathcal{C}^T$$

subject to (1), (2), and

$$0 \leq k(t) \leq K \qquad 0 \leq t \leq T \tag{6}$$
$$\delta_{i+1} \geq \delta_i \qquad i = 1,\ldots,\Gamma-1 \tag{7}$$
$$t_{i+1} \geq t_i \qquad i = 1,\ldots,N-1 \tag{8}$$
$$\Theta_i - \Omega\mu_i \leq 0 \qquad i = 1,\ldots,\Gamma \tag{9}$$
$$\mu_i - \Omega\Theta_i \leq 0 \qquad i = 1,\ldots,\Gamma \tag{10}$$
$$\mu_i = \{0,1\} \qquad i = 1,\ldots,\Gamma \tag{11}$$
$$\xi(t) \geq 0 \qquad 0 < t \leq \delta_\Gamma \tag{12}$$
$$x(t) \geq 0 \qquad 0 < t \leq t_N \tag{13}$$
$$Q_i \geq 0 \qquad i = 1,\ldots,N \tag{14}$$
$$\Theta_i \geq 0 \qquad i = 1,\ldots,\Gamma \tag{15}$$

where Ω is a positive number sufficiently high.

Problem 1 is a functional optimization problem with nonlinear cost function and nonlinear constraints, thus, it is a quite complex problem. The approach here followed to face the above problem is that of decomposing it into two sub-problems:

- the first sub-problem consists in minimizing the inventory cost for final products and the deviations from the external demand with respect to the production capacity (i.e., $k(t), 0 \leq t \leq T$) and to the exit process (i.e., variables $t_i, i = 1,\ldots,N$ and $Q_i, i = 1,\ldots,N$), assuming to have *unlimited available raw materials*
- the second sub-problem is relevant to the minimization of the inventory cost for raw materials and of order costs with respect to the arrival process (i.e., variables $\delta_i, i = 1,\ldots,\Gamma$ and $\Theta_i, i = 1,\ldots,\Gamma$), with the *fixed production capacity* solution of the first sub–problem.

By defining

$$\mathcal{C}_2 = P\int_0^{t_N} x(t)dt + \alpha\sum_{i=1}^{N}(t_i - t_i^\star)^2 + \beta\sum_{i=1}^{N}(Q_i - Q_i^\star)^2 \tag{16}$$

and

$$\mathcal{C}_3 = \sum_{i=1}^{\Gamma}(c_f\mu_i + c_v\Theta_i) + H\int_0^{\delta_\Gamma}\xi(t)dt \tag{17}$$

the first sub–problem can be stated as

Problem 2. Given the initial conditions $t_0 = 0$ and $x(0) \geq 0$, find

$$\min_{\substack{t_i,Q_i,\, i=1,\ldots,N\\ k(t),0\leq t\leq T}} \mathcal{C}_2$$

subject to (2), (6), (8), (13), and (14).

The second sub–problem is

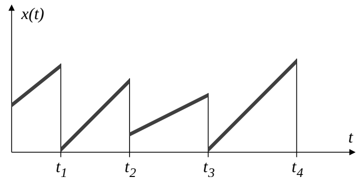

Fig. 4. Behaviour of the state variable $x(t)$ in the simplified version of Problem 2.

Problem 3. Given the initial conditions $\delta_0 = 0$ and $\xi(0) \geq 0$ find

$$\min_{\delta_i,\Theta_i,\mu_i\, i=1,\ldots,\Gamma} \mathcal{C}_3$$

subject to (1), (7), (9), (10), (11), (12) and (15).

4. A SIMPLIFIED VERSION OF PROBLEM 2

In this section, only the part relevant to the processing of raw materials to produce finite products will be considered. This means that the focus of this section will be on Problem 2. In this context, the N requests characterizing the external demand will be indicated as *orders*, since there cannot be ambiguity with orders of raw materials/parts not taken into account now. Problem 2 still is a functional optimization problem with nonlinear cost function and nonlinear constraints. A simplified, but still realistic, version of such a problem is here considered in which the production capacity is constant in the time intervals included between two subsequent exit instants, that is, the production capacity is a piecewise constant variable defined as

$$k(t) = k_i, \qquad t_i < t \leq t_{i+1}, \qquad i = 1, \ldots, N$$

With this assumption the dynamic behaviour of variable $x(t)$ can be represented as in Fig. 4

Moreover, it is imposed that the ordered quantities are always satisfied, which means $Q_i = Q_i^\star$, $i = 1, \ldots, N$. The cost function can, then, be restated as

$$\mathcal{C}_4 = P\sum_{i=1}^{N}\left(x(t_{i-1})\tau_i + \frac{qk_i\tau_i^2}{2}\right) + \\ + \alpha\sum_{i=1}^{N}(\tau_i + t_{i-1} - t_i^\star)^2 \quad (18)$$

where $\tau_i = t_i - t_{i-1}$, $i = 1, \ldots, N$. Problem 2 becomes

Problem 4. Given the initial conditions $t_0 = 0$ and $x(0) \geq 0$, find

$$\min_{k_i,\tau_i,\, i=1,\ldots,N} \mathcal{C}_4$$

subject to

$$x(t_{i-1}) + qk_i\tau_i - Q_i \geq 0$$
$$k_i \leq K \qquad k_i \geq 0 \qquad \tau_i \geq 0$$

for any $i = 1, \ldots, N$.

Problem 4 has the structure of a *mathematical programming problem* with nonlinear objective and nonlinear constraints; thus, it may be solved by standard mathematical programming techniques yielding the optimal control law in an open-loop form for a specific value of the initial conditions. In this work, Problem 4 is restated as a multistage optimal control and a optimal *feedback* control law is sought by applying dynamic programming techniques. The application of Kuhn–Tucker conditions to the problem yields some significant properties of the optimal solution of Problem 4. In the following, such properties will be stated together with sketches of the necessary proofs.

First of all, consider the following condition (which guarantees that the initial inventory level is all consumed at least during the first $N-1$ stages):

$$x(0) \leq \sum_{i=1}^{N-1} Q_i \quad (19)$$

The following result holds.

Proposition 1. If (19) holds true, in the optimal solution of Problem 4 *the inventory level at the end of the last stage is zero*, that is, $x(t_N) = 0$. □

Sketch of the proof. The application of dynamic programming allows to state the optimization problem to be solved at stage N as:

Problem 5. Given the initial conditions t_{N-1} and $x(t_{N-1}) \geq 0$, find

$$\min_{k_N,\tau_N} P\left(x(t_{N-1})\tau_N + \frac{qk_N\tau_N^2}{2}\right) + \\ + \alpha\left(\tau_N + t_{N-1} - t_N^\star\right)^2$$

subject to

$$x(t_{N-1}) + qk_N\tau_N - Q_N \geq 0$$
$$k_N \leq K \qquad k_N \geq 0 \qquad \tau_N \geq 0$$

By defining

$$f_N = P\left(x(t_{N-1})\tau_N + \frac{qk_N\tau_N^2}{2}\right) + \\ + \alpha\left(\tau_N + t_{N-1} - t_N^\star\right)^2$$
$$g_{N,1} = x(t_{N-1}) + qk_N\tau_N - Q_N$$
$$g_{N,2} = K - k_N$$

The following first-order Kuhn-Tucker conditions can be defined for Problem 5

$$k_N\left[\frac{\partial f_N}{\partial k_N} - \lambda_{N,1}\frac{\partial g_{N,1}}{\partial k_N} - \lambda_{N,2}\frac{\partial g_{N,2}}{\partial k_N}\right] = 0$$
$$\tau_N\left[\frac{\partial f_N}{\partial \tau_N} - \lambda_{N,1}\frac{\partial g_{N,1}}{\partial \tau_N} - \lambda_{N,2}\frac{\partial g_{N,2}}{\partial \tau_N}\right] = 0$$
$$\lambda_{N,1}\left[g_{N,1}\right] = 0 \qquad \lambda_{N,2}\left[g_{N,2}\right] = 0$$
$$\frac{\partial f_N}{\partial k_N} - \lambda_{N,1}\frac{\partial g_{N,1}}{\partial k_N} - \lambda_{N,2}\frac{\partial g_{N,2}}{\partial k_N} \geq 0$$
$$\frac{\partial f_N}{\partial \tau_N} - \lambda_{N,1}\frac{\partial g_{N,1}}{\partial \tau_N} - \lambda_{N,2}\frac{\partial g_{N,2}}{\partial \tau_N} \geq 0$$
$$g_{N,1} \geq 0 \qquad g_{N,2} \geq 0$$

$$k_N \geq 0 \qquad \tau_N \geq 0$$
$$\lambda_{N,1} \geq 0 \qquad \lambda_{N,2} \geq 0$$

The decision variables included in the above conditions are the problem control variables k_N and τ_N, together with the two multipliers $\lambda_{N,1}$ and $\lambda_{N,2}$. Each of these four variables can be either equal to 0 or not, giving rise to 16 possible configurations of the above set of equalities and inequalities. By analyzing in details such 16 configurations, it turns out that 4 cases boil down to not feasible solutions, 10 cases are feasible only if $x(t_{N-1}) \geq Q_N$, condition that can be easily seen as not included in the optimal solution due to the cost structure and to condition (19), and, finally two cases are actually to be taken into account. These two cases are characterized by having k_N, τ_N, and, above all, $\lambda_{N,1}$ possibly greater than zero (of course, the difference between the two cases stands in $\lambda_{N,2} = 0$ or $\lambda_{N,2} \geq 0$). The fact that in both cases $\lambda_{N,1} \geq 0$ makes it necessary that $g_{N,1} = 0$, thus, implying that the inventory level at the end of the N–th stage is zero. It is also to be noted that in the two cited cases, Problem 5 can be expressed as an *equivalent* quadratic programming problem, thus making the Kuhn–Tucker conditions necessary and also sufficient conditions for the definition of the optimal solution.■

The following two assumptions (which actually do not limit the generality of the proposed model) will be made concerning the data of Problem 4:

$$x(0) < Q_1 \tag{20}$$

which means that the initial inventory level is all "consumed" during the first order. If this condition is not verified, in the optimal solution of Problem 4 no production is realized ($k_i = 0$) for a certain number of orders starting from the first one and, then, the beginning of the sequence of orders can be simply shifted onward till meeting condition (20);

$$PqK < 4\alpha \tag{21}$$

which yields a *good balance* between the two terms of cost function $\mathcal{C}_4$ by avoiding a too strong influence of the inventory cost with respect to the cost relevant to deviations from the pre–defined due–dates.

Moreover, it can be observed that the optimal solution of Problem 4 yields a decomposition of the sequence of N orders into a finite number of *sub-sequences*. Let s, $1 \leq s \leq N$, be the number of sub-sequences; then, the indexes of orders identifying the beginning of a sub-sequence are gathered in set $\mathrm{DEC} = \{\nu_j, j = 1, \ldots, s\}$, (with $\nu_1 = 1$). In the optimal solution of Problem 4, sub-sequences have the following peculiar features:

- the inventory level at the end of each sub-sequence is zero, that is, $x(t_{j-1}) = 0, \forall j \in \mathrm{DEC} : j \neq 1$;
- the inventory level at the beginning of each sub-sequence, apart the first one, is zero;

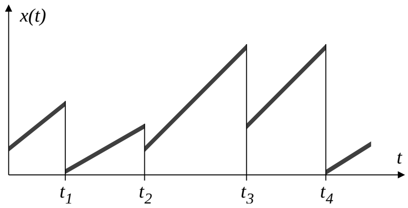

Fig. 5. Behaviour of the state variable $x(t)$ in the optimal solution of Problem 4

- in sub-sequences composed of $n > 1$ stages, production in the last $n-1$ stages is conducted at maximum production capacity.

The optimal behaviour of variable $x(t)$ can, then, be depicted as in Fig. 5.

The determination of sub-sequences greatly simplifies the application of dynamic programming to the solution of Problem 4 since it actually drives the choice of the problem to be solved at each stage (of course as regards the cost-to-go to be inserted in the cost function). Actually, it has not been possible to find an a-priori rule governing the definition of sub-sequences, but they can be easily derived by applying a mathematical programming software. Then, the proposed solution procedure is composed of two steps. The first step consists in finding, by mathematical programming, the set DEC. The second step of the procedure is devoted to he determination of the optimal solution by applying dynamic programming to the multistage control problem. The results of the second step are summarized in the following propositions (whose proofs, which consists in applying dynamic programming techniques and Kuhn–Tucker conditions for the solution of the control problem at each stage, are not reported here for the sake of brevity).

Proposition 2. In the generic k-th sub-sequence composed of $n > 1$ orders, that is orders indexed by $i = \nu_k, \ldots, \nu_{k+1} - 1$ ($\nu_k, \nu_{k+1} \in \mathrm{DEC}$), the optimal solution is

$$\tau_i^\circ = \frac{Q_i - x(t_{i-1})}{qK} \qquad k_i^\circ = K$$

for $i = \nu_{k+1} - 1$ when $\nu_{k+1} - 1 = N$;

$$\tau_i^\circ = t_i^\star - t_{i-1} - \frac{P}{2\alpha} Q_i \qquad k_i^\circ = K$$

for $i = \nu_k + 1, \ldots, \nu_{k+1} - 2$, and for $i = \nu_{k+1} - 1$ when $\nu_{k+1} - 1 \neq N$; and the optimal solution for the first of such n orders, that is for $i = \nu_k$, is

$$\tau_i^\circ = t_i^\star - t_{i-1} - \frac{P}{2\alpha} Q_i \qquad k_i^\circ = K$$

if

$$Q_i \geq \frac{2P}{8\alpha^2 - 4\alpha PqK - P^2q^2K^2}\Big(4\alpha C + - PqK(t_i^\star - t_{i-1})\Big)$$

and

$$\tau_i^\circ = \frac{2P}{16\alpha^2 - 8\alpha PqK - P^2q^2K^2}\Big[\big(Q_i + - \frac{4\alpha}{P}(t_i^\star - t_{i-1})\big)(Pqk - 2\alpha) - 2\alpha qKC\Big]$$

$$k_i^\circ = K + \frac{4\alpha}{qP}\left(1 - \frac{t_i^\star - t_{i-1}}{\tau_i^\circ}\right) + \frac{2Q_i}{q\tau_i^\circ}$$

otherwise, with

$$C = P\sum_{j=i+1}^{N-1} Q_j - \frac{2\alpha}{qK}\left[\sum_{j=i+1}^{N} Q_j - qKt_N^\star\right]$$

□

Proposition 3. Consider a sub-sequence composed of only one order. Let it be the i-th in the order sequence, with $i \neq 1$ and $i \neq N$. The optimal solution for such an order is

$$\tau_i^\circ = \frac{Q_i}{qK} \qquad k_i^\circ = K$$

if

$$\frac{4\alpha + PqK}{4\alpha - PqK} Q_i - \frac{PqK}{4\alpha - PqK}\sum_{j=i+1}^{N} Q_j + - \frac{4\alpha qK}{4\alpha - PqK}(t_i^\star - t_{i-1}) \geq 0$$

and

$$\tau_i^\circ = t_i^\star - t_{i-1} - \frac{P}{4\alpha} Q_i + \frac{P}{4\alpha}\left(\sum_{j=i+1}^{N} Q_j\right)$$

$$k_i^\circ = \frac{\frac{4\alpha}{q} Q_i}{4\alpha(t_i^\star - t_{i-1}) - PQ_i + \frac{P}{4\alpha}\left(\sum_{j=i+1}^{N} Q_j\right)}$$

otherwise. □

Proposition 4. If the first sub-sequence is composed of only one order, the optimal solution for the first order is

$$\tau_1^\circ = \frac{Q_1 - x(0)}{qK} \qquad k_1^\circ = K$$

if

$$\frac{4\alpha + PqK}{4\alpha - PqK} Q_1 - \frac{PqK}{4\alpha - PqK}\sum_{j=2}^{N} Q_j + - \frac{4\alpha qK}{4\alpha - PqK} t_1^\star \geq x(0)$$

and

$$\tau_1^\circ = t_1^\star - \frac{P}{4\alpha} Q_1 + \frac{P}{4\alpha}\left(\sum_{j=2}^{N} Q_j - x(0)\right)$$

$$k_1^\circ = \frac{\frac{4\alpha}{q}\left[Q_1 - x(0)\right]}{4\alpha t_1^\star - PQ_1 + \frac{P}{4\alpha}\left(\sum_{j=2}^{N} Q_j - x(0)\right)}$$

otherwise. □

Proposition 5. If the last sub-sequence is composed of only one order, the optimal solution for the last order is

$$\tau_N^\circ = \frac{Q_N}{qK} \qquad k_N^\circ = K$$

if

$$\frac{4\alpha + PqK}{4\alpha - PqK} Q_N - \frac{4\alpha qK}{4\alpha - PqK}(t_N^\star - t_{N-1}) \geq 0$$

and

$$\tau_N^\circ = t_N^\star - t_{N-1} - \frac{P}{4\alpha} Q_N$$

$$k_N^\circ = \frac{\frac{4\alpha}{q} Q_N}{4\alpha(t_N^\star - t_{N-1}) - PQ_N}$$

otherwise. □

5. CONCLUSIONS

This is a preliminary work funded by the Ministry of Education, University and Research, and, at current state, major attention has been directed to the definition of the model of the single node in a supply chain network. The proposed model is a hybrid model as includes continuous variables within a discrete-event dynamics. In addition, optimization problems have been considered by the authors and a first solution, relevant to a simplified, but realistic, version of one of the problems, has been provided. The current research work is devoted to the development of coordination strategies for decision makers behaving in a cooperative way within a supply chain, and the analysis of supply chains in presence of competing decision agents.

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