SENSORLESS CONTROL OF THE TRAPEZOIDAL BLDCM USING SLIDING MODE OBSERVER

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Abstract: This paper presents a method for speed, position and phase-to-phase back-EMF estimation in trapezoidal Brushless DC motor (BLDCM) through application of the sliding mode technique. The electrical dynamic equations of BLDCM are modified to obtain the sliding mode observer model where the phase-to-phase back-EMF, position and speed are estimated using measured stator currents and voltages. A theoretical study is developed and the results obtained by simulation are presented to show the effectiveness of the method.

Keywords: Permanent magnet, Trapezoidal Brushless DC Motor, Sliding Mode Observer, Sensorless Control.

1. INTRODUCTION

Today the demand for Permanent Magnet trapezoidal Brushless DC Motor (BLDCM) in the industrial and domestic applications is steadily rising. Their growing popularity is due to their high efficiency, silent operation, high power density, reliability and low maintenance. A BLDCM is one kind of synchronous motor, having permanent magnets on the rotor and trapezoidal shape back-EMF.

The most popular way to control BLDCM is through voltage-source current-controlled inverters. The inverter must supply a rectangular current waveform whose magnitude, I_{MAX}, is proportional to the motor shaft torque. Three Hall effect sensors are usually used as position sensors to perform current commutations; it is only required to know the position of commutation points, because the objective is to achieve rectangular current waveforms, with dead time periods of 60º. The sensors mentioned increase the cost and the size of the motor and reduce the reliability of the total system. Therefore, sensorless control of BLDCM has been receiving great interest in recent years. Many methods for obtaining rotor position and speed have been proposed in the literature. In most existing methods, the rotor position is detected every 60 electrical degrees, which is necessary to perform current commutations. These methods are based on:

1- Using the back-EMF of the motor (Becerra *et al*., 1991; Shao,2003).
2- Detection of the conducting state of freewheeling diodes in the unexcited phase (Ogasawara and Akagi,1991).
3- The stator third harmonic voltage components (Moreira,1994).

Since these methods cannot provide continual rotor position estimation, they are not applicable for the sensorless drives in which high estimation accuracy of the speed rotor is required. In other methods, the estimation of the instantaneous rotor position is proposed. These methods are based on:

1- Estimating flux linkage using measured motor voltages and currents (Bejerke,1996).
2- The Extended Kalman filter (EKF) (Terzic and Jadric,2001).

In the first method, the accuracy of the rotor position estimation depends significantly on the motor parameters variation and the accuracy of the measured voltages and currents. However in second method the Extended Kalman filter has some inherent disadvantages, such as the influence of noise characteristics, the computational burden, the absence of design and tuning criteria.

Despite of the drawbacks recalled in the previous paragraph, a state observer based sensorless controller represent an excellent solution for a wide range of rated-speed and low-cost applications.

Recently, in (Fakham and Djemai *et al.*,2004; Fakham *et al.*,2004) the sliding mode observer of the trapezoidal back-EMF is applied. This technique represents an attractive proposal because it is robust with respect to measurement noise and parametric uncertainties of the system. This method exploit the model of BLDCM in stationary reference frame *(a,b,c)*. The goal of this paper is to present, a new Sliding Mode Observer of the trapezoidal phase-to-phase back-EMF components (E_α, E_β) . The model of BLDCM in stationary reference frame *(α,β)* is applied. The proposed observer using this model is largely sufficient to provide the six positions of commutation and the speed of the rotor. However the continual estimation for the rotor position becomes unnecessary.

The organization of this paper is given hereafter. In section 2, sensorless control strategy for BLDC Motor is presented. In section 3, a formulation of the model of the BLDCM is given and a new Sliding mode observer of the trapezoidal phase-to-phase back-EMF components is developed. In section 4, a simple zero crossing detectors (ZCD) of the phase-to-phase back-EMF is presented to obtain the position of commutation points. In section5, the estimation rotor speed from the trapezoidal phase-to-phase back-EMF is presented. Finally, in section 6, the simulation results are presented and explained.

2. SENSORLESS CONTROL STRATEGY FOR BLDCM

The principle of the control strategy for BLDCM is illustrated in figure.1. The inverter must supply a rectangular current waveform (figure.2) whose magnitude I_{MAX} is proportional to the machine shaft torque. The equivalent dc current is obtained through the sensing of two among three armature currents. From these currents, the absolute value is taken, and a dc component, which corresponds to the amplitude I_{MAX} of the original phase currents, is obtained. This dc component is compared with a reference coming from the output of the speed regulator 1, and the error signal is processed through a PI controller 2. The output of the PI controller 2 is compared with a saw-teeth carrier signal, to generate the PWM for the power transistors. At the same time, the position sensor discriminates which couple of the six transistors of the inverter should receive this PWM signal, the several techniques of current control in such a system have been studied and described in the literature (Dixon, 2002). In this scheme, the position estimator is used to detect only six positions, which determine the switch commutations or commutations points (figure.2). From the outputs of the sliding mode observer the phase-to-phase back-EMF components in stationary reference frame *(α,β)* is observed and using the zero crossing detectors (ZCD), the positions of commutation points are estimated. The speed rotor is calculated using the relation mathematical between the magnitudes E_{max} of the phase-to-phase back-EMF observed and speed rotor.

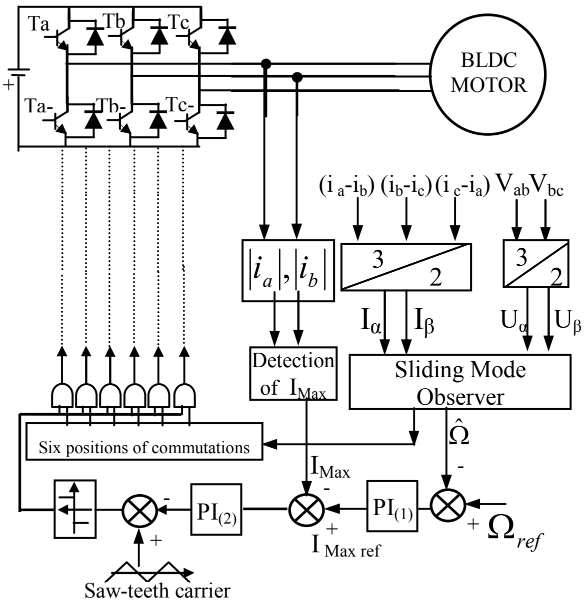

Fig. 1. The proposed sensorless control scheme

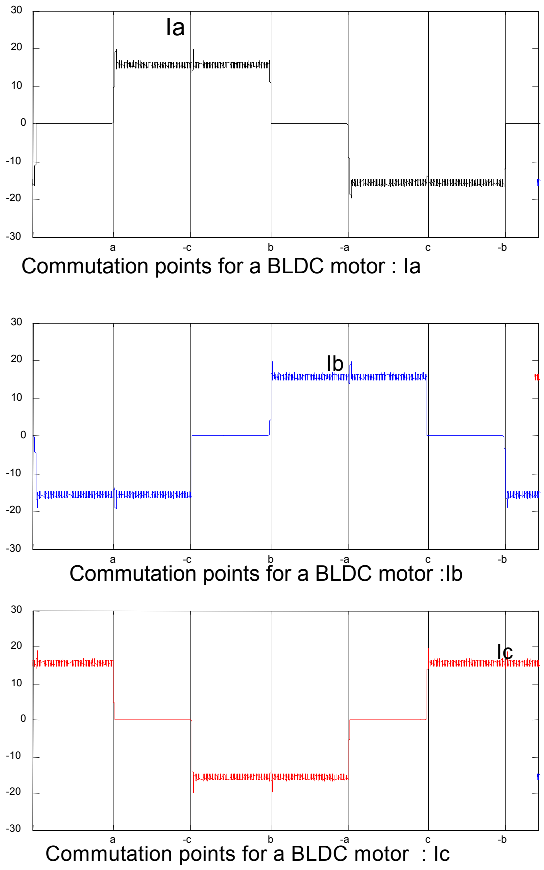

Fig. 2. The currents phases I_a, I_b and I_c

3. SLIDING MODE OBSERVER IN BLDCM

3.1 The trapezoidal Brushless DC motors model

In this kind of machine, this is only two of the three phases conducting at any time, as the stator winding neutral point of the machine is floating and not accessible, which makes it impossible to directly measure phase voltages. The BLDCM is modelled in stationary reference frame *(abc)* using the subtractions currents *(I_a-I_b, I_b-I_c, I_c-I_a)* which measured through tow phases, the phase-to-phase back-EMF *(E_{ab}, E_{bc}, E_{ca})* and the phase-to-phase voltages *(U_{ab}, U_{bc}, U_{ca})*. The following model has been derived:

$$\begin{cases} \dfrac{d(I_a-I_b)}{dt} = -\dfrac{R}{L}(I_a-I_b) - \dfrac{1}{L}E_{ab} + \dfrac{1}{L}U_{ab} \\ \dfrac{d(I_b-I_c)}{dt} = -\dfrac{R}{L}(I_b-I_c) - \dfrac{1}{L}E_{bc} + \dfrac{1}{L}U_{bc} \\ \dfrac{d(I_c-I_a)}{dt} = -\dfrac{R}{L}(I_c-I_a) - \dfrac{1}{L}E_{ca} + \dfrac{1}{L}U_{ca} \end{cases} \quad (1)$$

The following traditional assumptions are made:

- The distribution of the phase-to-phase back-EMF is trapezoidal, and its variation is very slow
- The motor is unsaturated.
- The armature reaction is negligible.

Transforming the model (1) in stationary reference frame *(α,β)* and neglecting the zero sequence component, the model (2) is written:

$$\begin{cases} \dfrac{dI_\alpha}{dt} = -\dfrac{R}{L}I_\alpha - \dfrac{1}{L}E_\alpha + \dfrac{1}{L}U_\alpha \\ \dfrac{dI_\beta}{dt} = -\dfrac{R}{L}I_\beta - \dfrac{1}{L}E_\beta + \dfrac{1}{L}U_\beta \end{cases} \quad (2)$$

In order to setup a back-EMF observer the phase-to-phase back-EMF components in (2) can be considered as disturbances with the following associated model.

$$\frac{dE_\alpha}{dt} = 0 \text{ and } \frac{dE_\beta}{dt} = 0 \quad (3)$$

from (2) and (3) the state model is obtained as follows:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

with:

$x = \begin{bmatrix} I_\alpha & I_\beta & E_\alpha & E_\beta \end{bmatrix}^T$: Vector of state variables;

$y = \begin{bmatrix} I_\alpha & I_\beta \end{bmatrix}^T$: Output vector;

$u = \begin{bmatrix} U_\alpha & U_\beta \end{bmatrix}^T$: Input vector;

$A = \begin{bmatrix} -\alpha_1 I & -\alpha_2 I \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -\alpha_2 I \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} I & 0 \end{bmatrix}$

$\alpha_1 = \dfrac{R}{L}$ and $\alpha_2 = \dfrac{1}{L}$

R, L: stator winding resistance and inductance, respectively;

I : identity matrix 2x2;

0 : zeros sub-matrices 2x2.

3.2 Sliding Mode Observer

The observer presented in this paragraph is a very simple case of the sliding mode observer (figure.3). It is a linear model disturbed by the phase-to-phase back-EMF components (external uncertainty) and the errors parametric (internal uncertainty). The sliding mode observer for phase-to-phase back-EMF expressed in stationary reference frame *(α,β)* can be easily described in the following form:

The sliding surface S is given by:

$$\dot{\hat{x}} = A\hat{x} + Bu + K\,\mathrm{sgn}(y - C\hat{x}) \quad (4)$$

$$S = \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

with ‘sgn’ is the sign operator defined as :

$$\mathrm{sgn}(S) = \begin{cases} 1 & if\ S > 0 \\ 0 & if\ S = 0 \\ -1 & if\ S < 0 \end{cases}$$

and $K \in R^{4\times 2}$ is a gain matrix

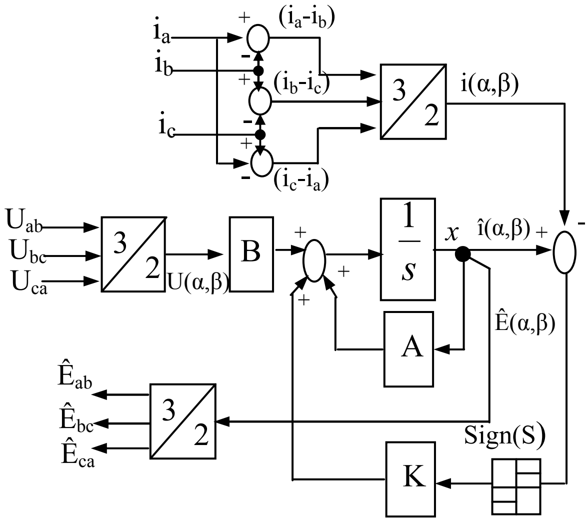

Fig. 3. Sliding mode observer

Let us define the tracking errors

$$e = \hat{x} - x \quad (5)$$

Its dynamic is governed by:

$$\begin{cases} \dot{e}_1 = -\alpha_2 e_3 - K_1\,\mathrm{sgn}(e_1) \\ \dot{e}_2 = -\alpha_2 e_4 - K_2\,\mathrm{sgn}(e_2) \\ \dot{e}_3 = -K_3\,\mathrm{sgn}(e_1) \\ \dot{e}_4 = -K_4\,\mathrm{sgn}(e_2) \end{cases} \quad (6)$$

In order to ensure the asymptotic convergence of the tracking errors $e = [e_1\ e_2\ e_3\ e_4]^T$ to zero, the following assumptions should be formulated:

A1: The system is uncoupled

$$K_{12} = K_{21} = K_{32} = K_{41} = 0.$$

A2: The gain k_{11}, k_{22} must be chosen so that

$$k_{11} > \alpha_2 |e_3|_{\max} \quad k_{22} > \alpha_2 |e_4|_{\max} \quad (7)$$

A3: The gain k_{31}, k_{42} must be chosen so that

$$\frac{k_{31}}{k_{11}} < 0 \text{ and } \frac{k_{42}}{k_{22}} < 0$$

Proof:

Define the Lyapunov function candidate:

$$V(t) = \frac{1}{2} e_{1,2}(t) e_{1,2}(t) \text{ With}$$
$$e_{1,2}(t) = \begin{bmatrix} e_1(t) & e_2(t) \end{bmatrix}^T$$

Its time derivative is calculated as:

$$\dot{V} = e_{1,2}{}^T \dot{e}_{1,2}$$

$$\dot{V} = \begin{bmatrix} e_1 & e_2 \end{bmatrix} \left[\begin{pmatrix} -\alpha_2 e_3 \\ -\alpha_2 e_4 \end{pmatrix} - \begin{pmatrix} k_{11} & 0 \\ 0 & k_{22} \end{pmatrix} \begin{pmatrix} sign(e_1) \\ sign(e_2) \end{pmatrix} \right] \quad (8)$$

Then:

$$\dot{V} = e_1(-\alpha_2 e_3 - k_{11} sign(e_1)) + e_2(-\alpha_2 e_4 - k_{22} sign(e_2)) \quad (9)$$

It is possible to find the conditions of K_{11} and K_{22} such as $\dot{V} < 0$ in order to force the asymptotic convergence of the error $e_{1,2}(t)$ to zero. According to the equation (9), these conditions are given by

- If the error $e_1 > 0$ the gain switching is $k_{11} > -\alpha_2 e_3$;
- If the error $e_1 < 0$ the gain switching is $k_{11} > \alpha_2 e_3$;
- If the error $e_2 > 0$ the gain switching is $k_{22} > -\alpha_2 e_4$;
- If the error $e_2 < 0$ the gain switching is $k_{22} > \alpha_2 e_4$.

The vector of the tracking errors *e(t)* is bounded all time *t*, then this condition can be written in the following form:

$$k_{11} > \alpha_2 |e_3|_{\max} \qquad k_{22} > \alpha_2 |e_4|_{\max} \quad (10)$$

Consequently $e_{1,2}(t)$ is stable.

When the sliding mode occurs on the sliding surface (5), then $e_{1,2}(t) = \dot{e}_{1,2}(t) = 0$, and therefore the dynamic behaviour of the tracking problem can be written as:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = -\alpha_2 \begin{pmatrix} e_3 \\ e_4 \end{pmatrix} - \begin{pmatrix} k_{11} & 0 \\ 0 & k_{22} \end{pmatrix} \begin{bmatrix} \mathrm{sgn}(e_1) \\ \mathrm{sgn}(e_2) \end{bmatrix} \quad (11)$$

$$I_s = \begin{bmatrix} \mathrm{sgn}(e_1) \\ \mathrm{sgn}(e_2) \end{bmatrix} = \begin{bmatrix} \frac{-\alpha_2 e_3}{k_{11}} \\ \frac{-\alpha_2 e_4}{k_{22}} \end{bmatrix} \quad (12)$$

The vector I_s represents the equivalent output error injection term necessary to maintain a sliding mode on *S*. Using the equivalent output (12), the reduced order sliding mode is governed by:

$$\dot{e}_3 = \frac{k_{31}\alpha_2}{k_{11}} e_3$$
$$\dot{e}_4 = \frac{k_{42}\alpha_2}{k_{22}} e_4 \quad (13)$$

Then, under assumption *A3* the tracking error *e(t)* converge to zero exponentially.

4. ROTOR POSITION ESTIMATION

In back-EMF based sensorless schemes for sinusoidal machines, the information on the rotor position is extracted by means of inverse trigonometric functions.In the case only six positions of commutation points are required and the presence of harmonic back-EMFs, makes this method not applicable. The problem can be overcome by using a zero crossing detectors (ZCD) of the phase-to-phase back EMFs (figure.5). From the observed phase-to-phase back EMF components, the phase-to-phase back-EMFs are obtained (figure.4) and the detection of six positions of the rotor can be determined easily. For example, during the switch commutation *'a'* of the transistor *'Ta'*, one can deduce that the phase-to-phase back-EMF E_{ab} is always positive and that the phase-to-phase back-EMF E_{ca} is always negative, this is mean that it is possible to obtain a sufficient condition to extract the state switch commutation *'a'* of the transistor *' Ta'*: it is thus enough to have $E_{ab} > 0$ and $E_{ca} < 0$. For the other switch commutations the same logic of analysis is followed.

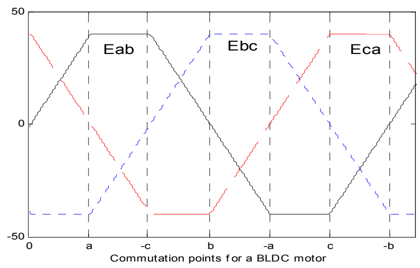

Fig. 4. The phase-to-phase back-EMFs

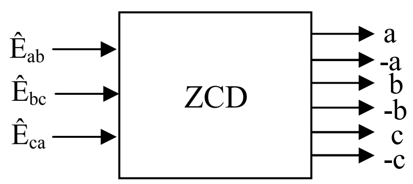

Fig. 5. The zero crossing detectors block

5. ROTOR SPEED ESTIMATION

For the application of the sensorless drives in which high estimation accuracy of rotor speed is required, the method in (Fakham *et all*. 2004b) is used, which based on the relation mathematical between the magnitude $E_{max(phase\text{-}to\text{-}phase)}$ and rotor speed (14). The magnitude $E_{max(phase\text{-}to\text{-}phase)}$ (figure.6) can be determined using the observed phase-to-phase back-EMF and six points of commutations.

$$\omega_r = \frac{E_{\max(phase-to-phase)}}{2K_{EMF}} \tag{14}$$

With:

$$E_{\max(Phase-to-neutral)} = K_{EMF}\,\omega_r$$

$$E_{\max(Phase-to-neutral)} = \frac{E_{\max(Phase-to-phase)}}{2}$$

Where K_{EMF} is the constant of the back EMF.

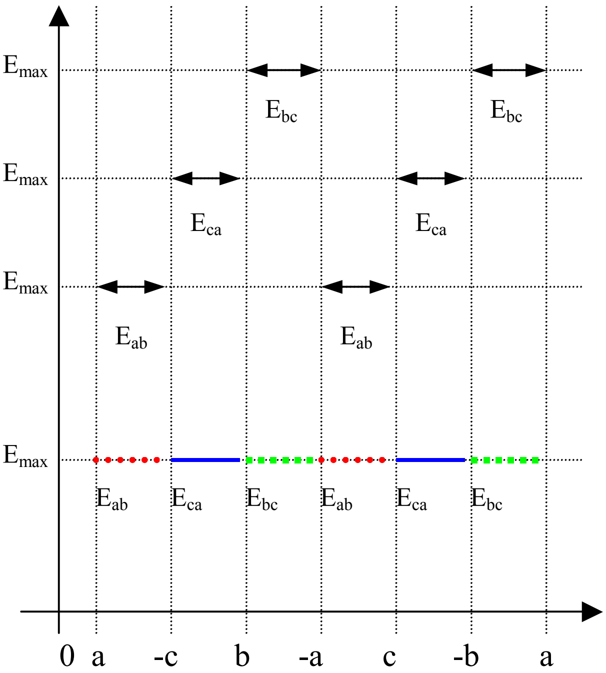

Fig. 6. The amplitude E_{max} of the phase-to-phase back-EMF

6. APPLICATION AND SIMULATIONS

The proposed sensorless control of the trapezoidal BLDC motors using sliding mode observer for phase-to-phase back-EMF has been verified through numerical simulation.

The parameters of the three-phase six-pole permanent magnet brushless DC motor are given by:

- permanent magnet brushless DC motor: $V=144V$, $I=120A$, $R=12m\Omega$, $L=150\mu H$, and $K_{EMF}=20V/krpm$

The figure.7 and figure.8 show the current components (I_α, I_β) obtained from Clark's transformation of three subtraction for three measured currents I_a , I_b and I_c . These current components are applied as magnitude measurement to the sliding mode observer. In the response of the sliding mode observer, the current component observed $\hat{I}_\alpha$ respectively $\hat{I}_\beta$ follow the same measured current component I_α respectively I_β. The phase-to-phase back-EMF components $(\hat{E}_\alpha, \hat{E}_\beta)$ follows closely the real trapezoidal phase-to-phase back EMF components (E_α, E_β) of the motor (figure.9 and figure.10).

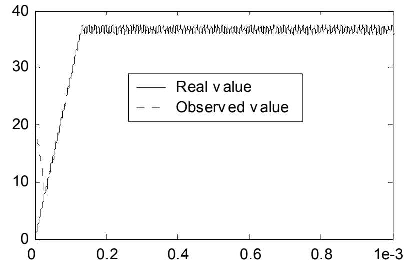

Fig. 7. The measured current component I_α and observed current component $\hat{I}_\alpha$.

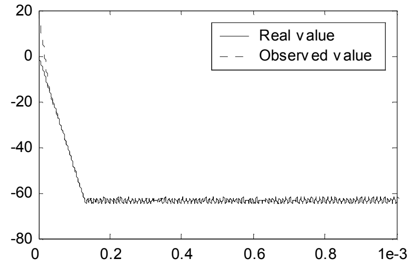

Fig. 8. The measured current component I_β and observed current component $\hat{I}_\beta$.

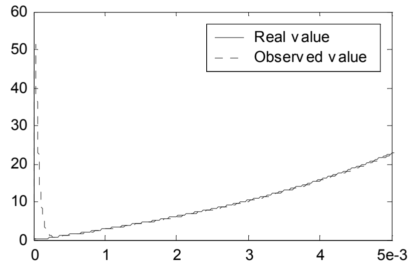

Fig. 9. The real phase-to-phase back-EMF component E_α and The observed phase-to-phase back-EMF component $\hat{E}_\alpha$

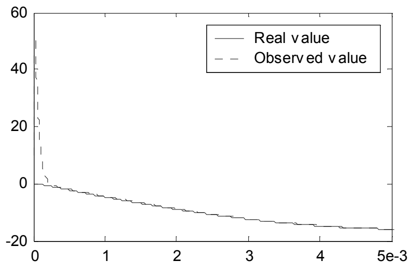

Fig. 10. The real phase-to-phase back-EMF component E_β and The observed phase-to-phase back-EMF component $\hat{E}_\beta$

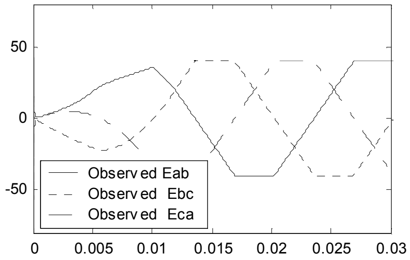

Fig. 11. The observed phase-to-phase back-EMFs: $\hat{E}_{ab}$, $\hat{E}_{bc}$ *and* $\hat{E}_{ca}$

After transforming the observed phase-to-phase back-EMF components $\hat{E}_{\alpha}$ and $\hat{E}_{\beta}$ in stationary reference frame *(a,b,c)*, the observed phase-to-phase back-EMFs are obtained (figure.11).Then the zero crossing detectors (ZCD) are given to the digital inputs (figure.12). These digital inputs forms a bus such as its digital value is used to change the firings sequence of transistors inverter and to detect the amplitude E_{max} of the trapezoidal phase-to-phase back-EMF observed which used to determine the speed estimation (figure.13). In this response the speed estimation $\hat{\omega}_r$ follows closely the speed measurement ω_r of the motor.

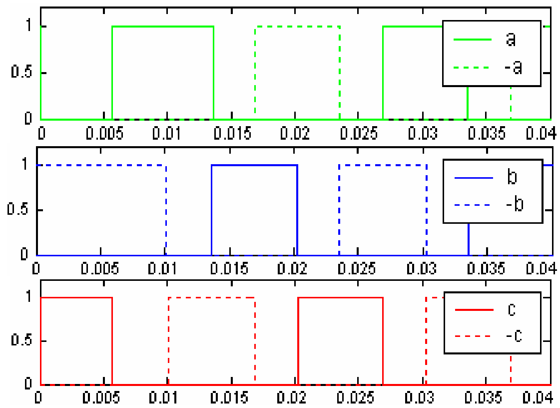

Fig. 12. The estimated position of commutation points

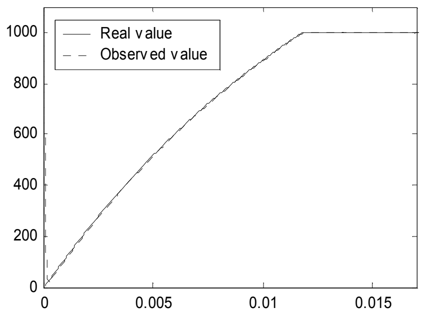

Fig. 13. The Actual speed ω_r (measured) and estimated speed $\hat{\omega}_r$ (observed). (rpm)

7. CONCLUSION

In the presented method, which applied the sliding mode observer, there are three essential factors that were respected in order to reduce cost and complexity. First, the speed and the rotor position of the BLDCM are estimated from the phase-to-phase back EMF components. Second the observer only uses the stator currents and voltages measurements. Finally the simplification of the observer's equations is made. The results obtained by simulation show the effectiveness of the method. In our future work we plan to make an experimental verification of the proposal method.

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