CONTROL OF PARABOLIC DISTRIBUTED SYSTEMS WITH DELAY IN FEEDBACK LOOP

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Abstract: The problem of regulation with guaranteed transient performances for parabolic distributed systems with delayed control is discussed. The representation of the parabolic distributed system by infinite-dimensional system of differential equations is used. The fast infinite-dimensional controller with the highest output derivative in feedback loop is introduced and hence, two-time-scale motions are induced in the closed-loop system. Stability conditions imposed on the fast and slow modes and sufficiently large mode separation rate can ensure that the full-order closed-loop system achieves the desired properties in such a way that the output transient performances are desired and insensitive to external disturbances and parameter variations in the system. Constraint set for selection of the controller parameters caused by delay is considered.

Keywords: parabolic distributed system, delay, distributed control, singular perturbation method.

1. INTRODUCTION

In order to controller design for distributed parameter systems various methods are widely used, for instance, such as pole assignment (Wang, 1972; Ray, 1981; Smagina *et.al.*, 2002), optimization technique (Toshkova and Petrov, 2003), adaptive approach (Demetriou and Rosen, 2002; King and Hovakimyan, 2003; Solo and Bamieh, 1998), discontinuous feedback stabilization (Orlov and Dochain, 2002). The problem of output regulation for distributed parameter systems is discussed as well, e.g., (Byrnes *et al.*, 2000). In particular, control systems with sliding mode (Utkin, 1992) and control systems with a high gain and the highest derivative in feedback loop (Vostrikov, 1977) are powerful tools for control system design under uncertainties of parameters and external disturbances.

The subject matter of this paper is the guaranteed cost control for parabolic distributed parameter systems based on fast infinite-dimensional controller with the highest output derivative in feedback loop as well as peculiarities and constraints caused by delay in feedback loop.

Note that the control problem of parabolic distributed parameter systems with the high gain and the highest derivative as well as with differentiating filter in feedback was discussed in (Yurkevich, 1992a; Yurkevich, 1992b). In the recent paper the modified control law structure (Yurkevich, 2004) in the form of the fast dynamical controller with the highest derivative of output signal in feedback loop is used, where the presented control law structure allows us to include the integral action in the control loop without increasing the controller's order in comparison with (Yurkevich, 1992b).

The paper is organized as follows. First, a model of the parabolic distributed parameter system with pure time delay in control is defined. Next, the method of controller design is presented and the influence of delay is investigated. Finally, the modified control law with compensation of delay is suggested and simulation results of an example are included as well.

2. ONE-DIMENSIONAL SYSTEM WITH DELAYED CONTROL

2.1 *One-dimensional equation*

Let us consider a heating or diffusion process described by a one-dimensional parabolic equation with delayed control and given by

$$\frac{\partial x}{\partial t}(z,t) = \alpha^2 \frac{\partial^2 x}{\partial z^2}(z,t) + c(t)x(z,t) \\ + w(z,t) + u(z,t-\tau), \qquad (1)$$

where t is time, $t > 0$, z is the spatial variable, $0 < z < 1$, $x(z,0) = x^0(z)$ is the initial condition, $[\partial x(z,t)/\partial z]|_{z=0} = 0$ and $[\partial x(z,t)/\partial z]|_{z=1} = 0$ are the boundary conditions, $c(t)$ is an unknown varying parameter, $|c(t)| \leq c_0 < \infty$, $w(z,t)$ is a distributed external disturbance unavailable for measurement, τ is a delay, $\tau > 0$, $u(z, t-\tau)$ is the distributed delayed control, α^2 is a constant (we shall take $\alpha^2 = 1$). We assume also that for all functions $x(z,t)$, $x^0(z)$, $w(z,t)$, and $u(z,t-\tau)$ the eigenfunction expansions

$$x(z,t) = \sum_{n=0}^{\infty} x_n(t)\varphi_n(z), \;\; x^0(z) = \sum_{n=0}^{\infty} x_n^0 \varphi_n(z),$$
$$u(z,t-\tau) = \sum_{n=0}^{\infty} u_n(t-\tau)\varphi_n(z),$$
$$w(z,t) = \sum_{n=0}^{\infty} w_n(t)\varphi_n(z)$$

hold where $\varphi_n(z) = \varphi_n^0 \cos(\sqrt{\lambda_n}\, z)$ are eigenfunctions (Wang, 1972) (also known as spatial modes or normal modes), $\lambda_n = n^2\pi^2$ are eigenvalues, $\varphi_0^0 = 1$, $\varphi_n^0 = \sqrt{2}\;\; \forall\, n = 1, 2, \ldots$.

In this case, the time functions $x_n(t)$ for $x(z,t)$ satisfy the equations (Wang, 1972; Ray, 1981)

$$\dot{x}_n(t) = \{c(t) - \lambda_n\}x_n(t) + w_n(t) + u_n(t-\tau), \; (2)$$
$$x_n(0) = x_n^0, \quad n = 0, 1, \ldots.$$

From (2), we get that some or all of the first n_0 equations can be unstable due to variations of the parameter $c(t)$, where $n_0 = \mathrm{int}(\sqrt{c_0}/\pi)$. Here $\mathrm{int}(y)$ is the integer part of y. Therefore, the first n_0 modes should be controlled in order to guarantee close-loop system stability.

So, distributed controller design is reduced to controller design for each separate mode. Note that $x_n^{(1)}(t)$ is the highest derivative of the system (2).

2.2 *Control problem statement*

The control problem is to provide a desired spatial distribution assigned by a function $x^d(z,t)$:

$$\lim_{t\to\infty} \sup_{0<z<1} \{x^d(z,t) - x(z,t)\} = 0, \qquad (3)$$

where $x^d(z,t)$ is defined by the eigenfunction expansion

$$x^d(z,t) = \sum_{n=0}^{\infty} x_n^d(t)\varphi_n(z). \qquad (4)$$

Moreover, the control transients $x(z,t) \to x^d(z,t)$ should have desired transient performance indices. These transients should not depend on the external disturbances $w(z,t)$ and varying parameter $c(t)$ of the system (1).

2.3 *Insensitivity condition*

Denote by $e_n(t) = x_n^d(t) - x_n(t)$ a realization error of the desired time function $x_n^d(t)$. Then the requirement (3) corresponds to

$$\lim_{t\to\infty} e_n(t) = 0, \qquad n = 0, 1, \ldots. \qquad (5)$$

So, the desired behavior of the transients $x(z,t) \to x^d(z,t)$ can be provided if the process $x_n(t) \to x_n^d(t)$ satisfies the desired differential equation

$$\dot{x}_n(t) = F_n(x_n(t), x_n^d(t)) \qquad (6)$$

for each time function $x_n(t)$. The parameters of (6) are selected in accordance with assigned transient performance indices and in such a way that the condition $x_n = x_n^d$ holds for the steady state of (6). For instance, the linear differential equation in the form

$$\dot{x}_n(t) = T_n^{-1}[x_n^d(t) - x_n(t)] \qquad (7)$$

is the most convenient in this case where T_n is selected in accordance with the desired settling time of the transients in (7).

Denote $e_n^F = F_n - \dot{x}_n(t)$, where e_n^F is the realization error of the desired dynamics assigned by (6). As a result, the control problem (3) with desired transient performance indices can be solved if

$$e_n^F = 0, \quad \forall\, n = 0, 1, \ldots. \qquad (8)$$

This is the insensitivity condition of the transients in the system (1) with respect to the external disturbances $w(z,t)$ and varying parameter $c(t)$.

By (2) and (6)–(7) the expression (8) can be rewritten in the form

$$[x_n^d(t) - x_n(t)]/T_n + [\lambda_n - c(t)]x_n(t) \\ -w_n(t) - u_n(t-\tau) = 0, \quad \forall\ n = 0,1,\dots \quad (9)$$

So, the discussed control problem has been reformulated as the requirement to provide the condition (9) or, in other words, to find a solution to (9) when its varying parameters are unknown.

The solution of (9) consists of the functions $u_n(t) = u_n^{id}(t)$ defined by

$$u_n^{id}(t-\tau) = [x_n^d(t) - x_n(t)]/T_n \\ +[\lambda_n - c(t)]x_n(t) - w_n(t),\ \forall\ n = 0,1,\dots, \quad (10)$$

where $u_n^{id}(t)$ is called the inverse dynamics (id) solution. As a result, we see that the distributed control function

$$u^{id}(z,t-\tau) = \sum_{n=0}^{\infty} u_n^{id}(t-\tau)\varphi_n(z) \quad (11)$$
$$= \sum_{n=0}^{\infty} \{[x_n^d(t) - x_n(t)]/T_n \\ +[\lambda_n - c(t)]x_n(t) - w_n(t)\}\,\varphi_n(z)$$

gives the desired behavior of transients $x(z,t) \to x^d(z,t)$. Note that (11) is the noncausel control function and, moreover, all parameters and external disturbances must be known and available for measurement. Hence, (11) cannot be used in practice for control. However, the expression (11) allows us to make estimate of control resource required for control problem solution. Thereto, we assume that the series (11) is absolutely convergent and a certain value M_u exists such that the requirement

$$|u(z,t)| \le M_u < \infty, \quad \forall\ t,z$$

is satisfied in a specified region of the system state space. Hence, M_u is the estimate of control resource required for control problem solution.

3. CLOSED-LOOP SYSTEM

3.1 *Control law*

In order to ensure that $e_n^F = 0$ when the parameter $c(t)$ is varying and unknown and the distributed external disturbance $w(z,t)$ is unavailable for measurement, let us consider the control law for the equation of the time function $x_n(t)$ with $x_n^{(1)}(t)$ in feedback, that is

$$\mu_n^{q_n} u_n^{(q_n)} + d_{n,q_n-1}\mu_n^{q_n-1}u_n^{(q_n-1)} + \cdots \\ +d_{n,1}\mu_n u_n^{(1)} + d_{n,0}u_n \quad (12) \\ = k_n\{[x_n^d(t) - x_n(t)]/T_n - x_n^{(1)}\},\ U_n(0) = U_n^0,$$

where

$$\mu_n > 0, \quad q_n \ge 1, \quad d_{n,j} > 0,\ \forall\ j = 1,\dots,q_n-1, \\ d_{n,0} = 1 \ \text{ or } \ d_{n,0} = 0 \\ U_n = \{u_n, u_n^{(1)}, \dots, u_n^{(q_n-1)}\}^T, \\ U_n \in \Omega_{U_n} \subset \boldsymbol{R}^{q_n}, \quad U_n(0) \in \Omega_{U_n}^0 \subset \Omega_{U_n}.$$

The closed-loop system equations for the nth mode are

$$x_n^{(1)}(t) = [c(t) - \lambda_n]x_n(t) \\ +w_n(t) + u_n(t-\tau), \quad (13)$$
$$\mu_n^{q_n} u_n^{(q_n)}(t) + d_{n,q_n-1}\mu_n^{q_n-1}u_n^{(q_n-1)}(t) \\ + \cdots + d_{n,1}\mu_n u_n^{(1)}(t) + d_{n,0}u_n(t) \\ = k_n\{[x_n^d(t) - x_n(t)]/T_n - x_n^{(1)}\}, \quad (14) \\ x_n(0) = x_n^0,\ U_n(0) = U_n^0,$$

where $n = 0,1,\dots$. Substitution of (13) into (14) yields the closed-loop system equations in the form

$$x_n^{(1)}(t) = [c(t) - \lambda_n]x_n(t) \\ +w_n(t) + u_n(t-\tau), \quad (15)$$
$$\mu_n^{q_n} u_n^{(q_n)}(t) + d_{n,q_n-1}\mu_n^{q_n-1}u_n^{(q_n-1)}(t) + \cdots \\ +d_{n,1}\mu_n u_n^{(1)}(t) + d_{n,0}u_n(t) + k_n u_n(t-\tau) \\ = k_n\{[x_n^d(t) - x_n(t)]/T_n \\ -[c(t) - \lambda_n]x_n(t) - w_n(t)\}, \quad (16) \\ x_n(0) = x_n^0,\ U_n(0) = U_n^0,$$

where $n = 0,1,\dots$. Since μ_n is a small parameter, the closed-loop system equations (15)–(16) are the singularly perturbed equations. If $\mu_n \to 0$, then fast and slow modes appear in the closed-loop system and the time-scale separation between these modes depends on μ_n.

3.2 *Fast-motion subsystem*

First, in order to enable usage of the well known standard technique for two-time-scale motions analysis (Tikhonov, 1952), we must represent the time delay τ in the normalized form $\tau = \tau_n^0\mu_n$ where τ_n^0 is the normalized time delay. Hence, from (15)–(16), we get the equations of the FMS

$$\mu_n^{q_n} u_n^{(q_n)}(t) + d_{n,q_n-1}\mu_n^{q_n-1}u_n^{(q_n-1)}(t) + \cdots \\ +d_{n,1}\mu_n u_n^{(1)}(t) + d_{n,0}u_n(t) + k_n u_n(t-\tau) \\ = k_n\{[x_n^d(t) - x_n(t)]/T_n \\ -[c(t) - \lambda_n]x_n(t) - w_n(t)\}, \quad (17) \\ U_n(0) = U_n^0,$$

where $x_n = \text{const}$ during the transients in (17) (x_n is the frozen variable) and $n = 0,1,\dots$.

3.3 *Slow-motion subsystem*

Assume that the asymptotic stability of the FMS unique equilibrium point holds and desired sufficiently small settling time of the transients of $u_n(t)$ can be achieved by a proper choice of the controller parameters $\mu_n, d_{n,j}, k_n$.

Let us obtain an equation of the slow-motion subsystem (SMS) under the condition of FMS stability. After the rapid decay of transients in (17), we have the steady state (more precisely, quasi-steady state) for the FMS (17). In particular, if $\mu_n \to 0$ in (17) with $d_{n,0} = 0$, then we obtain

$$u_n(t) = u_n^{id}(t)$$

where $u_n^{id}(t)$ is given by (10). Substitution of (10) into the right member of (15) yields the slow-motion subsystem (SMS) which is the same as the desired differential equation given by (7). However, the delay in control variable puts an additional restriction on the value of the small parameter μ_n.

3.4 *Phase margin of FMS with delay*

Inasmuch as the FMS (17) may be examined as a linear system, the Nyquist stability criterion can be applied. By the Nyquist stability criterion, the FMS (17) is marginally stable if the condition

$$\frac{k_n e^{-j\tau\omega}}{D(j\mu_n\omega)} = 1 \tag{18}$$

holds where

$$D(j\mu_n\omega) = \mu_n^{q_n} s^{q_n} + d_{n,q_n-1}\mu_n^{q_n-1}s^{q_n-1} + \cdots + d_{n,1}\mu_n s + d_{n,0}. \tag{19}$$

Hence, from (18), we get the lower bound on μ_n given by

$$\bar{\mu}_n = \tau a_n\{\pi - \mathrm{Arg}D(ja_n)\}^{-1}, \tag{20}$$

where a_n satisfies

$$|D(ja_n)| = k_n$$

and the FMS (17) is asymptotically stable if

$$\mu_n > \tau a_n\{\pi - \mathrm{Arg}D(ja_n)\}^{-1}. \tag{21}$$

Hence, a decrease in μ_n conflicts with the requirement on FMS stability, while an increase in μ_n conflicts with the requirement on time-scale separation degree of the fast and slow motions in the closed-loop system (15)–(16). So, the controller parameters should be selected in such a way that the both requirements are satisfied.

The parameters of the controller are selected in such a way that the FMS (17) be asymptotically stable with the required phase margin $\varphi_n(\tau)$ (by that we can provide an acceptable level of oscillation excited in the FMS) and the desired degree of time-scale separation between the fast and slow modes in the closed-loop system for the nth mode holds for all $n = 0, 1, \ldots$.

Let the polynomial $D(\mu_n s) + k_n$ be stable and the condition $k_n > d_{n,0}$ holds. Hence, the phase margin $\varphi_n(\tau)$ of the FMS (17) is given by

$$\varphi_n(\tau) = \pi - \mathrm{Arg}D(j\mu\omega_{n,c}) - \tau\omega_{n,c} \tag{22}$$

where the crossover frequency $\omega_{n,c}$ on the Nyquist plot of the FMS (17) is defined by the equation

$$|D(j\mu_n\omega_{n,c})| = k_n.$$

Hence, the controler parameters should be selected in such a way that the requirement

$$|\varphi_n| \le \varphi_n^0 \tag{23}$$

holds where φ_n^0 is an acceptable value of the phase margin φ_n in the FMS (17).

4. COMPENSATION OF DELAY

4.1 *Control law with compensation of delay*

In order to reduce the influence of the delay τ on the stability of the FMS, let us modify (12) and consider the control law given by

$$\begin{gathered}\mu_n^{q_n}u_n^{(q_n)} + d_{n,q_n-1}\mu_n^{q_n-1}u_n^{(q_n-1)} + \cdots \\ + d_{n,1}\mu_n u_n^{(1)} + d_{n,0}u_n + \gamma_n[u_n(t) - u_n(t-\tau)] \\ = k_n\{[x_n^d(t) - x_n(t)]/T_n - x_n^{(1)}\}, \\ U_n(0) = U_n^0, \end{gathered} \tag{24}$$

The modification of the controller (12) to a control law of the form (24) is related to the main idea of the time delay compensation scheme now known as the Smith predictor. This idea is widely used in controller design for processes with time delays (Palmor, 1996).

Let p be the operator given by $p = d/dt$. Then, the equation (2) of the nth mode and the new control law (24) can be rewritten in the operator form

$$p^n x_n = [c - \lambda_n]x_n + w_n + e^{-\tau p}u_n, \tag{25}$$

$$\{D(\mu_n p) + \gamma_n[1 - e^{-\tau p}]\}u_n = k_n\{[x_n^d - x_n]/T_n - p^n x_n\}. \tag{26}$$

Let $\gamma_n = k_n$. Substitution of (25) into the right member of (26) yields

$$p^n x_n = [c - \lambda_n] x_n + w_n + e^{-\tau p} u_n, \quad (27)$$
$$\{D(\mu_n p) + k_n\} u_n = k_n\{[x_n^d - x_n]/T_n - [c - \lambda_n] x_n - w_n\}. \quad (28)$$

Hence, from (27)–(28), we get the operator form of the FMS given by

$$\{D(\mu_n p) + k_n\} u_n = k_n\{[x_n^d - x_n]/T_n - [c - \lambda_n] x_n - w_n\}$$

where the characteristic equation of the FMS is given by

$$D(\mu_n s) + k_n = 0. \quad (29)$$

Hence, the stability of the FMS in this case does not depend on μ_n. So, the lower bound on μ_n has disappeared completely. From (28)–(27), we obtain

$$\lim_{\mu_n \to 0} e_n^F(\mu_n) = \left\{1 - \frac{k_n}{d_{n,0} + k_n} e^{-\tau p}\right\} \times\{[x_n^d - x_n]/T_n - [c - \lambda_n] x_n - w_n\}. \quad (30)$$

With $d_{n,0} = 0$, the time domain description of (30) yields

$$\lim_{\mu_n \to 0} e_n^F(\mu_n) = \sum_{i=1}^{\infty} \frac{(-\tau)^i}{i!} \frac{d^i}{dt^i}\{[c - \lambda_n] x_n + w_n - [x_n^d - x_n]/T_n\}. \quad (31)$$

So, the control law (24) allows us to provide compensation for the delay in the FMS. The drawback is the additional error (31) of the desired dynamics realization.

4.2 *Selection of controller parameters and simulation results*

Let $\gamma_n = k_n$, $d_{n,0} = 0$, $x_n^d(t) = \text{const}$, and $w_n(t) = w_n^v t1(t)$. Hence, from (27)–(28), we get the the relative velocity error due to a ramp external disturbance $w_n(t)$ given by

$$\overline{e}_{n,w}^v = \lim_{t \to \infty} \frac{x_n^d(t) - x_n(t)}{w_n^v} = -T_n \frac{\mu_n d_{n,1} + \tau \gamma_n}{k_n} \quad (32)$$

We can see that the additional term caused by the pure time delay τ exists, and this fact corresponds to (31). Hence, the controler parameter k_n should be selected in such a way that the requirement

$$|\overline{e}_{n,w}^v| < \overline{e}_{n,w}^{v,o} \quad (33)$$

holds where $\overline{e}_{n,w}^{v,o}$ is an acceptable level of the velocity error due to external disturbance $w_n(t)$.

Finally, as an example, take $q_n = 1$, $d_{n,1} = 1$, and $d_{n,0} = 0$. Then, from (12), we get the control law

$$\mu_n u_n^{(1)} = k_n\{[x_n^d - x_n]/T_n - x_n^{(1)}\} \quad (34)$$

where (34) corresponds to proportional-integral (PI) controller. Hence, based on the above results, we obtain the constraint set for selection of the controller parameters caused by delay.

First, from (21) and (23), we get

$$\mu_n > \frac{2\tau k_n}{\pi}, \quad \mu_n \geq \frac{2\tau k_n}{\frac{\pi}{2} - \varphi_n^0},$$

respectively, where $0 < \varphi_n^0 < \pi/2$. From (32) and (33) with $\gamma_n = 0$, we obtain

$$\mu_n \leq \frac{k_n}{T_n} \overline{e}_{n,w}^{v,o}.$$

Second, let η_n be the degree of time-scale separation between fast and slow modes in the closed-loop system equations (15)–(16). Then the desired degree of time-scale separation given by η_n^0 gives

$$\mu_n \leq \frac{T_n}{\eta_n^0}.$$

Usually, η_n^0 can be selected such that $5 \leq \eta_n^0 \leq 10$. Finally, we obtain the permissible set for parameter μ_n given by

$$\frac{2\tau k_n}{\frac{\pi}{2} - \varphi_n^0} \leq \mu_n \leq \min\left\{\frac{T_n}{\eta_n^0}, \frac{k_n}{T_n} \overline{e}_{n,w}^{v,o}\right\}.$$

Simulation results for the closed-loop system of the n-th mode given by (2), (24) are displayed in Fig. 1 and Fig. 2 for the time interval $t \in [0, 12]$ s, where $c = 6, \lambda_n = 1, k_n = 10, q_n = 1, \mu_n = 0.1\,\mathrm{s}, T_n = 1\,\mathrm{s}, \tau_n = 0.015\,\mathrm{s}, d_{n,0} = 0$ and the initial conditions are all zero.

5. CONCLUSIONS

Note that the control problem for systems governed by one-dimensional parabolic equation with delayed control has been discussed in the paper, while the presented results can be extended for other types of partial differential equations. For instance, the presented approach to control system design can be directly applied to stabilization of the reaction-convection-diffusion system with exothermic reactions of generic kinetics discussed in (Sheintuch *et.al.*, 2002), where the mathematical model for the set of the first controllable modes has the same structure as (2). The main advantage of the presented approach is that the desired transient performance indices and control accuracy for the controlled modes are guaranteed despite unknown external disturbances and varying parameters of the system.

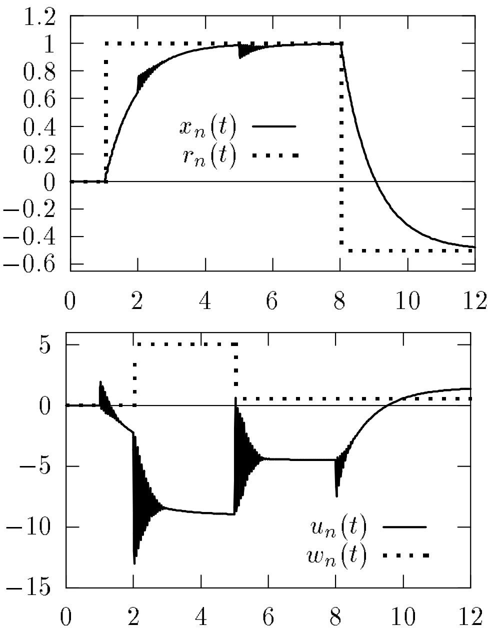

Fig. 1. Simulation results for the system (2), (24) without compensation of delay where $\gamma_n = 0$.

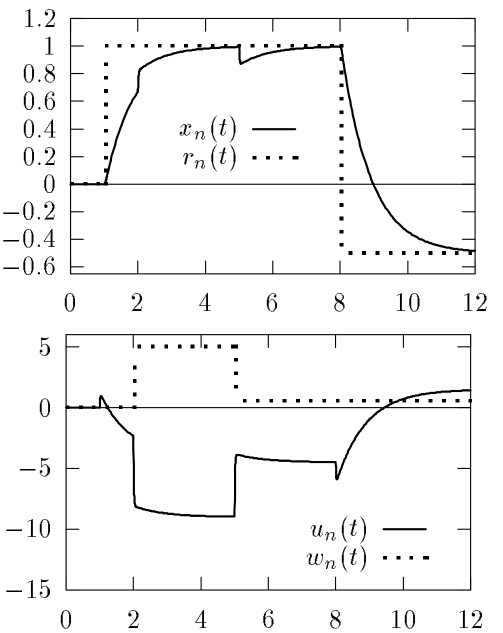

Fig. 2. Simulation results for the system (2), (24) with compensation of delay where $\gamma_n = k_n$.

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