A CONVEX METHOD FOR THE PARAMETRIC INSENSITIVE H_2 CONTROL PROBLEM

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Abstract: Contrary to the standard H_2 problem, the so-called insensitive H_2 problem makes use of a criterion that takes explicitly into account the parametric sensitivity of the closed loop system. This problem has already been re-formulated as a structured H_2 problem that is known to be equivalent to a specific BMI optimization problem when assuming full order controller. This paper presents a new formulation leading to a convex optimization problem. This is obtained thanks to the Youla parameterization and using the specific structure of the problem. The use of a finite dimensioned orthonormal basis for the structured Youla parameter leads to an LMI-based algorithm solving the insensitive H_2 problem. Its interest is shown by comparison with existing algorithms.

Keywords: Parametric sensitivity, H_2 Control, Youla parameterization, Structured control, LMI, BMI.

1. INTRODUCTION

The theory of sensitivity is not new ((Kreindler 1969, Rao and Soudak 1971, Eslami 1994) and references therein). Reducing the sensitivity is the main goal of feedback control. Some notable efforts have been made to enrich classical LQG/ H_2 control in that direction.

One can quote the parametric LQG/LTR method proposed in (Tahk and Speyer 1987) and the "desensitized LQG" control (Heniche and Bourlès 1995, Begovich 1992, Chevrel and Yagoubi 2004).

The Insensitive H_2 control (IH_2) approach considered in this paper is a design method able to manage the compromise between classical specifications on nominal performance, neglected dynamics and sensitivity with regards to parametric uncertainties.

In (Chevrel and Yagoubi 2004), IH_2 control problem has been formulated as a structured feedback H_2 control problem and solved using an iterative LMI algorithm dealing with the underlying BMI. Although efficient, it becomes time consuming when the system order increases.

The present paper introduces a new formulation of the IH_2 problem leading to an infinite but convex optimization problem. This will be possible thanks to the Youla parameterization (Youla *et al.*, 1976) by taking into account the specific structure of the problem. Using the projection on a finite orthonormal basis and convex optimization tools, a practical way to find the appropriate (structured) Youla parameter will be proposed. The underlying optimization problem can alternatively be treated by other methods which are also efficient such as the one proposed in (Qi *et al.*, 2003).

The paper is organized as follows: The IH_2 problem is first presented in section 2. In section 3, a new parameterization of the IH_2 controllers is given. The IH_2 problem is then formulated as an LMI optimization problem with respect to a structured Youla parameter. Finally, a new algorithm is proposed in section 4 to solve the IH_2 problem. This algorithm is applied to an automotive design problem. The results obtained are compared with existing ones in section 5. The conclusion takes place in section 6.

2. PROBLEM STATEMENT

2.1 Notations

Consider the scheme of Fig.1 in which G is an LTI operator with partitioned inputs and outputs, and Δ is an unknown operator related to the parametric uncertainties.

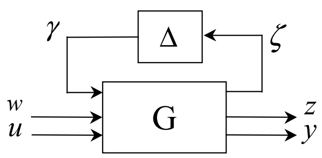

Fig.1. Parametric linear fractional representation

Let the transfer matrix $G(s)$ associated with G be defined by

$$G(s) := \left[\begin{array}{c|ccc} A & B_\gamma & B_w & B_u \\ \hline C_\zeta & D_{\zeta\gamma} & D_{\zeta w} & D_{\zeta u} \\ C_z & D_{z\gamma} & D_{zw} & D_{zu} \\ C_y & D_{y\gamma} & D_{yw} & D_{yu} \end{array}\right] \quad (1)$$

where $A \in \mathbb{R}^{n\times n}$, $B_\gamma \in \mathbb{R}^{n\times n_\gamma}$, $B_w \in \mathbb{R}^{n\times n_w}$, $B_u \in \mathbb{R}^{n\times n_u}$ $C_\zeta \in \mathbb{R}^{n_\zeta \times n}$, $C_z \in \mathbb{R}^{n_z\times n}$ and $C_y \in \mathbb{R}^{n_y\times n}$.

Let $\theta = (\theta_1,\ldots,\theta_q)$ and $\Delta \triangleq diag(\theta_1 I_{P1}, \theta_2 I_{P2},\ldots,\theta_q I_{Pq})$ in which $\theta_i \in \mathbb{R},\ i\in\{1,\cdots,q\}$ are the uncertain parameters.
Finally the feedback transfer matrix K is introduced according to Fig. 2.

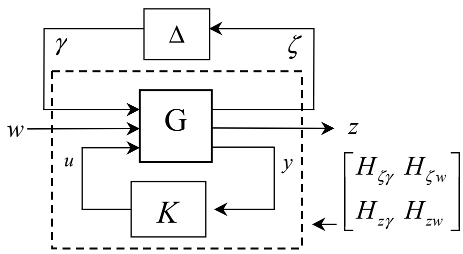

Fig. 2. The Closed loop system

The closed loop transfer matrix $F_l(G(s),K(s))$ has the partitioned form $\begin{bmatrix} H_{\zeta\gamma} & H_{\zeta w} \\ H_{z\gamma} & H_{zw}\end{bmatrix}$ ($F_l(\cdot,\cdot)$ denotes the lower linear fractional transformation (LFT)).

Indeed, all the transfers $H_{zw}, H_{z\gamma}, H_{\zeta\gamma}$ and $H_{\zeta w}$ depend on the feedback K.

Closing the "Δ-loop", the final transfer H depends on both s and θ and can be written as

$$H = H_{zw} + H_{z\gamma}(I - \Delta H_{\zeta\gamma})^{-1}\Delta H_{\zeta w} \quad (2)$$

The parametric sensitivity of the trajectory signals are denoted by

$$x_\theta = \frac{\partial x}{\partial\theta},\ z_\theta = \frac{\partial z}{\partial\theta},\ y_\theta = \frac{\partial y}{\partial\theta} \text{ and } u_\theta = \frac{\partial u}{\partial\theta}$$

In this paper the notation $\Re_{pr}^{m\times p}$ (respectively $\Re_{spr}^{m\times p}$) will designate the set of the proper (respectively strictly proper) real-rational transfer matrices.

2.2 The insensitive H_2 control problem

The insensitive H_2 control problem considers a special H_2 criterion. In addition to the standard H_2 norm, this criterion involves the H_2 norm of the closed loop parametric sensitivity function $\partial H/\partial\theta$.
In the following, the sensitivity function will be considered in the neighborhood of $\Delta = 0$ (nominal model).

$$\frac{\partial H}{\partial\theta} = [I_q \otimes \{H_{z\gamma}(I-\Delta H_{\zeta\gamma})^{-1}\}]\frac{\partial\Delta}{\partial\theta}(I-\Delta H_{\zeta\gamma})^{-1}H_{\zeta w} \quad (3)$$

$$\left.\frac{\partial H}{\partial\theta}\right|_{\theta=0} = I_q \otimes H_{z\gamma}\frac{\partial\Delta}{\partial\theta}H_{\zeta w} \quad (4)$$

Definition: IH_2 control Problem (IH_2P)
The IH_2 control problem consists in minimizing with respect to $K(s)$, the following criterion under the constraint of internal stability

$$J_{IH_2}(K) = \|H_{zw}\|_2^2 + \left\|\Sigma\otimes H_{z\gamma}\frac{\partial\Delta}{\partial\theta}H_{\zeta w}\right\|_2^2 \quad (5)$$

Remark:
Let $\Sigma = diag\ (\sigma_1,\ldots,\sigma_q)$; then,

$$J_{IH_2}(K) = \|H_{zw}\|_2^2 + \sum_{i=1}^{q}\sigma_i^2\left\|H_{z\gamma}\frac{\partial\Delta}{\partial\theta_i}H_{\zeta w}\right\|_2^2$$

and each $\sigma_i \in \Re$ may be considered as a weighting parameter associated with $\partial H/\partial\theta_i$, allowing a selective action on the parametric sensitivity. The minimization of the criterion above is not a standard H_2 optimization problem because of the additional term that containing the product of two transfer matrices, $H_{z\gamma}$ and $H_{\zeta w}$, depending both on $K(s)$.

3. A PARAMETRIZATION OF THE INSENSITIVE H_2 CONTROLLERS

Some assumptions will be made from now :

A1: For sake of simplicity (and without loss of generality), it is supposed that $\Sigma = I_q$.

A2: $D_{yu} = D_{zw} = 0$.

A3: Only small variations in the neighborhood of $\Delta = 0$ (nominal model) are considered.

Let us now consider some notations:

$$G_a(s) := \left[\begin{array}{cc|ccc} A & 0 & B_w & B_u & 0 \\ A_\theta & (I_q \otimes A) & B_{w\theta} & B_{u\theta} & (I_q \otimes B_u) \\ \hline C_z & 0 & 0 & D_{zu} & 0 \\ C_{z\theta} & (I_q \otimes C_z) & 0 & D_{zu\theta} & (I_q \otimes D_{zu}) \\ C_y & 0 & D_{yw} & 0 & 0 \\ C_{y\theta} & (I_q \otimes C_y) & D_{yw\theta} & 0 & 0 \end{array}\right] \qquad (6)$$

and A_θ, $B_{w\theta}$, $B_{u\theta}$, $C_{z\theta}$, $D_{zw\theta}$, $D_{zu\theta}$, $C_{y\theta}$, $D_{yw\theta}$, $D_{yu\theta}$ are given below

- $A_\theta = (I_q \otimes B_\gamma)\dfrac{\partial \Delta}{\partial \theta} C_\zeta$
- $C_{z\theta} = (I_q \otimes D_{z\gamma})\dfrac{\partial \Delta}{\partial \theta} C_\zeta$
- $B_{w\theta} = (I_q \otimes B_\gamma)\dfrac{\partial \Delta}{\partial \theta} D_{\zeta w}$
- $D_{zw\theta} = (I_q \otimes D_{z\gamma})\dfrac{\partial \Delta}{\partial \theta} D_{\zeta w}$
- $B_{u\theta} = (I_q \otimes B_\gamma)\dfrac{\partial \Delta}{\partial \theta} D_{\zeta u}$
- $D_{zu\theta} = (I_q \otimes D_{z\gamma})\dfrac{\partial \Delta}{\partial \theta} D_{\zeta u}$
- $C_{y\theta} = (I_q \otimes D_{y\gamma})\dfrac{\partial \Delta}{\partial \theta} C_\zeta$
- $D_{yw\theta} = (I_q \otimes D_{y\gamma})\dfrac{\partial \Delta}{\partial \theta} D_{\zeta w}$ (7)
- $D_{yu\theta} = (I_q \otimes D_{y\gamma})\dfrac{\partial \Delta}{\partial \theta} D_{\zeta u}$

Theorem 1:
Considering assumptions $(Ai)_{i\in\{1,2,3\}}$ and the notations below $F_l(G_a, I_{q+1} \otimes K) = \begin{pmatrix} H_{zw} \\ H_{z_\theta w} \end{pmatrix}$.

Proof:
By construction (cf. (Chevrel and Yagoubi 2004)).

Corollary 1:
The following holds:

- $J_{IH_2}(K) = \| F_l(G_a, I_{q+1} \otimes K) \|_2^2$.
- The IH_2 control problem is equivalent to the structured standard H_2 problem defined thanks to the augmented plant G_a (associated with Fig. 3).

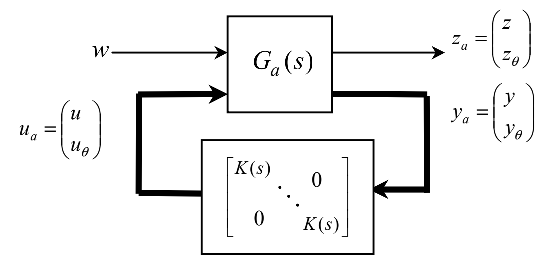

Fig. 3. A structured H_2 control problem

Proof:
The first equality results directly from *Theorem 1* and the H_2 norm property. The second proposition is obvious from *Theorem 1*.

Let us introduce the Youla parameterization (Youla *et al.*, 1976) thanks to the change of variable:

$$K = F_l(J, Q) \qquad (8)$$

with $J \triangleq \left[\begin{array}{c|cc} A - B_u K_c - K_f C_y & K_f & B_u \\ \hline -K_c & 0 & I \\ -C_y & I & 0 \end{array}\right]$.

The problem will consist from now on to find the best proper and stable Q according to the IH_2 criterion and the closed loop stability condition and to derive K according to (8). The standard H_2 problem will be solved for $Q = 0$ if K_c and K_f designate respectively the Kalman state feedback and observer gains.

Let us define $\bar{J}(s) = I_{q+1} \otimes J(s)$, $\bar{Q}(s) = I_{q+1} \otimes Q(s)$ and $\bar{G}(s) = F_l(G_a, \bar{J})$.

Theorem 2 :
The IH_2 control problem may be solved as follows:
Find $Q(s) \in RH_\infty$ such that $\| F_l(\bar{G}, I_{q+1} \otimes Q) \|_2^2$ is minimized. $K = F_l(J, Q)$ is then the optimal IH_2 controller.

Proof:
For $K(s) = F_l(J(s), Q(s))$, it is clear by construction (cf. Figure 4 and Figure 5) and from the result of *Corollary 1*, that $J_{IH_2}(K) = \| F_l(\bar{G}, I_{q+1} \otimes Q) \|_2^2$.

Moreover, for each stabilizing controller $K(s)$, there exists a stable[1] parameter Q such that $K(s) = F_l(J(s), Q(s))$.

Finally, the optimal parameter Q associated to $\| F_l(\bar{G}, I_{q+1} \otimes Q) \|_2^2$ leads to the optimal K for the IH_2 problem.

Corollary 2 :
The set of all IH_2 controllers is given by

$$I_{H_2} := \left\{ \bar{K}(s) \in \Re_{spr}^{(q+1)n_u \times (q+1)n_y} \;/\; \bar{K}(s) = F_l(\bar{J}, \bar{Q}(s)) \right\}$$

with

$$\bar{J} \triangleq \left[\begin{array}{c|cc} I_{q+1} \otimes A - B_u K_c - K_f C_y & I_{q+1} \otimes K_f & I_{q+1} \otimes B_u \\ \hline I_{q+1} \otimes (-K_c) & 0 & I \\ I_{q+1} \otimes (-C_y) & I & 0 \end{array}\right]$$

where $\bar{Q}(s) = \left[\begin{array}{c|c} \bar{A}_Q & \bar{B}_Q \\ \hline \bar{C}_Q & \bar{D}_Q \end{array}\right] = \left[\begin{array}{c|c} I_{q+1} \otimes A_Q & I_{q+1} \otimes B_Q \\ \hline I_{q+1} \otimes C_Q & I_{q+1} \otimes D_Q \end{array}\right]$;

[1] Necessary condition for the closed loop system to be stable

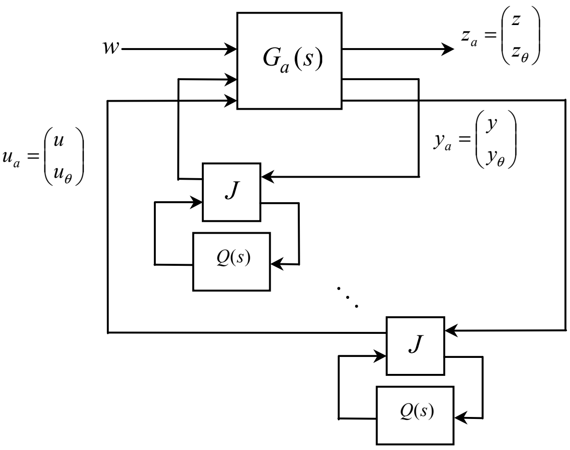

Fig. 4. The parametrized IH_2 problem

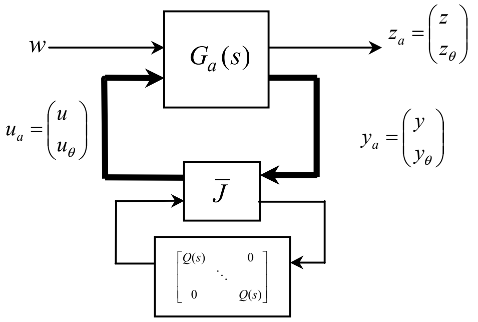

Fig. 5. A structured Youla parameter

4. SOLVING THE IH_2 PROBLEM IN THE STATE SPACE FRAMEWORK

For simplicity of presentation we will consider the following notations:

$$G_a(s) := \begin{bmatrix} A_a & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \qquad (9)$$

and T will denotes the closed-loop transfer matrix with

$$T \stackrel{\Delta}{=} F_l(G_a(s), F_l(\bar{J}, \bar{Q}) \stackrel{\Delta}{=} T_1 + T_2(I_{q+1} \otimes Q(s))T_3$$

The set of achievable stable closed loop maps is then given by

$$\Phi = \left\{ T_1 + T_2\bar{Q}T_3 \ / \ \bar{Q} = (I_{q+1} \otimes Q(s)),\ Q \in \Re_p^{n_u \times n_y} \right\}$$

where Q is a stable free parameter. The stability of the transfer matrices T_1, T_2 and T_3 is obvious since the central controller is the H_2 optimal controller.

It is then clear that a realization for T can be given by

$$T = \left[\begin{array}{cc|c} \bar{A}_1 & \bar{B}_2 M_Q \bar{C}_2 & \bar{B}_1 + \bar{B}_2 M_Q \bar{D}_2 \\ 0 & \bar{A}_2 & \bar{B}_3 \\ \hline \bar{C}_1 & \bar{D}_1 M_Q \bar{C}_2 & \bar{D}_1 M_Q \bar{D}_2 \end{array}\right] \qquad (10)$$

$$= \left[\begin{array}{c|cc} A_{cl} & B_{1cl} & B_{2cl} \\ \hline C_{1cl} & 0 & D_{12cl} \\ C_{2cl} & D_{12cl} & 0 \end{array}\right]$$

with:

$$\bar{A}_1 = \begin{bmatrix} A_a & 0 \\ 0 & A_a \end{bmatrix},\ \bar{A}_2 = \begin{bmatrix} \bar{A}_Q & \bar{B}_Q C_2 \\ 0 & A \end{bmatrix},\ \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$

$$\bar{B}_2 = \begin{bmatrix} 0 \\ B_2 \end{bmatrix},\ \bar{B}_3 = \begin{bmatrix} \bar{B}_Q D_{21} \\ B_1 \end{bmatrix},\ \bar{C}_1 = [C_1 \quad C_1],\ \bar{C}_2 = \begin{bmatrix} I & 0 \\ 0 & C_2 \end{bmatrix}$$

$$\bar{D}_1 = D_{12},\ \bar{D}_2 = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix},\ M_Q = \begin{bmatrix} \bar{C}_Q & \bar{D}_Q \end{bmatrix}$$

The state space matrices A_{cl}, B_{1cl}, B_{2cl}, C_{1cl}, C_{2cl}, D_{12cl} and D_{21cl} depend affinelly on M_Q.

In the following, the Q-parameter will be defined in an orthonormal basis (e.g. the Niness orthonormal basis (Niness and Gustafsson, 1997)):

$$Q(s) = \sum_{i=1}^{N_q} \theta_i Q_i(s) \qquad (11)$$

with: $$Q_i(s) = \frac{\sqrt{2\,\mathrm{Re}(a_i)}}{s + a_i} \prod_{k=1}^{i-1} \frac{s - \bar{a}_k}{s + a_k} \qquad (12)$$

The poles of the elements of the basis have to be chosen accordingly to the problem considered. This point will be discussed later when dealing with an example. Note that $A_Q(n_q \times n_q)$ and $B_Q(n_q \times n_u)$ are fixed from (12) (the poles of the Q-basis are chosen *a priori*). n_q denotes the size of A_Q.

The insensitive H_2 control problem now consists in finding a dynamic parameter Q solution of the optimization problem

$$\min_{M_Q} \left\| T_1 + T_2(I_{q+1} \otimes Q(s))T_3 \right\|_2^2 \qquad (13)$$

Using the state space formulation (10), the only free parameter is M_Q.

The structure constraint on the $\bar{Q}$-parameter $\left(\bar{Q} = I_{q+1} \otimes Q\right)$ is a convex structure constraint that reduces the number of decision variables.

Theorem 4:
For a given Q-basis projection the insensitive H_2 control problem is equivalent to the LMI problem (14).

$$\min_{R,S,T,M_Q} trace(Y)$$

$$\begin{cases} \begin{bmatrix} \bar{A}_1 R + R\bar{A}_1^T & \bar{A}_1 S - S\bar{A}_2 + \bar{B}_2 M_Q \bar{C}_2 & R\bar{C}_1^T & 0 & 0 & 0 \\ (.)^T & F\bar{A}_1 + \bar{A}_1^T F & S^T\bar{C}_1 + \bar{C}_2^T M_Q^T \bar{D}_1 & 0 & 0 & 0 \\ (.)^T & (.)^T & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y & \bar{B}_3^T S^T - \bar{D}_2^T M_Q^T \bar{B}_2^T - \bar{B}_1^T & -\bar{B}_2^T F \\ 0 & 0 & 0 & (.)^T & -R & 0 \\ 0 & 0 & 0 & (.)^T & 0 & -F \end{bmatrix} < 0 \\ M_Q = \begin{bmatrix} I_{q+1} \otimes C_Q & I_{q+1} \otimes D_Q \end{bmatrix} \end{cases} \qquad (14)$$

Proof:
According to Theorem 2, the insensitive H_2 control problem is equivalent to the following problem

Pb. Find the optimal *Q*-parameter such that

$$Q^* = \arg\min_{Q} \left\| F_l(\bar{J}, I_{q+1} \otimes Q) \right\|_2^2 \qquad (15)$$

Q^* may be obtained by solving the BMI optimization problem (16).

$$\begin{cases} \min_{X,Y,M_Q} trace(Y) \\ \begin{bmatrix} A_{cl}^T X + X A_{cl} & C_{2cl}^T & & 0 \\ C_{2cl} & -I & & \\ & & -Y & B_{2cl}^T X \\ 0 & & X B_{2cl} & -X \end{bmatrix} < 0 \\ \bar{Q} = I_{q+1} \otimes Q \end{cases} \qquad (16)$$

Let us consider the Lyapunov function X partitioned into $X = \begin{bmatrix} W & Z \\ Z^T & Y \end{bmatrix}$ according to $\begin{bmatrix} \bar{A}_1 & \bar{B}_2 M_Q \bar{C}_2 \\ 0 & \bar{A}_2 \end{bmatrix}$.

The matrix inequality (16) corresponding to the insensitive H_2 control problem applied to the closed loop system are non linear in the decision variables M_Q and X. It can, however, be transformed by a change of variable and a congruence transformation. Let us consider the change of variable (see (Scherer 1999, 2000)) :

$$\begin{bmatrix} W & Z \\ Z^T & Y \end{bmatrix} \to \begin{bmatrix} R & S \\ S^T & F \end{bmatrix} = \begin{bmatrix} W^{-1} & -W^{-1}Z \\ -Z^T W^{-1} & Y - Z^T W^{-1} Z \end{bmatrix} \qquad (17)$$

and define

$$N = \begin{bmatrix} R & 0 \\ S^T & I \end{bmatrix} \qquad (18)$$

Let $\Theta = diag(N, I, I, N)$ then pre-multiplying and post-multiplying the matrix inequality involved in (16) by Θ^T and Θ yields to the LMI problem (14).

Remark

The insensitive H_2 control problem can be formulated as the LMI optimization problems (14) which depends affinely on the variables R, S, F and M_Q.

Algorithm

Step 1- Synthesize the H_2 optimal controller (K_c, K_f).

Step 2- Derive the $\bar{J}$ parameter and then $F_l(G_a, \bar{J})$

Step 3- Choose the othonormal *Q*-basis $\left((a_i)_{1 \cdots N_q} \right)$.

Step 4- Solve the LMI problem (14).

Step 5- Reconstruct the controller $(K(s) = F_l(J, Q))$.

5. AN AUTOMOTIVE CONTROL EXAMPLE

The practical interest of the new method proposed in this paper is shown in this example through a robust vehicle dynamics control as considered in (Gay *et al.*, 2000). The lateral velocity V_y and the yaw velocity $\dot{\psi}$ have to be controlled through two control inputs: the yaw moment C_z that can be obtained by differential braking and the rear steering α_r (see Fig. 6). The vehicle must stay near to the desired trajectory as shown in Fig. 7. Disturbance efforts acting on the vehicle can be summarized into lateral force F and a yaw moment M.

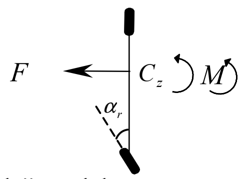

Fig. 6. The "bicycle" model

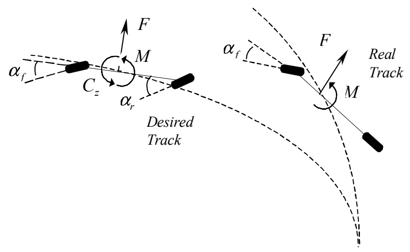

Fig. 7. The desired trajectory

The well known "bicycle model" given by (19) is used to describe the vehicle motions.

$$\begin{bmatrix} \dot{V}_y \\ \ddot{\psi} \end{bmatrix} = A \begin{bmatrix} V_y \\ \dot{\psi} \end{bmatrix} + B_r(\alpha_r + d_{\alpha r}) + B_c(C_z + d_{Cz}) \qquad (19)$$

with

$$A = \begin{bmatrix} -\frac{2\mu}{mV_x}(C_{yv} + C_{yr}) & -V_x + \frac{2\mu}{V_x}(l_2 C_{yr} - l_1 C_{yv}) \\ \frac{2\mu}{CV_x}(l_2 C_{yr} - l_1 C_{yv}) & -\frac{2\mu}{CV_x}(l_1^2 C_{yv} + l_2^2 C_{yr}) \end{bmatrix},$$

$$B_r = \begin{bmatrix} \frac{2\mu C_{yr}}{m} \\ \frac{-2\mu l_2 C_{yr}}{C} \end{bmatrix} \text{ and } B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

In this model, m denotes the mass of the vehicle, C the inertia, l_1 the front wheelbase, l_2 the rear wheelbase and (C_{yv}, C_{yr}) the nominal cornering stiffness. Note also that the model is parameterized by the road friction parameter μ which is, indeed, uncertain. The standard H_2 problem to be minimized to meet the control requirements is built following the Standard State Control methodology (Ph.De Larminat, 2000) as adopted in (Gay *et al.*, 2000). The augmented plant including the disturbances model is of order 4 (see (Gay *et al.*, 2000) for details).

The nominal model has been taken for a constant longitudinal speed $V_x = 80km/h$. The H_2 optimal controller is denoted by $K_0(s)$. For $\sigma = 0.75$ (a sensitivity weighting parameter), two controllers, namely $K_1(s)$ and $K_2(s)$, are derived using respectively the resolution of the BMI problem underlying the structured formulation proposed in

(Chevrel and Yagoubi 2004) and the new LMI algorithm presented in the previous section (based on a projection of the structured Youla parameter in the Niness orthonormal basis). Table 1 summarizes the results obtained.

Table I. Criterions for controllers K_0, K_1 and K_2

Controllers	K_0	K_1	K_2
IH_2 criterion	12,88	8.71	7.23
order	4	4	8

It appears that the proposed LMI algorithm gives better result in term of the IH_2 criterion. This comes probably from two facts:
i) the controller has not been restricted to be of the same order as the standard model;
ii) the problem to be solved is convex contrary to the BMI problem (only monotonic convergence to a local minimum is guaranteed (see (Chevrel and Yagoubi 2004)).

The computational time of the BMI resolution is higher than the computational time of the presently proposed LMI approach. This fact has to be analyzed more deeply since only a low order (N_q=2) *Q*-parameter was used to synthesize the controller K_2.

6. CONCLUSION

Finding an insensitive H_2 controller is a difficult problem. It was recently reformulated as a structured H_2 control problem. To the authors knowledge there is no algorithm that can solve efficiently such a problem. This paper proposes a new formulation of the insensitive H_2 control problem based on a structured Youla parameter. The projection of the Youla parameter in the Niness orthonormal basis allows approximating the insensitive H_2 control problem as a finite dimensional problem.
The resulting algorithm has been applied to an automotive control problem. It improves the results obtained previously. Although it has to be more widely tested, the approach is appealing on two points: optimality and computation time. It requires, however, some methodology to define a parsimonious basis for the Youla parameter description.

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