ACCURATE MODELLING AND IDENTIFICATION OF VEHICLE'S NONLINEAR LATERAL DYNAMICS

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Abstract: The use of model-based controller or fault diagnosis systems in automotive control requires reliable dynamics models. The reliability consists in accurate prediction of the vehicle's behavior, in order to evaluate current driving situations. Model parameters have to be precisely identified. This depends on the model's capability to reproduce the complex dynamics of the real system, by remaining as simple as possible, for online computation. In this article, a practical approach of modelling vehicle dynamics is presented. Conventional simplifications are removed by coupling lateral and roll motion. Besides, nonlinear tire characteristic is presented. The accuracy and performance of the identified model is demonstrated by tests on real automotive systems.

Keywords: Automotive Control, Vehicle Dynamics, Nonlinear Identification, Nonlinear Dynamics

1. INTRODUCTION

In the sector of automotive and transportation systems, the research is focused on improving passenger's safety and increasing driving comfort. Systems like the ESP (Electronic Stability Program) or ACC (Active Cruise Control) became standard equipment in modern vehicles. Such Features are generally real-time and embedded controller, which are based on mathematical dynamics models. It's obvious that the performance and therefore the commercial success of such systems depend on the accuracy of the implemented models and the correctness of their parameter settings. The parity of the last two mentioned factors is the subject of this paper. Modelling and identification of lateral dynamics models were treated in interesting publications, such as (Alloum *et al.*, 1997; Boerner and Isermann, 2002) and (Boros, 2002). Unfortunately the interaction with other DOF's (in particular with the roll motion) is neglected. This assumption leads mostly to the identification of nonconstant and velocity dependent parameters, especially for vehicles with high center of gravity (Boros, 2002; Abdellatif *et al.*, 2003). In (Wuertenberger *et al.*, 1994) an approach for the identification of roll dynamics was presented, but its influence on the lateral motion was not taken in account. The importance of coupling the roll and lateral dynamics for the purpose of motion controller's design were demonstrated in (Abdellatif *et al.*, 2003; Feng *et al.*, 1998). An other important factor for model accuracy is the description of tire lateral forces. Simple widespread linear approaches are also restricted to limited application. Since the monitoring of tire dynamics plays a decisive role in evaluating the current drive situation (Boerner and Isermann, 2002; Sienel, 1997), it is important to focus on appropriate methods for its description. In section 2 an accurate approach for modelling vehicle's lateral dynamics is presented. All important factors are taken into account, such the influence of the roll motion and the nonstationary and nonlinear tire dynamics. The nonlinear identification method of the model parameters follows in section 3. Its successful application to measurements of many car types is illustrated in section 4. The accuracy of the proposed model is demonstrated by comparison with classical approaches.

2. VEHICLE DYNAMIC MODEL

2.1 Linear Lateral Dynamics

For modeling the lateral dynamics, the well reputed bicycle model is introduced (Fig. 1). The

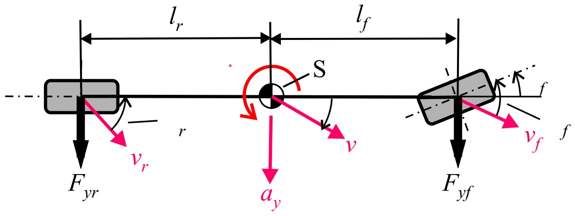

Fig. 1. Bicycle model.

equations of motion in terms of lateral acceleration a_y and yaw velocity $\dot{\psi}$ can be written as:

$$ma_y = F_{yf} + F_{yr} \quad (1)$$

$$J_z\ddot{\psi} = -l_f F_{y,f} + l_r F_{y,r} \quad (2)$$

In the stationary and linear case (small steering angles), the lateral forces are expressed as a linear relationship in respect to the related skew angle α: $F_y = c_\alpha \alpha$, where c_α is the tire cornering stiffness. The skew angle is the argument of the respective tire velocity vector:

$$\begin{aligned} \alpha_f &= \delta_f - \frac{l_f}{v}\dot{\psi} - \beta \\ \alpha_r &= \frac{l_r}{v}\dot{\psi} - \beta \end{aligned} \quad (3)$$

2.2 Linear Roll Dynamics

Vehicle's roll dynamics has been studied in many literature with varying complexity (Wuertenberger *et al.*, 1994; Feng *et al.*, 1998). Since our preliminary goal is the determination of the influence of the roll motions on the lateral dynamics, only a simple modeling is necessary. The schematic model is shown in Fig. 2. One distinguishes the mass of chassis m_a from the masses of front and rear axle m_f and m_r. m_a is suspended on four damper, four spring and two stabilisator devices. The differential equation of the roll motion κ is then :

$$\begin{aligned} &(J_x + m_a h_r^2)\ddot{\kappa} + d_\kappa \dot{\kappa} + c_\kappa \kappa - m_a g h_r \kappa = \\ &m_a h_r [\frac{\sum F_y}{m} - \frac{m_f + m_r}{m} h_r \ddot{\kappa}] \end{aligned} \quad (4)$$

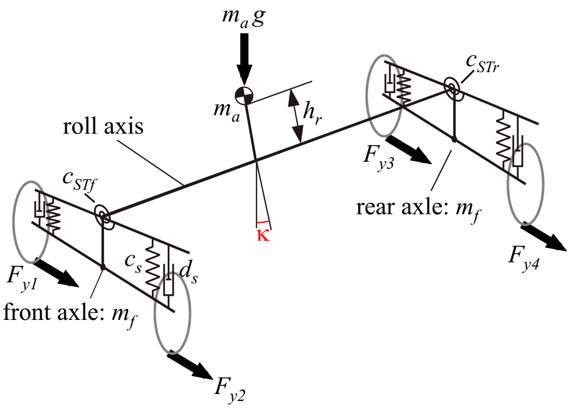

Fig. 2. Scheme of the roll dynamics model.

This equation can be simplified to:

$$J_\kappa \ddot{\kappa} + d_\kappa \dot{\kappa} + c_\kappa \kappa - m_a g h_r \kappa = m_a h_r a_y, \quad (5)$$

where

$$\begin{aligned} J_\kappa &= J_x + m_a h_r^2 + \frac{m_f + m_r}{m} h_r^2 \\ c_\kappa &= c_{STf} + c_{STr} + cs^2 \\ d_\kappa &= ds^2 \end{aligned} \quad (6)$$

The interaction of the roll motion with the lateral dynamics can be deduced by computing the skew angles at the front and rear axles:

$$\begin{aligned} \alpha_f &= \delta_f - \frac{l_f}{v}\dot{\psi} - \beta - \frac{h_r}{v}\dot{\kappa} \\ \alpha_r &= \frac{l_r}{v}\dot{\psi} - \beta - \frac{h_r}{v}\dot{\kappa} \end{aligned} \quad (7)$$

One notices the difference to eq. 3 with the appearance of the roll rate $\dot{\kappa}$ in the new formula. In fact the roll motion contributes to the scaling down of the tire skew angles.

2.3 Tire Model

The tire model is the most significant part of the lateral dynamics, which makes an accurate modelling of its physical behavior very necessary. One distinguishes between stationary and nonstationary tire behavior. Stationary properties describe nominal values for tire forces by constant or static physical variables (skew angle, tire slippage, vertical forces). Non stationary behavior considers the dynamics of the build up of tire forces.

2.3.1. Nonlinear Stationary Tire Properties It is known from practice, that the lateral dynamics are greatly affected by the tire's nonlinear properties. Many theoretic and empirical models have been proposed for the description of the tire nonlinear performance. The most popular approach is the magic formula (Pacejka and Besselink, 1997).

Other models express the relationship between cornering force and skew angle in polynomial form (Qu and Liu, 2000). A more interesting approach is the use of intelligent learning systems, such neural networks for the description and online adaptation of the complex tire behaviour (Holzmann *et al.*, 1999). In the following only analytical expressions of tire characteristics are discussed.

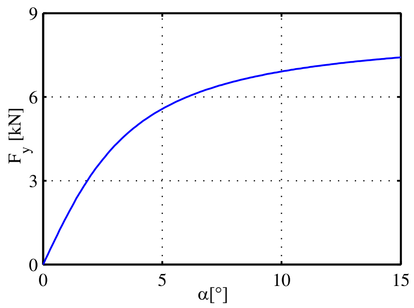

Fig. 3. Nonlinear tire characteristics for lateral forces.

The magic formula or the polynomial models are convenient for the simulation of vehicle's dynamics. The huge amount of the related parameters yields however bad conditioned identification. We propose the following simplification of the magic formula for the lateral forces:

$$F_y = f_T(\alpha) = c_\alpha \sin(C \arctan(B\alpha)) \qquad (8)$$

The degressive characteristics is depicted in Fig. 3 and corresponds to the known tire nonlinearity. Two quantitative properties of f_T are its limes in infinity:

$$\lim_{\alpha\to\infty} f_T = c_\alpha \sin(\frac{\pi}{2}C), \qquad (9)$$

and the value of its derivative at $\alpha = 0$, which corresponds to the linearization of the characteristics for small angles:

$$\frac{\partial f_T}{\partial \alpha}_{|\alpha=0} = c_\alpha CB, \qquad (10)$$

which is useful for the verification of the compatibility with the linear case ($CB \doteq 1$).

2.3.2. Tire Dynamics The consideration of tire dynamics is necessary to increase model accuracy for driving maneuver with high steering frequencies (usually≥1Hz). It's obvious that the stationary approach is only restricted for constant or very slow varying skew angles (Abdellatif *et al.*, 2003). The tire dynamics are modeled as a delay unit of the 1st order:

$$\frac{l_T}{v}\dot{F}_y + F_y = f_T(\alpha), \qquad (11)$$

where l_T is a tire delay constant and is the distance needed by the tire to build up the cornering force after a steering intervention from the driver.

2.4 Vehicle's Nonlinear Dynamics

The complete model of the vehicle's nonlinear dynamics results by combining all the above presented equations (eq. 1 to eq. 11). The block diagram in Fig. 4 displays in a clear way the building of the lateral dynamics. We summarize by the definition of the dynamics state vector:

$$\mathbf{x} = \left[\, a_y \;\; \dot{\psi} \;\; \kappa \,\right]^{\mathrm{T}}, \qquad (12)$$

and the unknown model parameter vector

$$\mathbf{p} = \left[\, \mathbf{p}_{lin}^{\mathrm{T}} \mid \mathbf{p}_{nonlin}^{\mathrm{T}} \,\right]^{\mathrm{T}}, \qquad (13)$$

which is subdivided in a parameter vector for the linear model part:

$$\mathbf{p}_{lin} = \left[\, J_z \;\; l_{Tf} \;\; l_{Tr} \;\; c_{\alpha f} \;\; c_{\alpha r} \;\; J_\kappa \;\; d_\kappa \;\; c_\kappa \,\right]^{\mathrm{T}}, (14)$$

and a parameter vector for the nonlinear tire characteristics:

$$\mathbf{p}_{nonlin} = \left[\, C_f \;\; B_f \;\; C_r \;\; B_r \,\right]^{\mathrm{T}}. \qquad (15)$$

The rest of the model parameter (such as l_f,l_r,h_r, etc..) are supposed to be known, since they can be directly and accurately measured.

3. NONLINEAR IDENTIFICATION METHOD

Since the model approach is high nonlinear in its states and in its parameters, it is reasonable to use nonlinear methods for its identification. A loss function is defined as a nonlinear least square formulation:

$$\mathbf{I}(\mathbf{p}) = \mathbf{f}^{\mathrm{T}}\mathbf{Q}\mathbf{f}. \qquad (16)$$

$\mathbf{f}(\mathbf{p})$ is a vector function of model observation errors in respect to the lateral accleration a_y, the yaw rate $\dot{\psi}$ and the roll angle κ. $\mathbf{Q}$ is a diagonal weight matrix, which is used to equalize the different order of magnitude of the three dynamics states. Minimizing the loss function $\mathbf{I}$ by implementing a nonlinear optimization technique yields an estimation for the model parameter $\hat{\mathbf{p}}$. Iteratively a new estimate is computed with the Newton's method:

$$\hat{\mathbf{p}}_k = \hat{\mathbf{p}}_{k-1} - \mathbf{H}_{k-1}^{-1}\mathbf{g}_{k-1}, \qquad (17)$$

where $\mathbf{H}$ and $\mathbf{g}$ are respectively the Hessian and the gradient of the loss function. The gradient can be formulated as:

$$\mathbf{g}_k = 2\mathbf{J}_k^{\mathrm{T}}\mathbf{f}_k. \qquad (18)$$

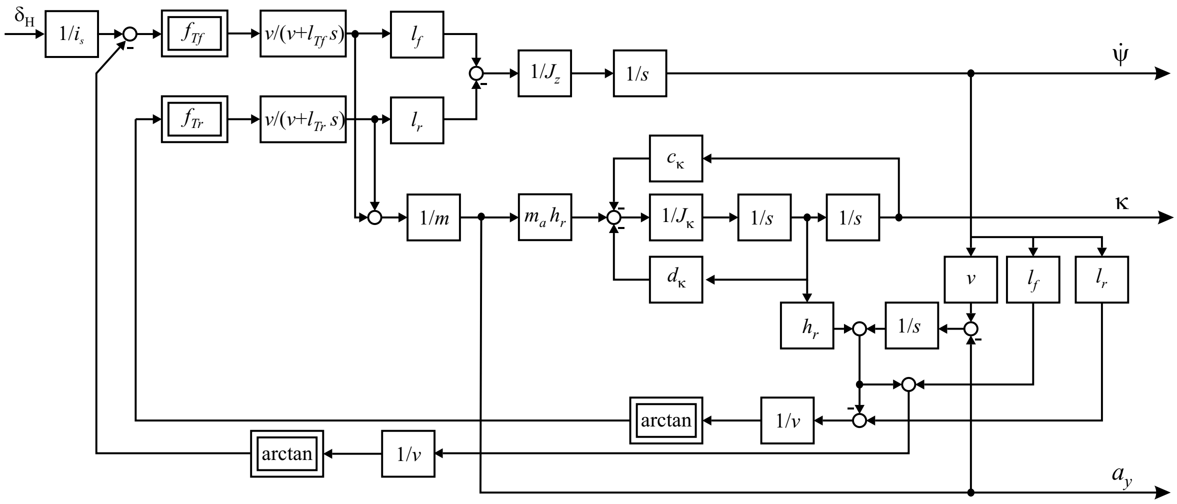

Fig. 4. Block diagram for nonlinear and coupled lateral and roll motion

$\mathbf{J}$ denotes the Jacobian of the loss function in respect to the parameter vector. The Hessian is obtained by:

$$\mathbf{H} = 2\mathbf{J}^{\mathrm{T}}\mathbf{J} + 2\mathbf{S}^{2nd}, \tag{19}$$

where $\mathbf{S}^{2nd}$ is a second order sensitivity matrix (Nelles, 2001), which is estimated with an auxiliary algorithm. Considering eq. 18 and 19, the estimate update of eq.17 becomes:

$$\begin{aligned}\hat{\mathbf{p}}_k &= \hat{\mathbf{p}}_{k-1} - \\ &(\mathbf{J}_{k-1}^{\mathrm{T}}\mathbf{J}_{k-1} + \mathbf{S}_{k-1}^{2nd})^{-1}\mathbf{J}_{k-1}^{\mathrm{T}}\mathbf{f}_{k-1}\end{aligned} \tag{20}$$

It is recommended to use the Cholesky factorization to inverse the Hessian Matrix.

4. EXPERIMENTAL RESULTS

The presented approach of modelling the lateral dynamics as well as the related identification method were successfully accomplished for 7 different test vehicles. Mid-size vehicles, limousines, transporter as well as compact cars were included (Abdellatif *et al.*, 2003). The efficiency and accuracy of our model approach is demonstrated by comparing the results to alternative classic approaches. For clarity reason the results are presented separately for the parameter of the linear submodel and for the nonlinear tire characteristics, although the identification was accomplished for the integral model.

4.1 Parameter Identification for Linear Submodel

The presented approach in this paper distinguishes itself by the systematic and correct consideration of the roll motion's influence on the vehicle's lateral dynamics. In our research, we found, that this influence is important for two major cases:

- for vehicles with high center of gravity or high chassis mass at every speed
- for all vehicles at small speed ($\leq$ 60km/h)

Neglecting the interaction of the lateral with the roll motion yields non reasonable velocity dependent parameter. The role of correct modeling should not be neglected to achieve good and accurate results. Fig. 5 shows the yaw velocity's fre-

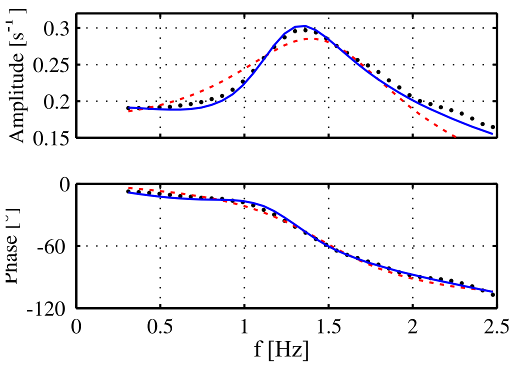

Fig. 5. Yaw velocity in frequency domain for a vehicle with high centre of gravity , (...) real vehicle, (- -) classic noncoupled model, (—) coupled model.

quency response for a vehicle with a high center of gravity. In contrast to the classical non-coupled bicycle model, the presented approach yields the improvement of the reproduction of the real vehicle's behavior. For mid-size cars or compact-vehicles, the influence of roll motion can be neglected only for velocities, which are higher than ≈60 km/h, without affecting greatly the model's accuracy. Fig. 6 shows the lateral acceleration's frequency response for a mid-size vehicle at a low velocity (40 km/h). It proves the importance of considering the roll motion, which affects also the accuracy of the identified parameters.Its Neglecting yields automatically velocity-dependent parameters, which make no physical sense (e.g. velocity-dependent yaw inertia). This effect is shown in Fig. 7. The car speed's dependency of the yaw

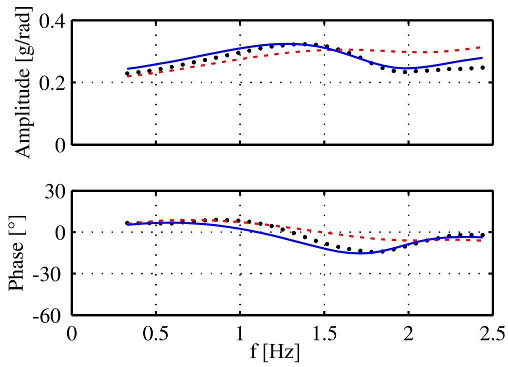

Fig. 6. Lateral acceleration in frequency domain for a mid-size car at low speed, (...) real vehicle, (- -) classic non coupled model, (—) coupled model.

inertia, which was also observed in (Boros, 2002) for commercial and heavy vehicles is eliminated by implementing our model approach. For such vehicles all parameters are affected. For mid-size cars however, parameter deviations were observed only at small vehicle's speed (Abdellatif *et al.*, 2003). The augmentation of the number of parameters

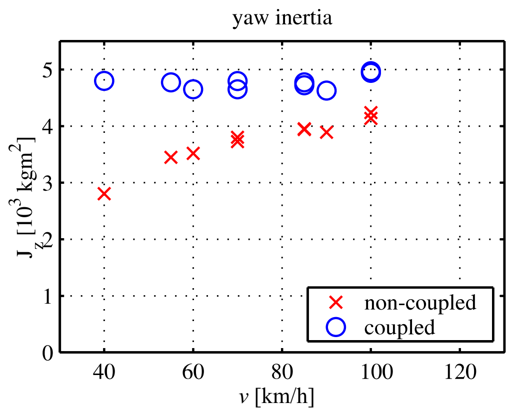

Fig. 7. Identified yaw Inertia for a heavy transporter.

in the presented approach does not decline their precise identification. All the additional parameter (in comparison to the classical approach) were identified accurately. Fig. 8 shows exemplarily the results for the roll damping constant d_κ.

4.2 Identification of Tire Characteristics

Vehicle's lateral dynamics can be considered as linear for lateral acceleration up to 4 m/s^2. Above this value, the dynamics are greatly affected by

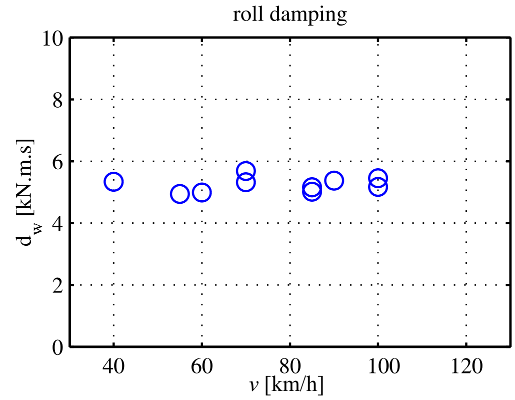

Fig. 8. identified roll damping ratio for a heavy transporter

the nonlinear tire behavior. For sufficient excitation of the parameter vector $\mathbf{p}_{nonlin}$, it is necessary to consider drive maneuver with high steering angles, such steer ramps or steer steps. Table 1 presents exemplarily the identification's results of the nonlinear submodel for 3 different vehicles. The characteristics of vehicle nr. 3 was identified for two different tire types (3-1 and 3-2).

Table 1. Identified parameter of the non-linear tire characteristics for 3 different vehicles

veh. nr.	C_f	B_f	C_fB_f	C_r	B_r	C_rB_r
1	0.06	17.34	1.04	0.04	24.45	0.98
2	0.09	11.38	1.02	0.05	22.67	1.13
3-1	0.08	13.04	1.04	0.04	25.90	1.04
3-2	0.09	11.02	0.99	0.04	24.48	0.98

It was mentioned in section 2.3, that the product CB is useful to verify the compatibility of the nonlinear model with the linear subpart. Table 1 shows, that the identified nonlinear model can be applied to all driving situations, since this product is always $\approx$ 1. Fig. 9 depicts the output of the linear and nonlinear models in comparison with real vehicle's data measured while a steer ramp maneuver (maximal steering angle of 75°). In the area of low lateral acceleration the nonlinear as well as the linear approach display accurate prediction. At high lateral acceleration, only the nonlinear approach is able to reconstruct the real vehicle behavior. Certainly it is possible to adjust the parameter of the linear model to reduce the model error during critical situations. This yields though non realistic parameter values and can besides never display the degressive character of the real dynamics. Finally, to demonstrate the accuracy of the modelling approach on practical driving situation, model errors of the lateral acceleration and yaw rate are recorded while a lane change maneuver. A comparison between the errors of the linear model and those of the nonlinear model is depicted by Fig. 10. The nonlinear approach yields as excepted high improvement of the model's accuracy.

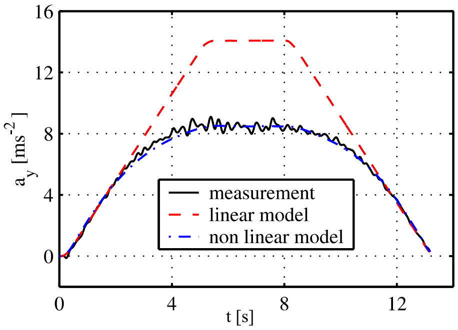

Fig. 9. Comparison of the lateral acceleration with linear and nonlinear model approach

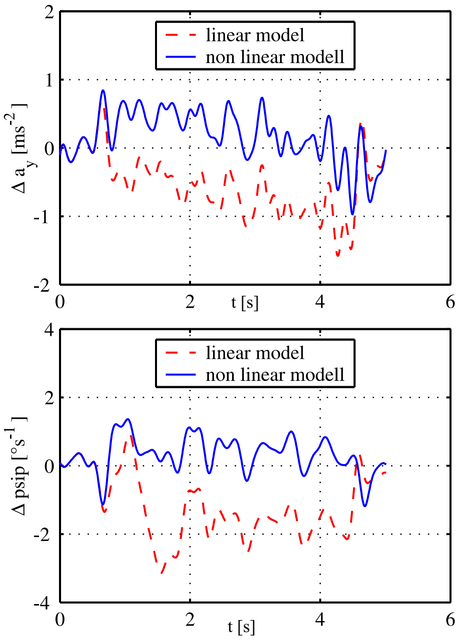

Fig. 10. Comparison of model error between linear and nonlinear approach. (Top: for lateral acceleration, down: for yaw rate)

5. CONCLUSIONS

The aim of this article was to present a practical and accurate approach for modeling vehicle's lateral dynamics for the application in vehicle's dynamic control. Many conventional simplifications were removed. Additionally to the consideration of tire dynamics, the influence of roll motion on the lateral dynamics was integrated. A simple and effective approach for the description of tire nonlinear characteristics was presented. The subsequent identification of the model proved the efficiency and high accuracy of the integral model. Successful experiments on wide range of vehicles was presented to demonstrate the practicability and general validity of the proposed model.

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NOMENCLATURA

ay	lateral acceleration	[ms^{-2}]
$\dot{\psi}$	yaw rate	[rad/s]
κ	roll angle	[rad]
β	vehicle slip angle	[rad]
v	vehicle velocity	[m/s]
v_f	front tire's velocity	[m/s]
v_r	rear tire's velocity	[m/s]
α	tire's skew angle	[rad]
δ_f	steering angle of front wheel	[rad]
F_y	tire lateral forces	[N]
s	track-width	[m]