NONLINEAR ADAPTIVE FUZZY CONTROL FOR HYDRAULIC ROBOTS

[2]Chih-Fu Chang, [2]Su-Chiun Wang and [1,2]Li-Chen Fu

[1]Department of Computer Science and Information Engineering
[2]Department of Electrical Engineering

National Taiwan University, R.O.C.
e-mail: d92921001@ntu.edu.tw

Abstract –In this paper, a novel adaptive fuzzy controller (AFC) is proposed to deal with the control problem of a parallel robot consisting of a Stewart platform and hydraulic actuators. Specifically, only two signals are measured from the robot system, namely, the leg displacements through Linear Variable Differential Transformer (LVDT) and the current change with Hall sensor. To cope with the difficulty of obtaining the full parametric model of the Stewart platform and that of the complete actuator dynamics, an adaptive fuzzy control method is then adopted. Simulations are done to compare the performance of the presented AFC and that of an appropriately designed adaptive controller. Apparently, the proposed control shows that the tracking errors of the present design are almost an order less than that of the otherwise designed adaptive controller.

Keywords – Adaptive fuzzy control, Stewart platform, robot, hydraulic, dynamics model

1. INTRODUCTION

Hydraulic systems have been used in industry in a variety of applications due to their low size-to-power ratio and the ability to apply very large force and torque. As illustrated in Fig.1, the Stewart platform is equipped with a payload and six hydraulic actuators whereby the six legs can change their lengths independently. For the structure of parallel actuators, the platform has rapid response and high power-to-weight ratio suitable for many applications. Normally, this platform is widely used in industry and/or in the fields of motion simulator and tele-surgery system.

The kinematics model for the platform was successfully developed (Tian Huang *et. al.*, 1999) to calculate the lengths of the six legs (actuators) given the basic position and attitude information of the platform, i.e., x, y, z, row, yow and pitch. Conceptually, since some of the applications which require more accurate location of the platform system, the nonlinear dynamic model becomes imperative and has been derived by (D. Li *et. al.*, 1997). Besides nonlinear nature of the hydraulic dynamics, hydraulic systems also suffer from considerable model uncertainty (Bio Yao *et. al.*, 2000). The uncertainties can be classified into two categories: parametric uncertainties and uncertain nonlinearities.

This paper aims to develop an adaptive fuzzy controller to successfully handle the situation with unknown actuator parameters and system configurations. Through Lyapunov analysis, the proposed closed-loop system is proved to be asymptotically stable. The paper is organized as follows. Brief description of Stewart platform are presented in Section 2 and 3, respectively. An adaptive controller taking Stewart platform dynamics into consideration is shown in Section 4. On the other hand, a fuzzy controller with actuator dynamics is introduced in Section 5. Finally, a novel AFC for the entire hydraulic robot system is designed in section 6.

The setup of the overall system and extensive simulation results are provided in Section 7, and conclusions are drawn in Section 8.

2. BRIEF DISCRIPTION OF PLATFORM

For the requirements on achieving precision motion control of the Stewart platform, the closed-form dynamics has to consider model uncertainties, actuator dynamics and uncertain nonlinearities. The configuration of the actual physical platform is shown in Figure 1.

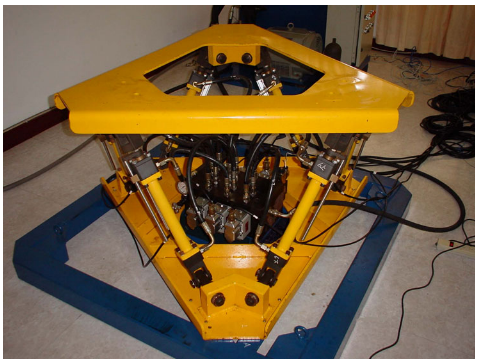


Fig. 1. Stewart platform with six hydraulic actuators

However, by taking into account both the platform dynamics and actuator dynamics, the controller design becomes much more complex. What is even worse is that some signals are hardly available or measurable, which causes a challenge to the design task. Nevertheless, the conceptual diagram of the high performance controller is shown as below :

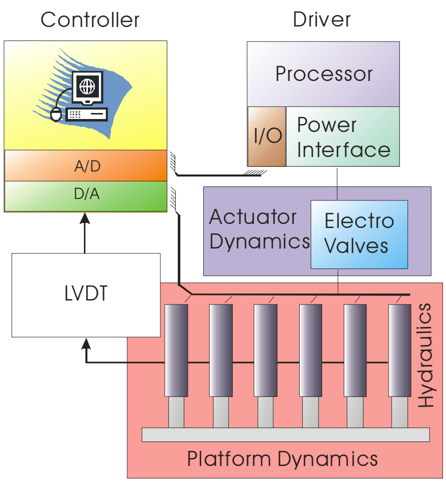


Fig. 2. Electrical structures of the conceptual diagram of the Stewart platform

The control command here is the DC voltage of the D/A output, which is proportional to the valve spool position of each hydraulic actuator. The relationship between valve and output voltage is provided in Appendix A. For sensing, the LVDT supplies the positional information of the platform legs, and hence the platform itself.

3. PLATFORM AND ACTUATOR DYNAMICS

The actuator is designed with servo valves instead of proportional ones for higher frequency response. To derive the kinematics model, we define a base coordinate frame $\{B\}$ affixed to the base platform, whereas the platform coordinate frame $\{p\}$ affixed to the top surface of the platform as shown below :

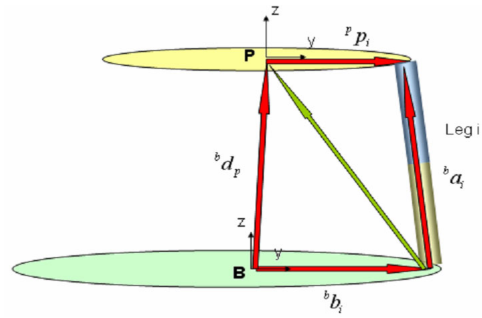


Fig. 3. Simplify structure of Stewart platform

2.1 Kinematics

The kinematics equation is expressed as

$$l_i = Rp_i + d - b_i \tag{1}$$

where $l_i \in R^3$ denotes the length with each i^{th} hydraulic actuators, $b \in R^6$ denotes the distance from the center of a base to the joint, $p_i \in R^{3\times1}$ denotes the distance from the center of platform to i^{th} joint, $R \in R^{3\times3}$ denotes the rotation matrix associated with the platform subjected to the rotation angles of row, yaw and pitch, and $\mathrm{d} = \begin{bmatrix} x_p & y_p & z_p \end{bmatrix}^T$ represents the translation vector associated with the platform. The length of the i^{th} actuator is given by $\{\|l_i\| i = 1 \sim 6\}$.

The control goal is to solve the inverse kinematics of the platform with control variable l_i. The following Jacobian transformation gives the velocity kinematics of the platform:

$$\dot{l}_i = J_i \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{2}$$

where $\mathrm{v} = \dot{\mathrm{d}}$ and ω are the translational and rotational velocities of the platform, respectively, and each row of J, Jacobian has the form

$$J_i = \frac{1}{l_i} \begin{bmatrix} Rp_i + d - b_i \\ (Rp_i) \times (d - b_i) \end{bmatrix}^T \tag{3}$$

2.2 Dynamic equation

Now, the system dynamics equation can be derived as follows, where the readers are referred to the paper by (D. Li *et al.*, 1999) for detail. In the task-space coordinate, the dynamics of a platform is governed by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G = (JL)^{\mathrm{T}} \tau \tag{4}$$

where $q = \begin{bmatrix} x & y & z & \psi & \theta & \phi \end{bmatrix}^T$, τ the torque and ϕ, θ, and ψ are Euler angles, respectively. Furthermore, the Jacobain $J = [J_1 \cdots J_6], J_i \in R^2$ and L is defined as

$$L = \begin{bmatrix} I_{3\times3} & 0 \\ 0 & T \end{bmatrix} \text{ with } T = \begin{bmatrix} \cos(\theta)\cos(\phi) & -\sin(\phi) & 0 \\ \cos(\theta)\sin(\phi) & \cos(\phi) & 0 \\ -\sin(\theta) & 0 & 1 \end{bmatrix}$$

where $L \in R^{6\times 6}$

2.3 Actuator dynamics

Actuator dynamics can be expressed as the following general form.

$$\dot{\tau} = \gamma_1 f_0\left(\zeta,\dot{\zeta}\right) + \gamma_2 g_0\left(\zeta,\tau,u\right) \quad (5)$$

where both $\gamma_1 \in R$ and $\gamma_2 \in R$ are the constant gain. and ζ denotes the load flow pumped by flow compressor, u actual control input of the actuator.

From equation (5), the first term is affected by dynamic flow rate whereas the second term is controlled by load flow, output torque and input control. Consequently, the gain could be got from the actual experiment.

4. NONLINEAR ADAPTIVE CONTROLLER

A nonlinear adaptive control law applied to the Stewart platform has been proposed by (Mohammad Reza Sirouspour *et. al.*, 2001). The indirect adaptive controller is derived based on Lyapunov theory. This controller can deal with the practical problems such as disturbance and model uncertainty. However, to realize such a controller suffers from several difficulties. First, the third order differentiation of position information read from LVDT should be derived. Second, the load flow of actuator is hard to measure from the electric-flow meter with high sampling rate. Third, the valve position is inherently difficult to measure essentially due to the limitations of the physical structure.

As explained above, the present adaptive fuzzy controller is designed to overcome the mentioned problems. The main strategy is that the direct adaptive controller is dedicated to control the platform dynamics whereas the fuzzy controller is concentrated to compensate the actuator control signal generated from the adaptive controller.

The dynamic equation of the platform with unknown parameters is expressed as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta \quad (6)$$

whereas the hydraulic actuator dynamics form eq. (5) is rewritten as

$$\dot{\tau} = \left(u_a + F\left(e_{q_q}, e_{\dot{q}_d}, \tilde{\tau}\right)\right)\gamma \quad (7)$$

where $q \in R^n$ is a vector of generalized joint positions, $\tilde{\tau}$ is the effect of both platform and actuator dynamics defined as $\tilde{\tau} = \tau - \tau_d$ with $\tau \in R^n$ being a vector of generalized joint torques, u_a is the control signal of the actuator which is compensated by fuzzy control output written as $u_a = -a\hat{\tau}$ with a being a positive scalar factor and $\hat{\tau} = Y(q,\dot{q}_r,\ddot{q}_r)\hat{\theta}$ being an estimated torque with $\hat{\theta}$ representing on-line estimates of θ, $F\left(e_{q_d}, e_{\dot{q}_d}, \tilde{\tau}\right)$ is a properly defined fuzzy approximation function, and $\gamma = \left[\gamma_1 \ \gamma_2\right]^T \in R^2$ stands for unknown parameter vector of the hydraulic dynamics.

By selecting a certain sampling number n, the inputs of the fuzzy tuner are given as follows:

1. Position error: $$e_{q_d} \cong \frac{1}{n}\sum_{k=1}^{n}\left(q_d(k) - \hat{q}_d(k)\right) \quad (8)$$
2. Velocity error: $$e_{\dot{q}_d} \cong \frac{1}{n}\sum_{k=1}^{n}\left(\dot{q}_d(k) - \dot{\hat{q}}_d(k)\right) \quad (9)$$
3. Torque error: $$\tilde{\tau} = \frac{1}{n}\sum_{k=1}^{n}\left(\tau(k) - \tau_d(k)\right) \quad (10)$$

Note that

1. $D(q), C(q,\dot{q}), G(q) \in R^n$ are fully unknown.
2. Parameters of actuator dynamics are fully unknown.

Let the position error be defined as

$$e = \hat{q} - q_d \quad (11)$$

$$\dot{q}_r = \dot{q}_d - \Lambda\left(\hat{q} - q_d\right) = \dot{q}_d - \Lambda e \quad (12)$$

$$s = \dot{q} - \dot{q}_r = \dot{e} + \Lambda e \quad (13)$$

where $\hat{q} \in R^n$ is the estimated value of q, e is the positional tracking errors, and $\Lambda > 0$ is diagonal matrix.

The system dynamics expressed in (6)-(7) satisfy the following properties:

(i) $x^T\left(D(q) - 2C(q,\dot{q})\right)x = 0 \quad \forall x \in R^n$

(ii) $F\left(e_{q_q}, e_{\dot{q}_d}, \tilde{\tau}\right)\gamma$ is bounded.

Theorem 1: Consider the system (6), and let Γ be a positive definite matrix. The control command τ_d is chosen as

$$\tau_d = \hat{D}(q)\ddot{q}_r + \hat{C}(q,\dot{q}_r) + \hat{G}(q) - \Gamma^{-1}s = Y(q,\dot{q}_r,\ddot{q}_r)\hat{\theta} - \Gamma^{-1}s \quad (14)$$

and define the parameter adaptation laws as

$$\dot{\hat{\theta}} = -\Gamma Y^T\left(q,\dot{q}_r,\ddot{q}_r\right)s \quad (15)$$

which leads to

$$Y(q,\dot{q}_r,\ddot{q}_r)\hat{\theta} = \hat{D}(q)\ddot{q}_r + \hat{C}(q,\dot{q}_r)\dot{q}_r + \hat{G}(q) \quad (16)$$

Due to the fact that

$$D(q)\dot{s} + C(q,\dot{q}_r)s + \Gamma^{-1}s = \tilde{\tau} \quad (17)$$

the closed-loop system is asymptotically stable.

Proof: Select a Lyapunov function candidate v_1 as

$$v_1 = \frac{1}{2}s^T D(q)s + \frac{1}{2}\hat{\theta}^T\Gamma^{-1}\hat{\theta} \quad (18)$$

Take the time derivative of v_1 to yield

$$\dot{v}_1 = s^T\left(\tilde{\tau} - \Gamma^{-1}s\right) + \dot{\hat{\theta}}^T\Gamma^{-1}\hat{\theta} \quad (19)$$

By selecting the control input

$$\tilde{\tau} = Y(q,\dot{q}_r,\ddot{q}_r)\hat{\theta} \quad (20)$$

Take into the Lyapunov candidate function to eq.(19). Updating law is obtained.

$$\dot{\hat{\theta}} = -\Gamma Y^T\left(q,\dot{q}_r,\ddot{q}_r\right)s^T \quad (21)$$

The Lyapunov function can be derived

$$\dot{v}_1 = -s^T\Gamma^{-1}s \quad (22)$$

where $\dot{v}_1$ is negative positive definite. The system is asymptotically stable in the Lyapunov sense.

Definition 1: Consider the system (7). For a real-valued input vector $\left(e_{q_q}, e_{\dot{q}_d}, \tilde{\tau}\right)^T$, the output $F\left(e_{q_d}, e_{\dot{q}_d}, \tilde{\tau}\right)$ of Takagi and Sugeno's fuzzy system is a weighted average of the y^l's:

$$F\left(e_{q_d}, e_{\dot{q}_d}, \tilde{\tau}\right) = \frac{\sum_{l=1}^{M} w^l y^l}{\sum_{l=1}^{M} y^l} \tag{23}$$

where y^l is the system output due to rule $L^{(l)}$, the weight w^l implies the overall truth value of the premise of rule $L^{(l)}$ for the input and is calculated as

$$w^l = \prod_{i=1}^{n} \mu_{F_i^l(x_i)} \tag{24}$$

Theorem 2: Consider the system (6)-(7) together with the fact that equation $F\left(e_{q_q}, e_{\dot{q}_d}, \tilde{\tau}\right)\gamma$ is finite and the initial estimation error is small enough relative to the initial desired point. For the fuzzy output function F limited by defuzzification factor and γ being a constant vector, the following equation will be promised

$$F\left(e_{q_q}, e_{\dot{q}_d}, \tilde{\tau}\right)\gamma = \delta \tag{25}$$

where the vector δ is always bounded, i.e., $\|\delta\| \le \varepsilon$ with ε being a small enough constant. Also, the adaptive control satisfies the requirements of Theorem 1. Moreover, u_a is chosen as $\left(-a\tilde{\tau} + \gamma^{+}\dot{\tau}_d\right)$ with $\gamma^{+} = \gamma^T\left(\gamma^T\gamma\right)^{-1}$ being the pseudo inverse matrix. Thus, the system with the adaptive fuzzy control is exponentially stable.

Proof:

From Definition 1, the output of the fuzzy system, let $u_f = F\left(e_{q_d}, e_{\dot{q}_d}, \tilde{\tau}\right)$ be bounded. The actuator control command u, excited persistently, is written as

$$u = u_a + u_f \tag{26}$$

with u being the actuator control command and u_f being a fuzzy control output value. Moreover, for $\tau_d \to \tau$, the estimation torque is represented as $\hat{\tau} = \tau_d + \Gamma^{-1}s$ by equation (14). We chose $u_a = -a\tilde{\tau} + \gamma^{+}\dot{\tau}_d$ carefully. When $\|\delta\| \le \varepsilon$, the equation (7) can be obtained

$$\dot{\tilde{\tau}} = -a\gamma\tilde{\tau} + \delta \tag{27}$$

where t is time. τ_0 is an initial value, such that the AFS is exponentially stable.

Corollary 1: Consider the system (6)-(7) with the large initial estimating error and the initial estimation error is large which means the point is far away from the desired point. Also, we do not consider the physical limitation of the output of system bandwidth. After selecting the proper fuzzy member functions, rules and defuzzy output factors , the design will be one of the design case of theorem 2.

Proof:

By equation (21), $\dot{\hat{\theta}}$ becomes negative large with large initial estimation error. For transient response of the system, adaptive control will win the chance to lose control which means $\hat{\tau}$ is far away from τ_d. After the fuzzy tuner is added, the fuzzy output F generates the maximum proper control values to modify the adaptive control problem by fuzzy inference mechanism for error e becoming large. Observing equation (26), the δ becomes one of the major auxiliary input to balance the transient response until the error pull back to small.

On the next section, the fuzzy tuning mechanism is proposed.

5. FUZZY CONTROLER CONTROLLER DESIGN WITH ACTUATOR DYNAMICS

The proposed fuzzy tuner is employed to monitor the actuator dynamic parameters. There are three inputs and one output for the fuzzy tuner. The position error, the velocity error, and torque error are considered as the inputs to monitor the degree of controller divergence.Figure 4. describes a conceptual diagram of the fuzzy controller :

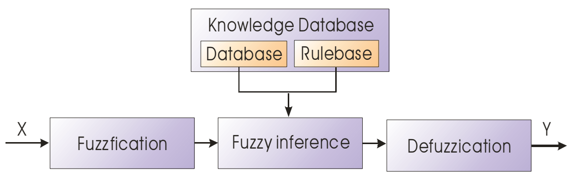
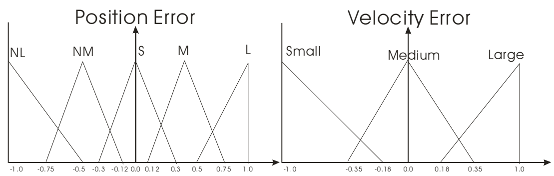
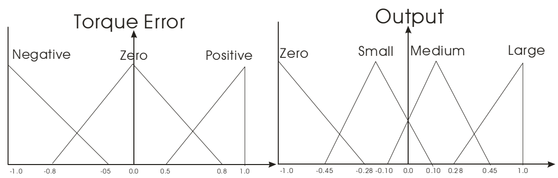


Fig. 4 Traditional fuzzy control scheme



Figs. 5,6. The membership functions for position and velocity error



Figs. 7,8. The membership functions for torque error and the weighting factor

V Err / T Err	NL	NM	S	M	L
S	Large	Small	Small	Small	Zero
M	Large	Large	Small	Zero	Zero
L	Large	Large	Medium	Zero	Zero

V Err / T Err	NL	NM	S	M	L
S	Large	Small	Medium	Small	Zero
M	Large	Medium	Small	Small	Zero
L	Large	Medium	Small	Small	Zero

Table 1,2. The fuzzy rule table for positive position error and zero position error

V Err / T Err	NL	NM	S	M	L
S	Medium	Small	Medium	Small	Small
M	Large	Small	Small	Zero	Zero
L	Large	Medium	Small	Zero	Zero

Table 3: The fuzzy rule table for negative position error.

The membership functions for input variables are trapezoidal functions since they are suitable for statistical data and are easy to implement. They are displayed in Figs. 5, 6 and 7, respectively.

The output of the fuzzy tuner is the predicted modified control signal of hydraulic, as can be seen in Fig. 3, lying within the neighborhood of 1. Let X be the input signal and Y be the output signal. By intuition and experience, the presented fuzzy tuner adopts 45 rules. A rule is listed below, for example:

- IF the position error is Medium AND the velocity error is Medium AND the torque error is Small, THEN u_f is Small

Table 1-3 summarizes those 45 fuzzy rules, in which NEGATIVE LARGE, NEGATIVE MEDIUM, SMALL, MEDIUM, LARGE, POSITIVE, ZERO and NEGATIVE stand for the corresponding fuzzy sets.

The basic tuning idea is as follows. When the position error e_{q_d} is near zero, u_f is selected according to velocity error and its output error to tune control value to eliminate the transient response. When the position error departs from zero and the velocity error becomes large, the control input will not respond to the transient response and a large value of u_f is then applied to modify the control torque and to avoid instability.

Note that actuator dynamics have a more direct effect on the control torque, thereby resulting in more confidence in the measured current. This ensures that the system can avoid divergence due to modeling errors or system uncertainties.

When the velocity and torque error are extremely large, the readings may have some problems such that the controller cannot trust the measurements any more, and a small value of u_f is preferred. By selecting appropriate u_f, the fuzzy tuner tunes the control input to actuator dynamics so as to keep the innovation sequence act as a zero-mean white noise with a suitable covariance.

6. NONLINEAR ADAPTIVE FUZZY ALGORITHM

In order to implement the above adaptive fuzzy control, we have to design the implementation algorithm. Actually, the information of the position of hydraulics is read from LVDT under the sampling rate 0.01s. Also, the hydraulic flow meter is not installed but the current of drivers is measured on our handmade driver board.

In what follows, we presents a real-time adaptive fuzzy controller algorithm. Fig. 9 depicts a block diagram of the algorithm.

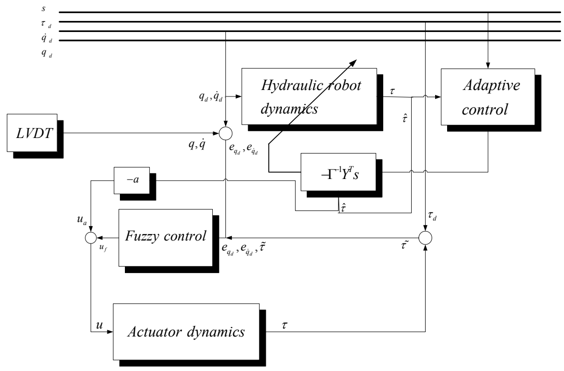


Fig. 9. A novel adaptive fuzzy control diagram

Step 1:Select initial value of $q(0)$, $\dot{q}(0)$, $\tau(0)$ and $s(0)$ at time $k = 0$.

Step 2:Read real position of each legs from LVDT.

Step 3:Calculate the updating law for $\hat{\theta}$ as shown in equation (15).

Step 4:Give a desired position q and velocity $\dot{q}$ and taken them into the adaptive control law to get the τ_d according to (14).

Step 5:Compute the actuator control signal u_a from $\hat{\tau}$.

Step 6:Generate the real torque τ which is measured from handmade driver and the torque error is expressed as $\tilde{\tau} = \tau - \tau_d$.

Step 7: Monitor the position, velocity and torque errors to produce the control output $\dot{u}$ by fuzzy control.

Step 8: Update the control command $u = u_a + u_f$.

Step 9:Repeat Step 2 to Step 9

Note that if the Stewart platform is powered on, calibration has to be done automatically. The initial value will be assigned after the self-calibration.

7. SIMULATION AND DISCUSSTION

The Stewart platform is characterized by:
$\min\{a_i\} = 100.0$ cm
$\max\{a_i\} = 160.0$ cm, $i = 1 \sim 6$
$M_p = 200$ kg,
sampling time =0.01 s
The actuator's parameters are
$\beta = 0.7\ Gpa$
$\rho = 800\ kg/m^3$
$c_d = 0.61$

A sinusoidal motion signal is inputted to leg parameters of hydraulics. Two kinds of the controllers, adaptive, and adaptive fuzzy controller, are adpoted conducted for simulations. All of them have the same system parameters and tracking path. Tracking signal u_d is commanded through digital to analog interface. The output from A/D channel lies between -10 to 10V. Hydraulics are held on if the output voltage is 0V after calibration.

Calibration process is needed because of the high nonlinear characteristic of hydraulics. The working space on the Stewart platform on vertical direction is 60 cm. The linear calibration function $y = ax + b$ is used for compensating position bias and mechanical errors. The vector of calibration parameters a, b are recorded into file after this step is adopted. The step is repeated till the 6 legs is well calibrated.

The tracking response of Stewart platform, under proposed adaptive fuzzy controller(AFC), are shown in Fig.10. The unit step response from 0 cm to 50 cm takes about 0.5 s. This response is obtained in the case where the nominal load torque and parameter variations are applied.

We conclude that the AFC gives smaller tracking error than traditional adaptive controller subjected to modeling error. Meanwhile, the tracking error is caused by 20% model. For qualifying the accuracy of the controllers, the tracking result with sinusoidal wave is shown in Fig.11.

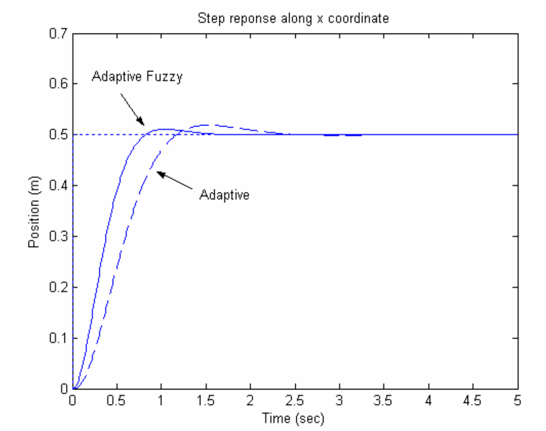


Fig. 10. Step response tracking result

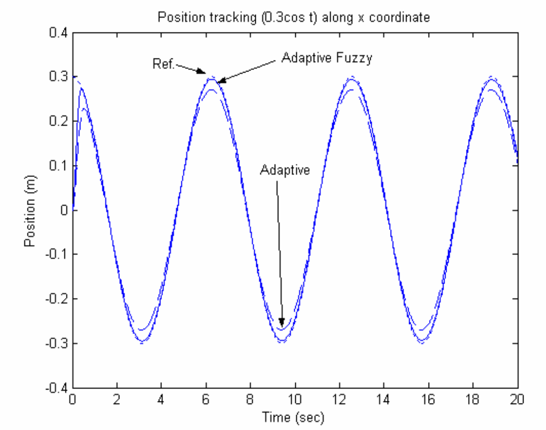


Fig. 11. Sinusoidal wave tracking result

8. CONCLUSIONS

In this paper, a novel adaptive fuzzy controller (AFC) has been proposed to deal with the control problem subjected to fully unknown parameters of both Stewart platform and actuator dynamics. To allow simple and low-cost implementation, minimal number of sensors are mounted on the hydraulic robot system. From above, AFC is free of taking the third-order time derivative of the position variable and second-order time derivative of the torque variable, which thus saves complicated computations, replacing the indirect adaptive control law with backstepping adaptive method.

From another point of view, AFC provided another way to solve the control problem of the Stewart platform with hydraulic actuators. The dynamic on-line tuning mechanism for both platform and actuator controller gave higher robustness than adaptive controller, despite not the solution with optimal parameters is provided. On the other hand, in comparison with traditional adaptive controller, the simulation data demonstrated significant improvement of tracking behavior over a well-tuned adaptive controller with this novel method.

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