CONSERVATIVE SYSTEMS WITH PORTS ON CONTACT MANIFOLDS

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Abstract: In this paper we propose an extension of port Hamiltonian systems, called *conservative systems with ports*, which encompass systems arising from the Irreversible Thermodynamics. Firstly we lift a port Hamiltonian system from its state space manifold to the thermodynamic phase space to a contact vector field with inputs and outputs. Secondly, we define a more general class of contact vector field (called conservative system with ports) generated by a function corresponding to the power of a physical system and illustrate it on a simple example of irreversible system.

Keywords: Port Hamiltonian systems, Irreversible Thermodynamics, contact forms, conservative systems

1. INTRODUCTION

Port Hamiltonian systems appeared to be a useful generalization of input-output Hamiltonian systems to deal with the modelling and control of a great variety of physical systems, essentially electro-mechanical systems (van der Schaft and Maschke, 1995) (Ortega *et al.*, 2002). Their definition is based on two objects. Firstly the *Hamiltonian function*, a smooth real valued function of the differential manifold $\mathcal{N}$ defining the state space and representing the total energy of physical systems. And secondly a geometric structure called *Dirac structure* (Courant, 1990) and defined on the product manifold of the state space manifold $\mathcal{N}$ with some vector space W of external variables. This Dirac structure is a vector subbundle of $T\mathcal{N} \times T^*\mathcal{N} \times W \times W^*$ where $T\mathcal{N}$ denotes the tangent bundle, $T^*\mathcal{N}$ the cotangent bundle of $\mathcal{N}$ and W^* the dual vector space of W. The skew-symmetry property of the Dirac structure implies the losslessness of the port Hamiltonian system (van der Schaft and Maschke, 1995). In terms of network modelling of physical systems, the Dirac structure represents the interconnection structure of the system and the skew-symmetry property corresponds to a generalization of Tellegen's theorem and implies the power continuity of the interconnection structure (Maschke and van der Schaft, 1997).

Irreversibility has been introduced by adding a dissipative term in the dynamics (Dalsmo and van der Schaft, 1999), however at the expense of loosing the conservation of the Hamiltonian function. Hence the Hamiltonian function does no more represent the total energy of the system and the system does no more express the first principle

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of Thermodynamics. The main obstacle for the simultaneous expression of both the irreversibility and the energy conservation in port Hamiltonian systems is the *linearity*, i.e. the vector bundle structure, of the Dirac structure (Eberard and Maschke, 2004).

In this paper we propose an extension of port Hamiltonian systems in order to cope with systems arising from the Irreversible Thermodynamics. These systems will be called *conservative systems with ports* and shall obey a power balance equation (leading to a lossless system), while relaxing the requirement of linearity and still defined on a manifold endowed with a differential-geometric structure, actually a contact manifold.

In the first part we shall recall how a canonical contact structure (Libermann and Marle, 1987) may be associated with the thermodynamic phase space associated with some differential manifold (Herman, 1973) (Mrugala, 1978). It will be recalled how Gibbs' relation, defining the thermodynamic properties of a system, may be expressed as a Legendre submanifold of the thermodynamic state space. In the second part, we shall lift a port Hamiltonian system from its state space manifold $\mathcal{N}$ to the thermodynamic phase space by defining some contact vector field with inputs and outputs. The generating function of this contact vector field is actually the expression of a virtual power of the port Hamiltonian system. In the third part, we shall generalize this contact vector field to the so-called conservative system with ports in such a way to cope with irreversible systems and shall illustrate this generalization with an example.

2. THE THERMODYNAMIC PHASE SPACE AND ITS PROPERTIES

In this section we shall define the manifold of the thermodynamic phases or thermodynamic phase space on which we shall define the conservative systems with ports. Consider a smooth (C^∞) differential manifold denoted by $\mathcal{N}$ and define the associated *thermodynamic phase space* $\mathcal{T}$ as follows (Mrugala, 1980) :

$$\mathcal{T} := \mathbb{R} \times T^*\mathcal{N} \ni (\varepsilon, x, e_x).$$

In accordance with the terminology of network modelling (Breedveld, 1984) and port-Hamiltonian systems (van der Schaft and Maschke, 1995), the manifold $\mathcal{N}$ is called the space of the energy variables, denoted by x. The elements e_x of the cotangent space $T^*_x\mathcal{N}$ are called the coenergy variables conjugated to the energy variable x.

It can be shown that $\mathcal{T}$ is a $(2n+1)$-dimensional manifold endowed with a *canonical contact form* θ. We shall recall the following proposition, characterizing a contact form (Libermann and Marle, 1987)

Proposition 1. A 1-form θ on a $(2n+1)$-dimensional manifold $\mathcal{M}$ is a contact form if and only if $\theta \wedge (\mathrm{d}\theta)^n$ is a volume form on $\mathcal{M}$. Then, $(\mathcal{M}, \theta)$ is called contact manifold.

Furthermore the canonical contact form θ is written in the canonical coordinates $(\varepsilon, x^i, e^i_x)$ as follows (Darboux's theorem) :

$$\theta = \mathrm{d}\varepsilon - \sum_{i=1}^{n} e^i_x \mathrm{d}x^i, \tag{1}$$

where d denotes the exterior derivative. The terminology *thermodynamic phase space* is borrowed from the formulation of thermodynamical systems. Following the pioneering work of Gibbs' (Gibbs, 1928) and Carathéodory (Carathéodory, 1909), there is now a well-established differential formulation of thermodynamic systems which uses the contact manifold $\mathcal{T}$ (Herman, 1973) (Mrugala, 1980). In the context of Thermodynamics, the energy variables $x \in \mathcal{N}$ may be identified with the extensive variables on which the conservation laws are written. The coenergy variables $e_x \in T^*\mathcal{N}$ may be identified with the intensive variables conjugated to the extensive variables x. And the variable $\varepsilon \in \mathbb{R}$ represents the internal energy.

Example 2. In the case of a simple thermodynamical system, (a single constituent, single phase system) the extensive variables are S the entropy, V the volume or N the number of moles of the system. The conjugated intensive variables are the temperature T, the pressure P (actually its opposite $-P$) and the chemical potential μ. Hence the *thermodynamic phase space* $\mathcal{T}$ is :

$$\mathcal{T} := \mathbb{R} \times \mathbb{R}^3 \times \mathbb{R}^3 \ni (U, S, V, N, T, (-P), \mu),$$

and the contact form is

$$\theta = \mathrm{d}U - T\mathrm{d}S + P\mathrm{d}V - \mu\mathrm{d}N, \tag{2}$$

where U is the internal energy. One recognizes that any submanifold of the thermodynamic phase space $\mathcal{T}$ satisfying $\theta = 0$, satisfies Gibbs' equations that is characterize the set of equilibrium states (i.e. the thermodynamic properties of the simple system). This illustrates that the contact structure is the fundamental structure of the thermodynamic phase space underlying Gibbs' equations.

It may be shown in general (Herman, 1973) that the thermodynamic properties (also called "equilibrium states") of any thermodynamic system are defined using the canonical contact form on the thermodynamic phase space. The thermodynamic properties are defined by a particular class submanifold of the thermodynamic phase space, called Legendre submanifolds and defined as follows (Libermann and Marle, 1987).

Definition 3. A *Legendre submanifold* of a $(2n+1)$-dimensional contact manifold $(\mathcal{M}, \theta)$ is an n-dimensional manifold of $\mathcal{M}$ that is an integral manifold of θ.

A Legendre submanifold may also be defined locally by using some canonical coordinates as follows.

Theorem 4. (Arnold, 1989) For a given set of canonical coordinates and any partition $I \cup J$ of the set of indices $\{1, ..., n\}$ and for a differentiable function $F(x^I, {e_x}^J)$ on a neighborhood $\mathcal{V}$ of $\mathcal{M}$, the following equations determine locally a Legendre submanifold $\mathcal{L}$ of $(\mathcal{M}, \theta)$

$$ {e_x}^I = \frac{\partial F}{\partial x^I} \quad x^J = -\frac{\partial F}{\partial {e_x}^J} \quad \varepsilon = F - {e_x}^J \frac{\partial F}{\partial {e_x}^J}. \tag{3} $$

Conversely, every Legendre submanifold of $\mathcal{M}^{2n+1}$ is defined in a neighborhood of every point by these equations for at least one of the 2^n choices of the subset I.

Now consider the particular case when the generating function F is a differentiable function on $\mathcal{N}$, that is $I = \{1, .., n\}$ and $J = \emptyset$. The Legendre submanifold is the set :

$$ \mathcal{L}_F := \left\{ f, x^1, \ldots, x^n, e_x^1 = \frac{\partial F}{\partial x^1}, \ldots, e_x^n = \frac{\partial F}{\partial x^n} \right\}. $$

If the function F is the (internal) energy of the system, then this expression amount to define the 1-form $\mathrm{d}_x F$. In general however the definition of the Legendre manifold, according to the definition 3 is coordinate-free which is of great practical importance in Thermodynamics. Indeed the experimental and numerical data of the thermodynamic properties are mostly *not* expressed in the extensive variables x but in terms of the intensive variables e_x. That means that the energy function F or its differential $\mathrm{d}_x F$ are not explicitly given, but the Legendre submanifold is given in some other coordinates.

3. LIFTING PORT HAMILTONIAN SYSTEMS ON THE THERMODYNAMIC PHASE SPACE

In this section, we shall lift a Port Hamiltonian System (PHS) defined on a pseudo-Poisson manifold to a particular vector *contact field* on the thermodynamic phase space $\mathcal{T}$.

Definition 5. (Libermann and Marle, 1987) A vector field X on the contact manifold $(\mathcal{T}, \theta)$ is called a *contact vector field* if and only if there exits a differentiable function ρ such that :

$$ L_X \theta = \rho \theta, \tag{4} $$

where L_X denote the Lie derivative with respect to the vector field X.

The definition that we shall mainly use, is the local definition of a contact field in some canonical coordinates as follows (Arnold, 1989).

Definition 6. For every function $\hat{f}$ defined on a contact manifold $(\mathcal{M}, \theta)$, one can associate a particular vector field $\widehat{X}_{\hat{f}}$ called *contact field* and defined in local coordinates as follows :

$$ \begin{pmatrix} \dot{\varepsilon} \\ x \\ e_x \end{pmatrix} = \begin{pmatrix} 0 & 0 & -e_x^T \\ 0 & 0 & -Id \\ e_x & +Id & 0 \end{pmatrix} \begin{pmatrix} \partial_\varepsilon \hat{f} \\ \partial_x \hat{f} \\ \partial_{e_x} \hat{f} \end{pmatrix} + \begin{pmatrix} \hat{f} \\ 0 \\ 0 \end{pmatrix} \tag{5} $$

where $\partial_x K$ denotes the partial derivative of the function K with respect to x. One says that $\widehat{X}_{\hat{f}}$ is generated by $\hat{f}$. Furthermore, the equation (4) is satisfied for $\rho = \partial_\varepsilon \hat{f}$.

Secondly, before performing the lift on the thermodynamic phase space, let us recall the definition of a port Hamiltonian system (van der Schaft and Maschke, 1995) defined on a pseudo-Poisson manifold (Marle, 2000).

Definition 7. Let $\mathcal{N}$ be a differential manifold endowed with a pseudo-Poisson bracket denoted by $\{.,.\}$. A *port Hamiltonian system* is defined by a Hamiltonian function $H_0(x) \in C^\infty(\mathcal{N})$, an input vector $u(t) = (u_1, \ldots, u_m)^T$ function of t, m input vector fields $g_1, \ldots, g_m$ on $\mathcal{N}$, and the equations :

$$ \begin{cases} \dot{x} = \Lambda^\#(\mathrm{d}_x H_0(x)) + \sum_{i=1}^m u_i(t)\, g_i(x) \\ {y_p}^j = L_{g_j}.H_0(x) \end{cases} \tag{6} $$

where $y_p = ({y_p}^1, \ldots, {y_p}^m)^T$ is called the *port output variable* (or *port conjugated variable*), L_X denote the Lie derivative with respect to the vector field X and Λ is the pseudo-Poisson tensor associated with the generalized Poisson bracket $\{.,.\}$.

In the sequel, we denote by $X_{H_0} = \Lambda^\#(\mathrm{d}_x H_0(x))$ the Hamiltonian vector field on $\mathcal{N}$ generated by H_0 (the drift vector field of a port Hamiltonian system).
Thirdly, we shall lift of a Port Hamiltonian System

into the contact vector field on the thermodynamic phase space $\mathcal{T} = \mathbb{R} \times T^*\mathcal{N}$ generated by the following map :

$$\widetilde{H}_{(u_i,u_i^*)}(x, e_x) := \widetilde{H}_0(x, e_x) + \sum_{i=1}^{m} u_i(t)\widetilde{H}_i(x, e_x) + \sum_{i=1}^{m} u_i^*(t)\widetilde{H}_i^*(x), \quad (7)$$

with $\widetilde{H}_0(x, e_x) = \langle e_x, X_{H_0}\rangle$, $\widetilde{H}_i(x, e_x) = \langle e_x, g_i(x)\rangle$ and $\widetilde{H}_i^*(x) = L_{g_i}.H_0(x)$, and where u_i are the input time functions and u_i^* are additional inputs time functions that we call the adjoint variational inputs.

Remark 8. Let us notice immediately that the generating function $\widetilde{H}$ has the dimension of **power** and not of energy like in the Hamiltonian function of port Hamiltonian systems corresponding to physical systems.

The function (7) generates a contact field on the thermodynamic phase space, denoted by $\widehat{X}_{\widetilde{H}}$ and expressed as follows in the canonical coordinates of $\mathcal{T}$:

$$\begin{pmatrix} \dot{\varepsilon} \\ x \\ e_x \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{m} u_i^* \, L_{g_i}.H_0 \\ X_{\widetilde{H}} \end{pmatrix} \quad (8)$$

where $X_{\widetilde{H}}$ denotes the Hamiltonian vector field generated by $\widetilde{H}$ on the *cotangent space* $T^*\mathcal{N}$ with respect to the canonical *symplectic* Poisson bracket $\{\cdot,\cdot\}_s$ (of a cotangent bundle (Libermann and Marle, 1987)). The dynamics corresponding to the vector field $X_{\widetilde{H}}$ may be decomposed in the following way :

$$\begin{aligned} \dot{x} &= X_{H_0}(x) + \sum_{i=1}^{m} u_i \, g_i(x) \\ \dot{e_x} &= -\left\langle e_x, \frac{\partial}{\partial x} X_H \right\rangle (x) - \sum_{i=1}^{m} u_i^* \frac{\partial}{\partial x} \left(L_{g_i}.H_0\right)(x) \end{aligned}$$

It may be seen that the dynamics of the energy variables x is precisely the port Hamiltonian system on $\mathcal{N}$, hence the lifted system indeed projects on it. The dynamics of the co-energy variables e_x is precisely state equation of the adjoint variational systems (Fujimoto *et al.*, 2002; van der Schaft and Crouch, 1987) of the port Hamiltonian system (6).

However the contact vector field $\widehat{X}_{\widetilde{H}}$ does not, in general, leave invariant the Legendre submanifold generated by the Hamiltonian function H_0 and denoted by $\mathcal{L}_{H_0}$. This means that the thermodynamic properties of the system (corresponding to the first principle of Thermodynamics, the energy properties) are not conserved by this contact vector field. Therefore we shall now characterize the conditions under which the Legendre submanifold $\mathcal{L}_{H_0}$ is invariant. And we shall use the following theorem proposed in (Mrugala *et al.*, 1991) :

Theorem 9. Let $\mathcal{L}$ be a Legendre submanifold of a contact space $(\mathcal{M}, \theta)$. Then a contact vector field $\widehat{X}_f$ is tangent to $\mathcal{L}$ if and only if f vanishes on $\mathcal{L}$.

Therefore let us compute the restriction of the generating function $\widetilde{H}_{(u_i,u_i^*)}(x, e_x)$ defined in (7) to the Legendre submanifold $\mathcal{L}_{H_0}$:

$$\widetilde{H}_{|\mathcal{L}_{H_0}} = \sum_{i=1}^{m} (u_i + u_i^*) L_{g_i(x)}.H_0(x), \forall \; u_i^* \in \mathcal{U}_i^*. \quad (9)$$

This leads to the following sufficient condition in order to leave the thermodynamic properties invariant.

Proposition 10. A sufficient condition for the lifted functional $\widetilde{H}$ defined in (7) to generate a contact field $\widehat{X}_{\widetilde{H}}$ preserving the Legendre submanifold associated with the energy $\mathcal{L}_{H_0}$ is that $u_i^* = -u_i$ for all i in $\{1, \ldots, m\}$.

It is interesting to relate the invariance condition given in (9) with the power balance equation of the port Hamiltonian system on the base manifold $\mathcal{N}$. Indeed assume that proposition 10 is satisfied, then the condition of invariance amount to the power balance equation of the PHS on $\mathcal{N}$:

$$\widetilde{H}_{|\mathcal{L}_{H_0}}(\tilde{x}) = \overbrace{\langle \mathrm{d}_x H_0(x), \Lambda_J^{\#}(x) \mathrm{d}_x H_0(x)\rangle + \sum_{i=1}^{m} u_i \underbrace{\langle \mathrm{d}_x H_0(x), g_i(x)\rangle}_{L_{g_i(x)}.H_0(x)}}^{\mathrm{d}H_0/\mathrm{d}t} - \sum_{i=1}^{m} u_i L_{g_i(x)}.H_0(x). \quad (10)$$

Hence $\widetilde{H}_{|\mathcal{L}_{H_0}} \equiv 0$ is equivalent to the power balance :

$$\frac{\mathrm{d}H_0}{\mathrm{d}t} - \sum_{i=1}^{m} u_i \underbrace{L_{g_i}.H_0}_{y_p{}^i} = 0. \quad (11)$$

In conclusion, we shall define the lift of the port Hamiltonian systems of the definition 7 on the thermodynamic phase space $\mathcal{T}$ as the contact vector field generated by the function (7) which satisfies the conditions of the proposition 10.

Definition 11. The lift of the port Hamiltonian system of definition 7 on the thermodynamic phase space $\mathcal{T}$ is the contact vector field generated by the function:

$$\widetilde{H} = \Lambda_J(e_x, \mathrm{d}_x H_0) + \sum_{i=1}^{m} u_i \langle e_x - \mathrm{d}_x H_0, g_i \rangle. \quad (12)$$

4. CONSERVATIVE SYSTEMS WITH PORTS WITH RESPECT TO A CONTACT STRUCTURE

In this section we shall propose the definition of conservative systems with ports that extend port Hamiltonian systems in the sense that they are generated by a more general class of function than the functions of the lifted port Hamiltonian systems given in (12).

Definition 12. A conservative system with ports is defined by the thermodynamic phase space $\mathcal{T} = \mathbb{R} \times T^*\mathcal{N} \ni (\varepsilon, x, e_x) = \widehat{x}$ associated with some differentiable manifold $\mathcal{N} \ni x$, a Legendre submanifold $\mathcal{L}$ (associated with the canonical contact form θ on $\mathcal{T}$), a space of external variables $W \ni f_p$ and its dual $W^* \ni e_p$ and the contact vector field $\widehat{X}_{\widehat{H}}$ on $\mathcal{T}$ generated by the function $\widehat{H} \in \mathcal{C}^\infty(\mathcal{T} \times W \times W^*)$ such that :

$$\widehat{H}(\varepsilon, x, e_x, e_p, f_p) := \langle e_x, f_x \rangle + \langle e_p, f_p \rangle + \varphi(\varepsilon) \quad (13)$$

where f_x is a tangent vector of $\mathcal{N}$ at x depending on (x, e_x, e_p, f_p) and $\varphi \in \mathcal{C}^\infty(\mathbb{R})$, satisfying the invariance condition

$$\widehat{H}_{|\mathcal{L}} \equiv 0 \quad (14)$$

and the differential equation :

$$\dot{\widehat{x}} = \widehat{X}_{\widehat{H}} \quad (15)$$

Remark 13. Notice that, for a certain pseudo-Poisson tensor $\Lambda_J^\#$ on $\mathcal{N}$, by choosing $f_x = \Lambda_J^\#(\mathrm{d}H_0) + g_i$, $f_p = u \in \mathbb{R}^m$ and $e_p = -\langle \mathrm{d}_x H_0, g_i \rangle$, one recovers immediately the dynamics (8) when φ is identically zero.

We shall now illustrate this definition on the very simple system of two fluids in thermal interaction that have been already presented in the port Hamiltonian frame in (Maschke, 1998). Here, we want to illustrate how the definition of conservative system with ports on a contact structure allows to formalize the thermodynamic properties and the generation of the heat and entropy fluxes in an independent way.

Example 14. Consider a closed system Σ constituted by two media in contact, only exchanging thermal energy with no volume variation. Σ is characterized by its internal energy U, and the pair of conjugated variables (S_i, T_i) (entropy, temperature) of each medium i. Let $\mathcal{N} = \mathbb{R}^2$ and consider the contact space $(\mathbb{R} \times T^*\mathcal{N}, \theta)$ with the contact form $\theta = \mathrm{d}\varepsilon - e_x^i \mathrm{d}x^i = \mathrm{d}U - T_i \mathrm{d}S_i$.

A particular choice of the energy function U will define the thermodynamic properties of the system. On the other hand the heat and entropy fluxes in the two media will be defined independently by, for instance, Fourier's conduction law which leads to the following definition of the vector field:

$$f_x = R(x, e_x) \Lambda_s^\#(\mathrm{d}U), \quad (16)$$

where $\Lambda_s^\#(\mathrm{d}U)$ denotes the Hamiltonian vector field, generated by the internal energy with respect to the intrinsic Poisson tensor on $\mathcal{N}$ (derived from the canonical symplectic structure on a 2-dimensional manifold) and with

$$R(x, e_x) = \lambda(e_x)(1/e_x^1 - 1/e_x^2), \quad (17)$$

$e_x^i \neq 0$, λ being Fourier's heat conduction coefficient. Then consider the function $\widehat{H} = \langle e_x, f_x \rangle$ and its contact field :

$$\begin{cases} \dot{\varepsilon} &= -e_x^T \langle e_x, \dfrac{\partial f_x}{\partial e_x} \rangle \\ \dot{x} &= f_x + \langle e_x, \dfrac{\partial f_x}{\partial e_x} \rangle \\ \dot{e_x} &= -\langle e_x, \dfrac{\partial f_x}{\partial x} \rangle, \end{cases} \quad (18)$$

one obtains a contact field describing our system on the whole contact space (excepted the zero section for e_x). $\widehat{H}$ fulfills the conditions given in the definition 12.
Now consider the dynamics of the system *restricted on the Legendre submanifold* of energy $\mathcal{L}_U$. Then

$$\langle e_x, \frac{\partial f_x}{\partial e_x} \rangle_{|\mathcal{L}_U} = \langle \mathrm{d}U, \frac{\partial R}{\partial e_x} \Lambda_s^\#(\mathrm{d}U) \rangle = 0 \quad (19)$$

and the dynamics of (18) becomes $\dot{\varepsilon} = 0$ that we expected (conservation of energy), $\dot{e_x} = -\langle \mathrm{d}U, R\frac{\partial}{\partial x} \Lambda_s^\#(\mathrm{d}U)$ and $\dot{x} = f_{x|\mathcal{L}_U}$. The dynamics of the extensive variables then becomes :

$$\begin{cases} \dfrac{\mathrm{d}S_1}{\mathrm{d}t} &= \dfrac{-1}{T_1} \lambda(T_1 - T_2) \\ \dfrac{\mathrm{d}S_2}{\mathrm{d}t} &= \dfrac{1}{T_2} \lambda(T_1 - T_2). \end{cases} \quad (20)$$

It is worth noting that the equation (17) is indeed defining a *nonlinear* relation, between the vector field f_x and the one-form e_x, which is zero on the Legendre submanifold (i.e. satisfying the power continuity) but does not define a Dirac structure.

5. CONCLUSION

In this paper we have proposed a generalization of port Hamiltonian systems defined on pseudo-Poisson manifolds to some contact fields with inputs defined on a contact manifold. This contact manifold corresponds to the thermodynamic phase space associated with a set of extensive variables of a physical system.

Therefore we have firstly lifted the port Hamiltonian system on the contact manifold associated with its state space. This has lead to a contact vector field with inputs generated by a function associated to the Hamiltonian function and defined with respect to the pseudo-Poisson bracket. For physical systems this function has the dimension of power. We have then shown that this contact fields leaves invariant the Legendre submanifold associated with the Hamiltonian function of the port Hamiltonian systems if the power balance equation is satisfied.

Secondly we have proposed the definition of a class contact fields with inputs that generalize the lifted port Hamiltonian systems. Its generating function is defined by the canonical duality product on the thermodynamical phase space. It has also the dimension of power, however it should only meet the requirement to leave some Legendre submanifold invariant. Finally the definition is illustrated on a very simple example of heat conduction.

This definition of conservative systems with ports may also be extended to constrained systems and leads to a very similar property of composition as for port Hamiltonian systems on Dirac structures (Maschke and van der Schaft, 1997) what will be the object of future work. It is also of interest to investigate in which respect this opens the way of larger classes of stabilizing feedback laws.

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