LOW EFFORT CONTROL FOR CHAOTIC SYSTEMS VIA A FUZZY MODEL-BASED APPROACH*

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Abstract: In this paper,we propose a low effort control scheme for chaotic systems by using fuzzy model-based design method. First, we represent nonlinear systems into T-S fuzzy models in a working region covering the point to be regulated. The stability condition of the overall system is formulated into (LMIs). To guarantee the stability, the region of attraction is also investigated. According to topologically transitive property for chaotic systems, the feedback control force is activated only when the trajectory passes through the neighboring region of the regulated point. Compared to purely fuzzy model-based controller, the control force for the fuzzy chaos hybrid controller is extremely low.

Keywords: Low effort control, T-S fuzzy model, Chaotic systems.

1. INTRODUCTION

emptyempty Recently, the Takagi-Sugeno (T-S) fuzzy approach has been extensively used to model nonlinear systems. The basic idea is to decompose a nonlinear system into a set of linear subsystems with associated nonlinear weighting functions. Two methods are often employed to construct T-S fuzzy models, namely approximated modeling using a local linearization technique (Bergsten, *et al.*, 2002) and exact modeling (Lian, *et al.*, 2001) using nonlinear combination technique. Since chaotic systems are sensitive to parameter variation, the exact modeling is adopted in this paper. Once the linear models are obtained, the local linear controller for each subsystem can be designed and inferred to an overall controller. Many algorithms of T-S fuzzy control have been developed recently (Lian, *et al.*, 2001). There are also wide applications in the control of complex systems by using T-S fuzzy model. In addition, the T-S fuzzy model-based controller analysis and synthesis rely on a linear matrix inequality (LMI) (Boyd, *et al.*, 1994) approach. In tracking control, using the T-S fuzzy model based approach, has been limited to model following, where the reference input is considered as disturbance and attenuated using a robust criterion. However, in the control process, the control inputs sometimes are large. This, of course, is not desired.

Chaos has been found in many different physical systems (Ott, *et al.*, 1990). Analyzing and predicting the behavior of a chaotic system is beneficial, but to maximize the benefit, one has to be able to control it (Joo, *et al.*, 1999). Nowadays, most conventional control methods and many specific techniques can be used for chaos control. On the other hand, a nonlinear dynamical system can exhibit chaotic dynamics by using some of the latent characteristics of chaotic attractors, which can be achieved by appropriate usages of open-loop control. When employing chaos in developing control algorithms, there are many advantages such as low energy consumption, robustness of the controller performance and simplicity in the original system itself (Udawatta, *et al.*, 2002).

In this paper, we give guidelines to exact model a general nonlinear system into a nonlinear combination of linear dynamical subsystems. According to these linear dynamical subsystems, we design a hybrid type controller which includes an open-loop

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control and closed-loop control. The open-loop control is used to make the controlled system exhibit chaotic phenomenon, whereas the closed-loop control formed by T-S fuzzy method is to drive the neighboring system states to approach desired state. An output tracking control based on T-S fuzzy model is also proposed. Then, the trajectory of chaotic systems can be steered to track specific orbits.

2. STABILIZATION USING FUZZY CHAOS HYBRID CONTROLLER

2.1 Fuzzy Chaos Hybrid Controller

In this section, we consider a stabilization problem for a chaotic system. According to the topologically transitive property, a chaotic system has dense orbits in its working space. In light of this, we will not activate the control force until that the trajectory enters in a neighbor region of equilibrium point to be stabilized. Therefore, the control force can be kept small. The concept is illustrated in Fig. 1. The system trajectory, shown in the figure, starts from initial state x_0 and is to be steered to a desired state x_d. The region Ω denotes a neighborhood including x_d. For a system with no chaotic feature, we will apply a small signal as open-loop control input to induce a chaotic attractor.

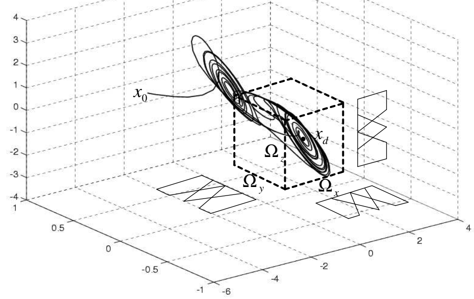

Fig. 1. The region of interest is chosen

Accordingly, the hybrid control law is temporarily assumed to take the following form:

$$u = \begin{cases} u_o, & \text{if } x \notin \Omega \\ u_c, & \text{if } x \in \Omega \end{cases} \tag{1}$$

where u_o denotes the open-loop control and u_c denotes the closed-loop control. If the original nonlinear system did have chaotic feature, it is natural to let the open-loop control be zero, i.e., $u_o = 0$. The closed-loop control u_c is to drive the system states to approach the desired state x_d. To this end, an LMI-based controller will be derived. First, we consider a nonlinear system where its fuzzy model over the universe of discourse Ω is with the following rules:

$$\begin{aligned} &\textbf{Rule } i: && \text{IF } z_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{and } z_g(t) \text{ is } F_{gi} \\ &\text{Then} && \dot{x}(t) = A_i x(t) + B_i u(t) \\ & && y(t) = C_i x(t), \; i = 1, 2, \cdots, r \end{aligned}$$

which yields the following inferred output:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(z(t)) \left(A_i x(t) + B_i u(t) \right) \\ y(t) &= \sum_{i=1}^{r} \mu_i(z(t)) C_i x(t), \end{aligned} \tag{2}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{bmatrix}^T \in R^n$ is the state vector; A_i, B_i, C_i are system matrices of appropriate dimensions; $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_g \end{bmatrix}^T$ are the premise variables of the T-S fuzzy model which would consist of the states of the system; $\mu_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{r} \omega_i(z(t))$ with $\omega_i(z(t)) = \prod_{k=1}^{g} F_{ki}(z(t))$ where F_{ki} for $k = 1, 2, \ldots, g$ are fuzzy sets. Note that $\sum_{i=1}^{r} \mu_i(z(t)) = 1$ for all t, where $\mu_i(z) \geq 0$ are normalized weights.

According to parallel distribution compensation (PDC), the controller consists of the following rules:

$$\begin{aligned} &\textbf{Rule } i: && \text{IF } z_1(t) \text{ is } F_{1i} \text{ and } \ldots \text{ and } z_g(t) \text{ is } F_{gi} \\ &\text{THEN} && u(t) = -K_i x(t), i = 1, 2, \cdots, r \end{aligned} \tag{3}$$

where K_i is a feedback gain. The inferred output of (3) is

$$u(t) = u_c(t) = -\sum_{i=1}^{r} \mu_i(z(t)) K_i x(t) \tag{4}$$

Applying PDC on (2), we obtain the overall closed-loop system

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(z) \mu_j(z) \left\{ A_i - B_i K_j \right\} x(t) \tag{5}$$

Notice that a dummy index j has been introduced to represent the coupling indices between the local linear subsystem and the PDC. As a similar result of (Kim, *et al.*, 2000), the control gains determined by the following theorem has a stronger decay rate which is important in practical applications.

Theorem 1 *The augmented system (5) of closed-loop controller is exponentially stable if there exists a common positive definite matrix $P = P^T > 0$, a diagonal positive definite matrix D* and *X_{ij} such that*

$$\Lambda_{ii}^T P + \Lambda_{ii} P + X_{ii} + DPD < 0, i = 1, ..., r \tag{6}$$

$$\Lambda_{ij}^T P + \Lambda G_{ij} + X_{ij} \leq 0, i < j \leq r \tag{7}$$

$$\begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1r} \\ X_{12} & X_{22} & \cdots & X_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1r} & X_{2r} & \cdots & X_{rr} \end{bmatrix} \equiv \tilde{X} > 0 \tag{8}$$

where $G_{ij} \equiv A_i - B_i K_j$, $\Lambda_{ii} \equiv G_{ii}$, $i = 1, \cdots, r$, $\Lambda_{ij} \equiv (G_{ij} + G_{ji})/2$.

Proof. Choose the Lyapunov function candidate as $V = x^T(t) P x(t)$. Taking the derivative of V with respect to t, we have

$$
\begin{aligned}
\dot{V} =& \dot{x}^T\ Px + x^T P\ \dot{x} \\
=& \sum_{i=1}^{r} \mu_i^2(z) \left\{ x^T (G_{ii}^T P + P G_{ii}) x \right\} \\
&+ 2 \sum_{i<j\leq r} \mu_i(z)\mu_j(z) \left\{ x^T \left(\left(\frac{G_{ij}+G_{ji}}{2} \right)^T P \right.\right. \\
&\left.\left. +\ P \left(\frac{G_{ij}+G_{ji}}{2} \right) \right) x \right\}
\end{aligned}
$$

According to (6) and (7), it follows that

$$
\begin{aligned}
\dot{V} &\leq -\mu_i^2 x^T (X_{ij} + DPD) x - 2 \sum_{i<j\leq r} \mu_i \mu_j x^T X_{ij} x \\
&= x^T H^T (-\tilde{X}) H x - x^T H^T DPDHx
\end{aligned}
$$

where $H = \begin{bmatrix} \mu_1 I & \mu_2 I & \cdots & \mu_r I \end{bmatrix}^T$

Therefore, once the inequality (8) is satisfied, we have $\dot{V} < -x^T H^T DPDHx$ which further results in $V(x(t)) \leq V(0) e^{-\frac{\lambda_{\min}(DPD)}{r\lambda_{\max}(P)} t}$ where $\lambda_{\min}(M)$, $\lambda_{\max}(M)$ denote the minimal and maximal eigenvalue of matrix M, respectively. Therefore,

$$
\|x\|^2 \leq \frac{V(0)}{\lambda_{\min}(P)} e^{-\frac{\lambda_{\min}(PDP)}{r\lambda_{\max}(P)} t}
$$

is concluded. □

Remark: To obtain control gains K_i by using efficient toolbox, we may transform (6) $\sim$ (8) into LMIs. Let $M_i = K_i X$ *and* $X = P^{-1} > 0$, we rewrite (8) the following LMI:

$$
\begin{aligned}
H_{ii} &< 0,\ i = 1, \cdots, r \\
Z_{ij} + 2Y_{ij} &\leq 0,\ i < j \leq r
\end{aligned}
$$

$$
\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1r} \\ Y_{12} & Y_{22} & \cdots & Y_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1r} & Y_{2r} & \cdots & Y_{rr} \end{bmatrix} \equiv \tilde{Y} > 0
$$

where $Z_{ij} = XA_i^T + A_i X + XA_j^T + A_j X - M_j^T B_i^T - B_i M_j - M_i^T B_j^T - B_j M_i$ and $KK_i = A_i^T X + A_i X - B_i M_i - M_i^T B_i^T + X_{ii}$

$$
H_{ii} = \begin{bmatrix} KK_i & XD^T \\ DX & -X \end{bmatrix}.
$$

In the derivation, the region Ω denotes the domain where the T-S fuzzy model (2) is well defined. However, it is not the region of attraction, an interesting region where the trajectory starts form the region will lie in it forever. Notice that even though a trajectory crossing and entering Ω will move from a Lyapunov surface $V(x) = c_1$ to an inner Lyapunov surface $V(x) = c_2$, with $c_2 < c_1$, there is no guarantee that the trajectory will remain forever in Ω. Specifically, when a trajectory crosses Ω at the corner region, it will often leave Ω, for instance, the Path I in Fig. 2. When the trajectory evolves in Ω, we see that Lyapunov function value decreases until it reaches the boundary (point B). However, it goes away from the region. Hence, it is apparently that the region (Ω) does not stand for the convex attractive region.

2.2 Analysis of Attraction Region

Lyapunov functions can be used to estimate the domain of attraction. Let $\sigma(\Omega)$ denote the boundary of Ω. The simplest estimate is provided by the set

$$
\Omega_c = \{x \in R^n \mid V(x) \leq c\} \tag{9}
$$

where $c = \min\{V(x) \mid x \in \sigma(\Omega)\}$. The attraction region Ω_c is illustrated in Fig. 2. As shown by Path II, the trajectory into Ω_c at C is with function value less than C and decreasing. Hence the state trajectory can not leave this region. If we want to avoid the bounce of the trajectory, the hybrid control law is modified to the following form:

$$
u = \begin{cases} u_o, & \text{if } x \notin \Omega_c \\ u_c, & \text{if } x \in \Omega_c \end{cases}
$$

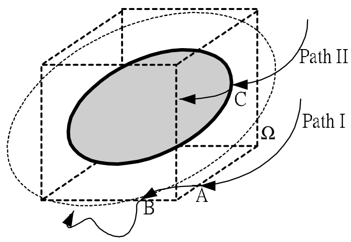

Fig. 2. The Lyapunov functions are used to estimate the domain of attraction

2.3 Predefined Attraction Region by Minimizing Ellipsoid Volume

In the above subsection, the level set of Lyapunov functions is used to estimate the domain of attraction. The attraction region is determined until we get P. Here, we want the attraction region containing a predefined region described by polytope $Co\{v_1, ..., v_r\}$, where Co denotes the convex hull and $v_i \in R^n$ are designed vectors. Notice that the polytope centers at the equilibrium point x_d. The convex minimization problem is simply by solving

$$
\begin{aligned}
&\text{minimize}\ \ \log\det P^{-1} \\
&\text{subject to } P > 0,\ v_i^T P v_i \leq 1,\ i = 1, ..., L
\end{aligned}
$$

Let R_1 denote the ellipsoid centered at the equilibrium point x_d determined by P, i.e., $R_1 = \{x | (x - x_d)^T P (x - x_d) \leq 1\}$. The constraints imply $(v_i + x_d) \in R_1$. An alternative expression to minimize ellipsoid volume is by solving

$$
\begin{aligned}
&\text{minimize}\ \ c^2 \\
&\text{subject to } (6), (7), (8)\ P > 0,\ v_i^T P v_i \leq c^2, \\
&i = 1, ..., L
\end{aligned} \tag{10}
$$

Then the minimum volume ellipsoid containing the polytope $R_c = \{x|(x-x_d)^T P(x-x_d) \le c^2\}$. The Schur complement procedure implies that the conditions in (10) yields

$$\begin{bmatrix} c^2 & v_i^T \\ v_i & X \end{bmatrix} > 0,\ i=1,...,L \tag{11}$$

2.4 Constraints on Control Force

In this subsection, the attraction region R_c is further modified to guarantee $\|u(t)\| \le \beta$, for a given β.

Theorem 2 *Consider the augmented system (5) with closed-loop controller (4). The input constraint $\|u_c(t)\| \le \beta$ is enforced in the attraction region R_c if the LMIs (10) and attraction region R_c*

$$\begin{bmatrix} \beta^2 I & cM_i \\ cM_i^T & X \end{bmatrix} \ge 0 \tag{12}$$

are feasible.

Proof. According to (4) and considering the state lies in R_c, we can rewrite

$$\begin{aligned} &\max_{x(t)\in R_c} \Big\| -\sum_{i=1}^{r} \mu_i(z) K_i x(t) \Big\| \\ = &\max_{\|X^{-\frac{1}{2}}x(t)\|\le c} \Big\| -\sum_{i=1}^{r} \mu_i(z) K_i X^{\frac{1}{2}} X^{-\frac{1}{2}} x(t) \Big\| \\ \le &\Big\| -\sum_{i=1}^{r} \mu_i(z) K_i c X^{\frac{1}{2}} \Big\| \end{aligned}$$

for all $t \ge 0$. Then, the inequality $\|u(t)\| < \beta$ is satisfied under the condition of

$$\Big\| -\sum_{i=1}^{r} \mu_i(z) c K_i X^{\frac{1}{2}} \Big\| < \beta$$

which is equivalent to

$$\Big[-\sum_{i=1}^{r} \mu_i(z) c K_i X \Big] X^{-1} \Big[-\sum_{i=1}^{r} \mu_i(z) c (K_i X)^T \Big] < \beta^2 I \tag{13}$$

The inequality (13) is transformed into (12) by the Schur complement. □

3. OUTPUT TRACKING CONTROL

For a chaotic system, it is an interesting issue to track a specific signal such as a periodic signal. For this purpose, we consider the output tracking control problem. The control objective is required to satisfy

$$y(t) - y_d(t) \to 0 \text{ as } t \to \infty$$

where $y_d(t)$ denotes the desired trajectory. In order to convert the output tracking problem into a stabilization problem, we introduce a set of virtual desired variables x_d which are to be tracked by the state variables x. According to $y(t) = h(x)$, it is natural to require

$$y_d(t) = h(x_d). \tag{14}$$

Let $\tilde{x}(t) = x(t) - x_d(t)$ denote the tracking error for the state variables. The time derivative of $\tilde{x}(t)$ yields

$$\begin{aligned} \dot{\tilde{x}}(t) &= \dot{x}(t) - \dot{x}_d(t) \\ &= \sum_{i=1}^{r} \mu_i(z)(A_i x(t) + B_i u(t)) - \dot{x}_d(t) \end{aligned} \tag{15}$$

If the control input is assumed to satisfy the following equation

$$\sum_{i=1}^{r} \mu_i(z) B_i u = \sum_{i=1}^{r} \mu_i(z)(B_i \tau - A_i x_d) + \dot{x}_d \tag{16}$$

where $\tau(t)$ is a new control to be designed, then the tracking error system (15) results in the following form:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} \mu_i(z) A_i \tilde{x}(t) + \sum_{i=1}^{r} \mu_i(z) B_i \tau(t) \tag{17}$$

For the error system (17), we can find that the tracking control design for $\tau(t)$ is similar to solve a stabilization problem. Our control purpose is to steer $\tilde{x}(t)$ to zero, which means that the state $x(t)$ tracks $x_d(t)$. The new fuzzy controller $\tau(t)$ is designed based on PDC and represented as follows:

Rule i : IF z_1 is F_{1i} and ... and z_g is F_{gi}

THEN $\tau(t) = -K_i \tilde{x}(t)$

The inferred output of the PDC controller is with the following form:

$$\tau(t) = -\sum_{i=1}^{r} \mu_i(z(t)) K_i \tilde{x}(t) \tag{18}$$

Substituting (18) into (17), we obtain the closed-loop system

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))(A_i - B_i K_j)\tilde{x}(t) \tag{19}$$

Theorem 3 *The output tracking is achieved if the virtual desired variables x_d and control input u satisfy (14) and (16) whereas the LMIs defined in (6) $\sim$ (8) are feasible.*

Proof. The proof is similar to that of **Theorem 1**. Choose the Lyapunov function candidate as $V = \tilde{x}^T P \tilde{x}$. Then $\dot{V} < 0$ once the inequalities (6) $\sim$ (8) is satisfied. □

3.1 Hybrid Control for Output Tracking

Consequently, the hybrid control law considering low effort force for output tracking is with the following form:

$$u = \begin{cases} u_o, & \text{if } x \notin \Omega_c \\ u_c, & \text{if } x \in \Omega_c \end{cases} \tag{20}$$

The open-loop control input u_o is the same as in stabilization problem. The purpose of u_c is to drive the system states to achieve the desired states when the trajectory entering Ω_c. According to Thm. (3), the closed-loop input is designed as the following form:

$$\sum_{i=1}^{r} \mu_i(z) B_i u_c(t) = \dot{x}_d(t) - \sum_{i=1}^{r} \mu_i(z) \left(A_i x_d(t)\right) - \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z)\mu_j(z) B_i K_j \left(x(t) - x_d(t)\right) \quad (21)$$

3.2 Attractive Region for Output Tracking Controller

The attracting region Ω_c, however, is now more complex. In this subsection, the attractive region Ω_c of tracking problem is estimated by minimizing ellipsoid volume. Let $\tilde{\varepsilon}$ denote the ellipsoid centered at the x_d determined by P, $\tilde{\varepsilon} = \{\tilde{x} | \tilde{x}^T P \tilde{x} \le c^2\}$. The constraints are simply $\tilde{v}_i \in \tilde{\varepsilon}$.

$$\begin{aligned} &\text{minimize } c^2 \\ &\text{subject to } (6), (7), (8)\ P > 0,\ \tilde{v}_i^T P \tilde{v}_i \le c^2, \\ &i = 1, ..., L \end{aligned} \quad (22)$$

According to (22), notice that even though a trajectory crossing and entering $\tilde{\varepsilon}$ will move from one Lyapunov surface $V(\tilde{x}) = c_1$ to an inner Lyapunov surface $V(\tilde{x}) = c_2$, with $c_2 < c_1$. However, T-S fuzzy modeling usually focus on x-domains. So that, we need mapping $\tilde{x}$-domains to x-domains. The concept is illustrated in Fig. 3. In this Figure, $\tilde{\varepsilon}$ is the ellipsoid centered at the x_d. We denoted $R_\varepsilon = \{conv(x) | (x - x_d)^T P (x - x_d) \le c^2\}$. If the bounded interval Ω of T-S fuzzy model include R_ε, then this fuzzy model arrive tracking control.

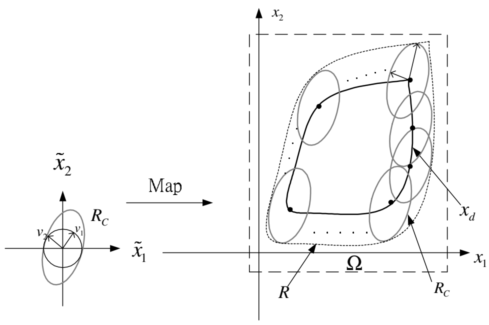

Fig. 3. The domain of attraction for tracking control

4. SIMULATION RESULTS

The dynamic equation of a mass-spring system are given by

$$m\ddot{X}_p + c\dot{X}_p + kX_p + ka^2 X_p^3 = u \quad (23)$$

Let $m = 1,\ c = 0.4,\ k = 1.1,\ a^2 = 0.9$. The state equations are written as follows:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -1.1x_1(t) - x_1^3(t) - 0.4x_2(t) + u(t) \end{aligned}$$

where $u(t)$ is external control input. Using the exact T-S fuzzy modeling method, the system is represented by the fuzzy rules:

$$\textbf{Plant Rule } i: \quad \text{IF } x_1(t) \text{ is } F_i \text{ THEN } \dot{x}(t) = A_i x(t) + Bu(t)$$

where the system matrices are

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1.1 - d & -0.4 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ -1.1 & -0.4 \end{bmatrix}$$

and the common $B = [\ 0 \quad 1\ ]^T$. The corresponding fuzzy sets are $F_1 = \frac{x_1^2}{d}$ and $F_2 = 1 - F_1$ and the initial values of states are set as $x^T(0) = [\ 4 \quad 4\ ]$.

Purely T-S fuzzy model-based controller

According to (6) $\sim$ (8), the $d = 25$ are chosen. The control gains obtained via LMI toolbox of Matlab are given below:

$$K_1 = [\ -15.6714 \quad 0.6714\ ]; K_2 = [\ 0.3286 \quad 0.6714\ ]$$

Based on only T-S fuzzy model-based controller, we choose the simulation time is 20 s and control result is shown in Fig. 4. In this case, we focus on the maximum of control input $u = 113.6258$.

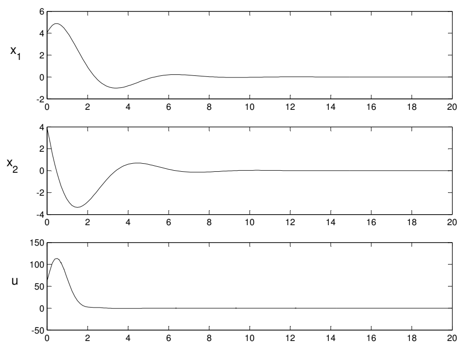

Fig. 4. Purely T-S fuzzy model-based controller

Fuzzy chaos hybrid stabilization controller

Let the open-loop control be $u_o = 1.8\cos(1.8t)$. The region Ω, denotes $-3 < x_1 < 3$, $-3 < x_2 < 3$ and $d = 9$. According to (9), the attraction region $\Omega_c = 0.0145x_1^2 + 0.0116x_1x_2 + 0.0145x_2^2 - 0.092$. The control result is shown in Fig. 5. The maximum of control input is $u = 19.9838$.

Fuzzy chaos hybrid stabilization controller with input constraints

According to Thm. 2, the input constraint $\beta = 100$ and $c = 1$ are chosen. The control gains obtained via LMI toolbox of Matlab are given below:

$$K_1 = [\ -1.0928 \quad 1.1428\ ]; K_2 = [\ 2.6743 \quad 1.2050\ ]$$

The control result is shown in Fig. 6. The maximum of control input is $u = 5.4859$.

Fuzzy chaos output tracking Control

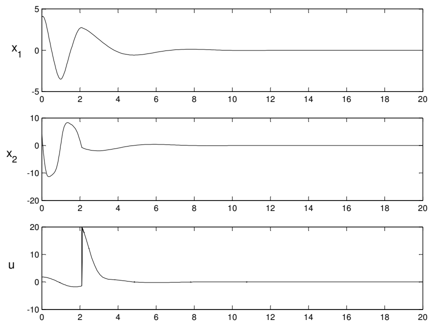

Fig. 5. Stabilization using fuzzy chaos hybrid controller

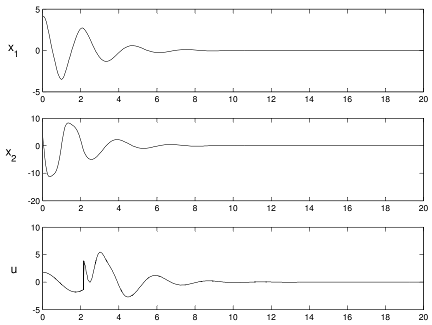

Fig. 6. Stabilization with input constraint

Before the state entering the universe of discourse Ω, the fuzzy chaos hybrid tracking controller is the same as stabilization controller. When the state of system enters Ω, the tracking controller is activated. The desired state x_d is $\begin{bmatrix} 2\sin(t) & 2\cos(t) \end{bmatrix}^T$. According to (21), the tracking control input is

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}(u-\tau)=\begin{bmatrix} \dot{x}_{1d} \\ \dot{x}_{2d} \end{bmatrix}-\begin{bmatrix} 0 & 1 \\ -1.1-x_1^2 & -0.4 \end{bmatrix}\begin{bmatrix} x_{1d} \\ x_{2d} \end{bmatrix}$$

where $\tau = \Sigma_{i=1}^{2} K_i(x - x_d)$. The simulation results are shown in Fig. 7.

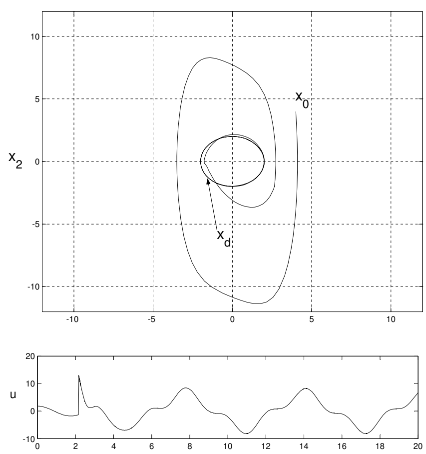

Fig. 7. Output tracking control for fuzzy chaos hybrid controller

5. CONCLUSIONS

In this paper, we first introduce stabilization using fuzzy chaos hybrid controller. The attraction region can be derived simply by using the level set of Lyapunov function. Furthermore, minimizing ellipsoid volume and constraints on control input containing a predefined region is presented by LMI condition. Furthermore, the attraction region for output tracking problem is given. In the simulation, we choose a mass-spring mechanical system to verify the theoretical results. From theoretical and numerical simulations, the fuzzy chaos hybrid controller are shown to have low effort.

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