ISE–OPTIMAL NONMINIMUM–PHASE COMPENSATION FOR NONLINEAR PROCESSES

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Abstract: This work concerns the optimal regulation of single-input-single-output nonminimum-phase nonlinear processes of relative order one. The problem of calculation of an ISE-optimal, statically equivalent, minimum-phase output for nonminimum-phase compensation is formulated using Hamilton-Jacobi theory and the normal form representation of the nonlinear system. A Newton-Kantorovich iteration is developed for the solution of the pertinent Hamilton-Jacobi equations, which involves solving a Zubov equation at each step of the iteration. The method is applied to the problem of controlling a nonisothermal CSTR with Van de Vusse kinetics, which exhibits nonminimum-phase behavior.

Keywords: Process Control, Nonlinear control, Nonlinear systems, Nonminimum phase systems, Optimal Control.

1. INTRODUCTION

Controlling processes with unstable zero dynamics is a significant challenge. For linear systems, the system is typically factored into minimum-phase and nonminimum-phase parts, with the minimum-phase part inverted for controller design. For nonlinear systems, the decomposition into minimum-phase and nonminimum-phase parts is an extremely difficult problem. In the special case of second-order systems, Kravaris and Daoutidis (1990) solved the decomposition problem and calculated ISE-optimal control laws. Ball and van der Schaft (1996) proposed a mathematical formulation for the decomposition problem for higher-order nonlinear systems. Alternative approaches were also developed by Doyle *et al.* (1992; 1996), which include approximate stable/anti-stable factorization of the zero dynamics, an inner-outer based approximation, and a multiple-input approach. All of these aforementioned methods are applicable only to limited classes of nonlinear systems.

Wright and Kravaris (1992) and Kravaris *et al.* (1994) developed a nonminimum-phase compensation structure for nonlinear systems, which is based upon a synthetic output that is statically equivalent to the original output and makes the system minimum-phase. This methodology bypassed the difficulty of decomposition and reduced the problem to the construction of an appropriate synthetic output.

One approach to the problem of construction of synthetic minimum-phase outputs is by formulating it as a zeros assignment problem. In the work of d'Andrea and Praly (1988), a linear synthetic output was constructed for prescribed zeros for the linearization of the system. In Kravaris *et al.* (1998) and Niemiec and Kravaris (2003), the synthetic output was constructed to be statically equivalent to the original process output, in addition to having prescribed zeros in the linearization of the system. All these zeros-assignment approaches are of general applicability, leading to controllers with reasonable performance, but they don't provide an answer to the question of optimal selection of the synthetic output.

The present work will study the problem of construction of an ISE-optimal minimum-phase output for single-input-single-output nonlinear systems of relative order one, using a Hamilton-Jacobi formulation.

2. PRELIMINARIES

Consider a single-input-single-output nonlinear process described by a state-space model of the form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where x denotes the vector of state variables, u denotes the manipulated input, and y denotes the controlled output. In order to simplify the development of the ideas, assume that the relative order of the system (1) is equal to 1, i.e.

$$\frac{\partial h}{\partial x}(x)g(x) \neq 0 .$$

For a system of the form (1) of relative order 1, a simple choice of feedback is the input/output linearizing state feedback law:

$$u = \frac{v - h(x) - \tau \frac{\partial h}{\partial x}(x) f(x)}{\tau \frac{\partial h}{\partial x}(x) g(x)} \tag{2}$$

where τ is a positive adjustable parameter. Under this state feedback, the input / output behaviour of the closed-loop system is linear, with time constant τ :

$$\tau \frac{dy}{dt} + y = v \tag{3}$$

As long as the system (1) is hyperbolically minimum-phase around the steady state of interest, (2) induces local asymptotic stability in closed loop.

When the system to be controlled is nonminimum-phase, the foregoing simple controller design method is not applicable. However, it can potentially be modified to “compensate” for the nonminimum-phase nature of the system. Generally speaking, the term “nonminimum-phase compensation” refers to a control methodology by which the control problem for a nonminimum-phase system reduces to controller design of an auxiliary minimum-phase system. In a state-space design context, this involves using an auxiliary output (also called “synthetic output”)

$$y' = h'(x) \tag{4}$$

such that

(i) The system:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y' &= h'(x) \end{aligned} \tag{5}$$

is locally hyperbolically minimum-phase.

(ii) The outputs $y = h(x)$ and $y' = h'(x)$ are statically equivalent in the sense that they assume the same values when the system (1) is at steady state.

If such an output map can be found, controlling the output y to a constant set point can be accomplished by controlling the auxiliary output y' to the identical set point. In this case, the input/output linearizing state feedback may be based on the minimum-phase auxiliary output:

$$u = \frac{v - h'(x) - \tau \frac{\partial h'}{\partial x}(x) f(x)}{\tau \frac{\partial h'}{\partial x}(x) g(x)} \tag{6}$$

inducing linear input/output behaviour with time constant τ , with respect to the auxiliary output y' .

The present work will study the problem of selection of the synthetic output $y' = h'(x)$ in an optimal fashion, in terms of the ISE criterion.

3. PROBLEM STATEMENT

Consider a nonlinear system of the form (1) of relative order 1. Without loss of generality, the system will be considered in Byrnes–Isidori normal form (Byrnes and Isidori, 1985; Isidori, 1989):

$$\begin{aligned} \dot{\zeta} &= F_0(\zeta, y) \\ \dot{y} &= F_y(\zeta, y) + G(\zeta, y)u \end{aligned} \tag{7}$$

where $u \in \mathbb{R}$ is the manipulated input, $y \in \mathbb{R}$ is the output, $\begin{bmatrix} \zeta \\ y \end{bmatrix} \in \mathbb{R}^n$ is the state vector, $F_0 : \mathbb{R}^{n-1} \times \mathbb{R} \to \mathbb{R}^{n-1}$, $F_y : \mathbb{R}^{n-1} \times \mathbb{R} \to \mathbb{R}$ and $G : \mathbb{R}^{n-1} \times \mathbb{R} \to \mathbb{R}$ are real analytic functions.

Denote by $\upsilon \in \mathbb{R}$ the set point value at which the output y must be regulated and $\zeta_s \in \mathbb{R}^{n-1}$ the corresponding steady state for ζ at which the system must operate. (Hence, ζ_s and υ are related via $F_0(\zeta_s, \upsilon) = 0$)

The zero dynamics of the system at $y = \upsilon$ is

$$\dot{\zeta} = F_0(\zeta, \upsilon) \tag{8}$$

and its local stability characteristics determine the local minimum- or nonminimum–phase behaviour of the system.

The following standing assumptions will be made concerning the system (7):

i) $\frac{\partial F_0}{\partial \zeta}(\zeta_s, \upsilon)$ does not have any eigenvalues on the imaginary axis

ii) $\left(\frac{\partial F_0}{\partial \zeta}(\zeta_s, \upsilon), \frac{\partial F_0}{\partial y}(\zeta_s, \upsilon) \right)$ form a controllable pair

iii) $G(\zeta, y) \neq 0$ for all ζ and y .

With regard to regulation of the output y to the set point value of υ , the <u>I</u>ntegral of the <u>S</u>quare of the <u>E</u>rror (ISE) is a meaningful performance measure:

$$ISE = \frac{1}{2} \int_0^\infty [\upsilon - y(t)]^2 \, dt \tag{9}$$

Thus, the optimal control problem under consideration is the minimization of the performance index (9) under the dynamics (7) and subject to the constraint of closed-loop stability. This is a singular optimal control problem, since, as can be easily verified, its Hamiltonian function is linear in the input u (Bryson and Ho, 1975).

Because of the structure of the Byrnes–Isidori normal form, the solution of the foregoing singular optimal control problem can be accomplished by solving the following regular optimal control problem:

$$\boxed{\begin{array}{l} \min \qquad ISE = \frac{1}{2}\int_0^\infty [\upsilon - y(t)]^2\, dt \\ \text{subject to the dynamics} \\ \qquad \dot{\zeta} = F_0(\zeta, y) \\ \text{and closed-loop stability} \end{array}} \tag{10}$$

The solution to the above optimal control problem, when expressed in the form of state feedback law

$$y = y^*(\zeta;\upsilon) \tag{11}$$

represents exactly the singular surface for the original singular optimal control problem. Note that, by construction, the function y^* will be such that the dynamics $\dot{\zeta} = F_0\left(\zeta, y^*(\zeta;\upsilon)\right)$ is stable and $y^*(\zeta_s;\upsilon) = \upsilon$.

When (11) is solved with respect to υ ,

$$\upsilon = h'(\zeta, y) \tag{12}$$

this defines an auxiliary output map

$$y' = h'(\zeta, y) \tag{13}$$

which has the following properties:

i) $h'(\zeta_s, \upsilon) = \upsilon$, which implies that y' is statically equivalent to y .
ii) the zero dynamics of system (7) with output (13) is $\dot{\zeta} = F_0\left(\zeta, y^*(\zeta;\upsilon)\right)$ which is stable, therefore y' is a minimum-phase output.
iii) Since the ISE–optimal trajectories for $\zeta(t)$ and $y(t)$ will satisfy (11) for every t, this means they will also satisfy $h'\left(\zeta(t), y(t)\right) = \upsilon$ for every t, i.e. will correspond to perfect control of y' to υ .

Consequently, $y' = h'(\zeta, y)$ is the ISE–optimal choice of statically equivalent minimum–phase output in the sense that its perfect control to set point corresponds to ISE–optimality in the original output.

In the linear case, where

$$F_0(\zeta, y) = A_0\zeta + \gamma y \tag{14}$$

where A_0 and γ are $(n-1)\times(n-1)$ and $(n-1)\times 1$ matrices respectively, the solution to the above optimal control problem is standard (Kailath, 1980):

$$y^*(\zeta;\upsilon) = \upsilon - \gamma^{\mathrm{T}} P\left(\zeta + A_0^{-1}\gamma\upsilon\right) \tag{15}$$

where P is the solution of the quadratic matrix equation:

$$A_0^{\mathrm{T}} P + P A_0 - P\gamma\gamma^{\mathrm{T}} P = 0 \tag{16}$$

that makes $\left(A_0 - \gamma\gamma^{\mathrm{T}} P\right)$ Hurwitz.

Consequently, the singular surface for the original singular optimal control problem is:

$$y = \upsilon - \gamma^{\mathrm{T}} P\left(\zeta + A_0^{-1}\gamma\upsilon\right) \tag{17}$$

or equivalently

$$\frac{y + \gamma^{\mathrm{T}} P\zeta}{1 - \gamma^{\mathrm{T}} P A_0^{-1}\gamma} = \upsilon \tag{18}$$

The function

$$h'(\zeta, y) = \frac{y + \gamma^{\mathrm{T}} P\zeta}{1 - \gamma^{\mathrm{T}} P A_0^{-1}\gamma} \tag{19}$$

is the ISE–optimal choice for auxiliary output for nonminimum–phase compensation.

4. HAMILTON–JACOBI FORMULATION OF THE OPTIMAL CONTROL PROBLEM

Consider the optimal control problem (10). The Hamiltonian function associated with this problem is (Bryson and Ho, 1975):

$$H(\zeta, y, \lambda) = \frac{1}{2}(\upsilon - y)^2 + \lambda^{\mathrm{T}} F_0(\zeta, y) \tag{20}$$

where $\lambda \in \mathbb{R}^{n-1}$ is the vector of multipliers.
Denoting

$$\kappa(\zeta, \lambda) = \arg\min_y H(\zeta, y, \lambda) \tag{21}$$

the Hamilton–Jacobi equation is (Lee and Markus, 1967):

$$\begin{gathered} \frac{1}{2}\left(\upsilon - \kappa\left(\zeta, \frac{\partial V}{\partial \zeta}(\zeta)\right)\right)^2 + \\ + \frac{\partial V}{\partial \zeta}(\zeta) F_0\left(\zeta, \kappa\left(\zeta, \frac{\partial V}{\partial \zeta}(\zeta)\right)\right) = 0 \end{gathered} \tag{22}$$

and the optimal control can be derived from the solution to the above equation, as:

$$y^* = \kappa\left(\zeta, \frac{\partial V}{\partial \zeta}(\zeta)\right) \tag{23}$$

Equivalently, since $\frac{\partial H}{\partial y}(\zeta, y, \lambda)$ must vanish at $\arg\min_y H(\zeta, y, \lambda)$, the functions $V(\zeta)$ and $y^*(\zeta)$ must satisfy the coupled equations:

$$\boxed{\begin{array}{c}\frac{1}{2}\left(\upsilon - y^*(\zeta)\right)^2 + \frac{\partial V}{\partial \zeta}(\zeta) F_0\left(\zeta, y^*(\zeta)\right) = 0 \\ \upsilon - y^*(\zeta) - \frac{\partial V}{\partial \zeta}(\zeta)\frac{\partial F_0}{\partial y}\left(\zeta, y^*(\zeta)\right) = 0\end{array}} \quad (24)$$

The above equations must be solved with initial conditions:

$$\boxed{V(\zeta_s) = 0 \quad , \quad y^*(\zeta_s) = \upsilon} \quad (25)$$

Under the assumptions stated in the previous section, there exists a unique analytic solution in a neighbourhood of $\zeta = \zeta_s$, such that the dynamics $\dot{\zeta} = F\left(\zeta, y^*(\zeta)\right)$ is locally asymptotically stable (Lukes, 1969). The solution for $V(\zeta)$ is locally positive semidefinite. Given the local analyticity property of the solution, it is possible to seek for the solution in the form of a Taylor series expansion and, recursively try to determine the Taylor coefficients up to a certain truncation order.

When this approach is applied to the leading terms of the Taylor series expansion (quadratic terms in $V(\zeta)$ and linear terms in $y^*(\zeta)$), one obtains the result for the linear–quadratic approximation of the problem:

$$V(\zeta) = \frac{1}{2}(\zeta - \zeta_s)^{\mathrm{T}} P(\zeta - \zeta_s) + O(\zeta^3) \quad (25)$$

$$y^*(\zeta) = \upsilon - \left[\frac{\partial F_0}{\partial y}(\zeta_s, \upsilon)\right]^{\mathrm{T}} P(\zeta - \zeta_s) + O(\zeta^2) \quad (26)$$

where P is the solution of the quadratic matrix equation:

$$\begin{array}{l}\left[\frac{\partial F_0}{\partial \zeta}(\zeta_s, \upsilon)\right]^{\mathrm{T}} P + P\left[\frac{\partial F_0}{\partial \zeta}(\zeta_s, \upsilon)\right] - \\ -P\left[\frac{\partial F_0}{\partial y}(\zeta_s, \upsilon)\right]\left[\frac{\partial F_0}{\partial y}(\zeta_s, \upsilon)\right]^{\mathrm{T}} P = 0\end{array} \quad (27)$$

that makes

$$\left[\frac{\partial F_0}{\partial \zeta}(\zeta_s, \upsilon)\right] - \left[\frac{\partial F_0}{\partial y}(\zeta_s, \upsilon)\right]\left[\frac{\partial F_0}{\partial y}(\zeta_s, \upsilon)\right]^{\mathrm{T}} P$$

Hurwitz.

The procedure can, in principle, continue, to determine the coefficients of the higher-order terms of the Taylor series expansion of the solution, even though the resulting algebraic equations become extremely complex. In the following section, a Newton-type iteration will be developed to enable the calculation of higher-order terms, using symbolic computing.

5. A NEWTON–KANTOROVICH ITERATION FOR THE SOLUTION OF THE HAMILTON–JACOBI EQUATIONS

Given a general nonlinear operator equation $\aleph(x) = 0$, the Newton–Kantorovich iteration (Kantorovich and Akilov, 1964) involves solving the linear operator equation

$$\aleph'\left(x_N\right) \cdot \left(x_{N+1} - x_N\right) = -\aleph\left(x_N\right) \quad (28)$$

at each step of the iteration, where $\aleph'\left(x\right) \cdot \delta x$ represents the Fréchet differential of the operator $\aleph$.

For the particular nonlinear operator of the Hamilton–Jacobi equations,

$$\aleph\left(V, y^*\right) = \begin{bmatrix} \frac{1}{2}\left(\upsilon - y^*\right)^2 + \frac{\partial V}{\partial \zeta} F_0\left(\zeta, y^*\right) \\ -\left(\upsilon - y^*\right) + \frac{\partial V}{\partial \zeta}\frac{\partial F_0}{\partial y}\left(\zeta, y^*\right)\end{bmatrix} \quad (29)$$

the Fréchet differential is:

$$\aleph'\left(V, y^*\right) \cdot \begin{bmatrix}\delta V \\ \delta y^*\end{bmatrix} = $$
$$= \begin{bmatrix} \frac{\partial(\delta V)}{\partial \zeta} F_0\left(\zeta, y^*\right) + \left(y^* - \upsilon + \frac{\partial V}{\partial \zeta}\frac{\partial F_0}{\partial y}\left(\zeta, y^*\right)\right)\delta y^* \\ \frac{\partial(\delta V)}{\partial \zeta}\frac{\partial F_0}{\partial y}\left(\zeta, y^*\right) + \left(1 + \frac{\partial V}{\partial \zeta}\frac{\partial^2 F_0}{\partial y^2}\left(\zeta, y^*\right)\right)\delta y^*\end{bmatrix} \quad (30)$$

Given the above expression for the Fréchet differential, the Newton–Kantorovich iteration (28), after some algebraic manipulations, finally takes the form:

$$\begin{array}{l}\frac{\partial V_{N+1}}{\partial \zeta}\left[F_0\left(\zeta, y_N^*\right) - \frac{\partial F_0}{\partial y}\left(\zeta, y_N^*\right)\vartheta_N\right] + \\ + \frac{1}{2}\left(y_N^* - \upsilon\right)^2 - \left(y_N^* - \upsilon\right)\vartheta_N = 0\end{array} \quad (31)$$

$$y_{N+1}^* = y_N^* - \frac{y_N^* - \upsilon + \frac{\partial V_{N+1}}{\partial \zeta}\frac{\partial F_0}{\partial y}\left(\zeta, y_N^*\right)}{1 + \frac{\partial V_N}{\partial \zeta}\frac{\partial^2 F_0}{\partial y^2}\left(\zeta, y_N^*\right)} \quad (32)$$

where

$$\vartheta_N = \frac{y_N^* - \upsilon + \frac{\partial V_N}{\partial \zeta}\frac{\partial F_0}{\partial y}\left(\zeta, y_N^*\right)}{1 + \frac{\partial V_N}{\partial \zeta}\frac{\partial^2 F_0}{\partial y^2}\left(\zeta, y_N^*\right)} \quad (33)$$

Notice that (31) is a linear partial differential equation with unknown $V_{N+1}(\zeta)$. In particular, it is a Zubov equation, for which a recursive series solution algorithm is available (Kazantzis, *et al.*, 2005). Once the solution of (31) is computed, (32) determines directly $y_{N+1}^*(\zeta)$.

The Newton–Kantorovich iteration can be initialized with $V_1(\zeta)$ quadratic and $y_1^*(\zeta)$ linear, obtained from the linear–quadratic approximation of the problem, given in the previous section.

6. EXAMPLE

Consider a nonisothermal continuous stirred tank reactor (CSTR) of constant volume V, in which the following series/parallel reaction takes place:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$
$$2A \xrightarrow{k_3} D$$

The mass and energy balances that describe the dynamics of the reactor are:

$$\begin{aligned}
\frac{dC_A}{dt} &= -k_1 C_A - k_3 C_A^2 + \left(C_A^0 - C_A\right)\frac{F}{V} \\
\frac{dC_B}{dt} &= k_1 C_A - k_2 C_B - C_B \frac{F}{V} \\
\frac{dT}{dt} &= \frac{-\left(\Delta H_1\right)k_1 C_A - \left(\Delta H_2\right)k_2 C_B - \left(\Delta H_3\right)k_3 C_A^2}{\rho_{mix} C_P} + \\
&\quad + \frac{Q_H}{\rho_{mix} C_P} + \left(T^0 - T\right)\frac{F}{V} \\
y &= C_B
\end{aligned} \tag{34}$$

where C_A, C_B are the molar concentrations of A and B respectively, T the temperature of the reactor, $\frac{F}{V}$ the dilution rate, ρ_{mix} the density of the mixture, C_P the heat capacity, ΔH_i the heats of the reaction and Q_H the constant rate of the heat removed per unit volume. The rate coefficients are given by the Arrhenius equation $k_i(T) = k_{i0}\exp(-E_i/RT)$, $i = 1,2,3$. All the constants and parameters are given in Table 1 (Kravaris, *et al.*, 1998).

Table 1. Constants and parameters of the system

$k_{10} = 1.287 \cdot 10^{12} h^{-1}$	$k_{20} = 1.287 \cdot 10^{12} h^{-1}$	$k_{30} = 9.043 \cdot 10^{9} L/(mol \cdot h)$
$E_1/R = 9758.3K$	$E_2/R = 9758.3K$	$E_3/R = 8560K$
$\Delta H_1 = 4.2kJ/mol$	$\Delta H_2 = -11kJ/mol$	$\Delta H_3 = -41.85kJ/mol$
$\rho_{mix} = 0.9342kg/L$	$C_P = 3.01kJ/(kg \cdot K)$	$Q_H = -451.51kJ/(L \cdot h)$
$C_A^0 = 5gmol/L$	$T^0 = 403.15K$	

The control objective is the optimal regulation of the output $y = C_B$ at set point by manipulating the dilution rate $\frac{F}{V}$. In particular, the controller must bring the system to the final steady state of $C_{As} = 1.0774 mol/l$, $C_{Bs} = 0.8181 mol/l$ and $T_s = 403.15K$, which corresponds to $\left(F/V\right)_s = 12.5418\, h^{-1}$.

The coordinate transformation

$$\zeta_1 = \frac{C_A^0 - C_A}{C_B}, \quad \zeta_2 = \frac{T^0 - T}{C_B}, \quad y = C_B \tag{35}$$

transforms (34) to Byrnes-Isidori normal form:

$$\begin{aligned}
\frac{d\zeta_1}{dt} &= k_1(1-\zeta_1)\frac{C_A^0 - \zeta_1 y}{y} + \zeta_1 k_2 + k_3 \frac{\left(C_A^0 - \zeta_1 y\right)^2}{y} \\
\frac{d\zeta_2}{dt} &= k_1\left(\frac{\Delta H_1}{\rho_{mix} C_p} - \zeta_2\right)\frac{C_A^0 - \zeta_1 y}{y} + k_2\left(\zeta_2 + \frac{\Delta H_2}{\rho_{mix} C_p}\right) + \\
&\quad + \frac{k_3 \Delta H_3}{\rho_{mix} C_p}\frac{\left(C_A^0 - \zeta_1 y\right)^2}{y} - \frac{Q_H}{\rho_{mix} C_p}\frac{1}{y} \\
\frac{dy}{dt} &= k_1\left(C_A^0 - \zeta_1 y\right) - k_2 y - y\frac{F}{V}
\end{aligned} \tag{36}$$

where $k_i = k_{i0}\exp\left\{-E_i/R(T^0 - \zeta_2 y)\right\}$, $i = 1,2,3$.

Coordinate transformation (35) maps the desirable final steady state to $\zeta_{1s} = 4.7948$, $\zeta_{2s} = 0$ and $y_s = 0.8181 mol/l$. A straightforward calculation of the eigenvalues of the Jacobian of the zero dynamics shows that the system is locally nonminimum – phase at the desirable final steady state.

Using deviation variables $\bar{\zeta}_1 = \zeta_1 - \zeta_{1s}$, $\bar{\zeta}_2 = \zeta_2 - \zeta_{2s}$ and $\bar{y} = y - y_s$, the problem becomes the one of regulating the given nonminimum phase system to the origin.

The pertinent Hamilton - Jacobi equations were solved using the symbolic program MAPLE, applying the iterative solution method described in the previous section. The solution up to 5th order was found to be:

$$\begin{aligned}
V(\bar{\zeta}_1,\bar{\zeta}_2) &= 0.1156\, 10^{-3}\bar{\zeta}_1^2 + 0.2\, 10^{-6}\bar{\zeta}_1\bar{\zeta}_2 + \\
&- 0.644601\, 10^{-4}\bar{\zeta}_1^3 + 0.545595\, 10^{-5}\bar{\zeta}_1^2\bar{\zeta}_2 + \\
&+ 0.112623\, 10^{-7}\bar{\zeta}_1\bar{\zeta}_2^2 - 0.749841\, 10^{-11}\bar{\zeta}_2^3 + \\
&+ 0.207583\, 10^{-4}\bar{\zeta}_1^4 - 0.459909\, 10^{-5}\bar{\zeta}_1^3\bar{\zeta}_2 + \\
&+ 0.138649\, 10^{-6}\bar{\zeta}_1^2\bar{\zeta}_2^2 + 0.365272\, 10^{-9}\bar{\zeta}_1\bar{\zeta}_2^3 - \\
&- 0.791093\, 10^{-12}\bar{\zeta}_2^4 - \\
&- 0.554933\, 10^{-5}\bar{\zeta}_1^5 + 0.196181\, 10^{-5}\bar{\zeta}_1^4\bar{\zeta}_2 - \\
&- 0.161327\, 10^{-6}\bar{\zeta}_1^3\bar{\zeta}_2^2 + 0.253904\, 10^{-8}\bar{\zeta}_1^2\bar{\zeta}_2^3 + \\
&+ 0.963585\, 10^{-11}\bar{\zeta}_1\bar{\zeta}_2^4 - 0.593702\, 10^{-13}\bar{\zeta}_2^5
\end{aligned} \tag{37}$$

$$\begin{aligned}
\bar{y}^*(\bar{\zeta}_1,\bar{\zeta}_2) &= -0.241444\bar{\zeta}_1 - 0.208779\, 10^{-3}\bar{\zeta}_2 - \\
&- 0.668295\, 10^{-2}\bar{\zeta}_1^2 + 0.202038\, 10^{-3}\bar{\zeta}_1\bar{\zeta}_2 - \\
&- 0.150569\, 10^{-5}\bar{\zeta}_2^2 - \\
&- 0.016334\bar{\zeta}_1^3 - 0.211563\, 10^{-2}\bar{\zeta}_1^2\bar{\zeta}_2 - \\
&- 0.126407\, 10^{-4}\bar{\zeta}_1\bar{\zeta}_2^2 - 0.321561\, 10^{-7}\bar{\zeta}_2^3 - \\
&- 0.014529\bar{\zeta}_1^4 - 0.870863\, 10^{-3}\bar{\zeta}_1^3\bar{\zeta}_2 - \\
&- 0.517792\, 10^{-5}\bar{\zeta}_1^2\bar{\zeta}_2^2 - 0.278626\, 10^{-6}\bar{\zeta}_1\bar{\zeta}_2^3 - \\
&- 0.581388\, 10^{-9}\bar{\zeta}_2^4
\end{aligned} \tag{38}$$

Figure 1 depicts the solution $V(\bar{\zeta}_1,\bar{\zeta}_2)$ for different truncation orders N=2,3,4,5. In Figure 1, the N=2 approximation is at the top, N=3 at the bottom, and N=4,5 essentially coincide in the middle. This indicates that convergence of the Taylor series is achieved for N>3 within the ranges of ζ_1 and ζ_2

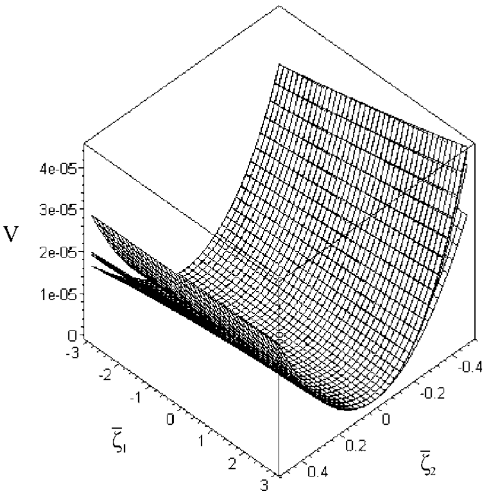

Fig. 1. Solution for $V(\bar{\zeta}_1,\bar{\zeta}_2)$ for different truncation orders N=2, 3, 4, 5.

shown. A similar diagram was constructed for $\bar{y}^*(\bar{\zeta}_1,\bar{\zeta}_2)$ (not shown), that indicated numerical convergence for N>2 within the same ranges.

For nonminimum-phase compensation, the synthetic output

$$y' = \bar{y} - \bar{y}^*(\bar{\zeta}_1,\bar{\zeta}_2) \quad (39)$$

is used, which is a statically equivalent minimum–phase output and, moreover, perfect control of y' to 0 corresponds to ISE–optimality in y. The state feedback law (6), with the synthetic output map (39) is then applied to regulate y' to 0.

Figures 2 and 3 show the resulting closed-loop responses of output $y = C_B$ and synthetic output y', for a step change in the set-point from 0.85 to 0.8181 and for different time constants, τ=10^{-2}, 10^{-3}, 10^{-4} and 10^{-5}. The calculations were made for 5-th order approximation. As the closed-loop time constant τ tends to zero, the resulting closed-loop responses converge to the ISE-optimal responses.

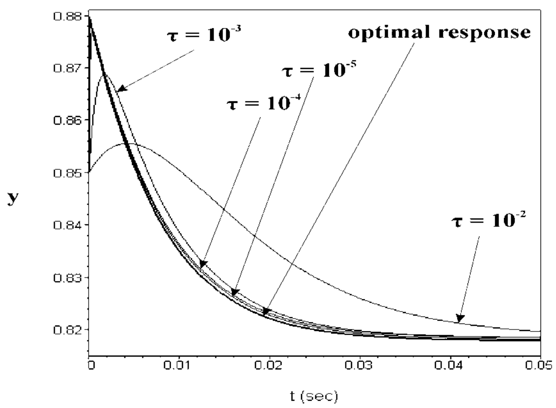

Fig. 2. Closed-loop responses of output $y = C_B$ for a step change in the set point from 0.85 to 0.8181, for N=5 (τ=10^{-2}, τ=10^{-3}, τ=10^{-4}, τ=10^{-5})

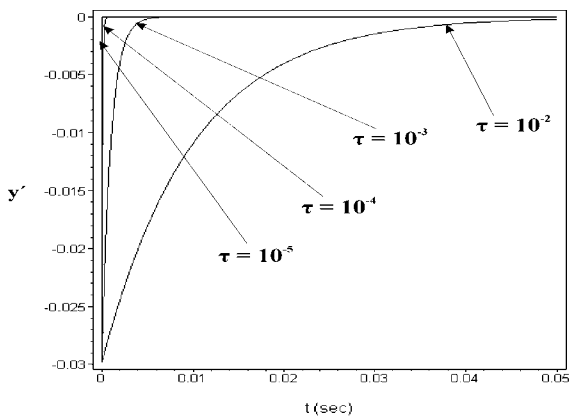

Fig. 3. Closed-loop responses of synthetic output y' for a step change in the set point from 0.85 to 0.8181, for N=5 (τ=10^{-2}, τ=10^{-3}, τ=10^{-4}, τ=10^{-5})

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