CONSTRUCTIVE DESIGN OF UNKNOWN INPUT NONLINEAR OBSERVERS BY DISSIPATIVITY AND LMIS [2]

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Abstract: The existence of Unknown Input Observers (UIO) for nonlinear systems has been characterized recently by the first authors of this note as an abstract incremental dissipativity property of the system. In this paper, it is shown that for a large class of systems this condition can be made computable by the use of LMI techniques, leading to a constructive design method of nonlinear UIOs.

Keywords: Unknown input observers, passivity, dissipativity, passive design, passivity by output injection, LMI.

1. INTRODUCTION

The standard state estimation problem consists in reconstructing the state of the plant using the available measurements of inputs and outputs and the complete model of the plant. However, it is usual to lack the complete information on the system: some unmeasurable disturbances are present or there are uncertainties in the knowledge of the model. In these cases, a robust observation problem has to be solved, if despite of these uncertainties the state of the system is to be reconstructed perfectly.

If the uncertainties are interpreted as unknown inputs the existence of Unknown Input Observers (UIO) gives a solution to this robust observation problem (Moreno, 2000). Although their existence conditions are strong, UIOs are important for robust observation schemes, for fault detection and isolation (FDI) problems (see for example (Seliger and Frank, 2000)) and in other fields. For linear systems, existence conditions are well-known (Hautus, 1983), whereas for nonlinear systems the problem has not yet been completely solved.

Recently, (Moreno, 2001) has shown that the existence of UIO for linear time-invariant systems is equivalent to the possibility of rendering the plant dissipative by output injection. For nonlinear systems, a similar incremental dissipativity property has been derived to be sufficient for the existence of UIOs by (Rocha-Cózatl and Moreno, 2004). This result, although theoretically appealing and new, requires the knowledge of a storage function and an output injection in order to design a UIO. Since this is a difficult task in general, the result is not constructive and of less practical interest.

The objective of this paper is to improve this situation. It is shown that for a wide class of nonlinear systems the approach developed in (Rocha-Cózatl

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and Moreno, 2004) can be made computable. The required storage function and the output injection can be calculated, under generic conditions, by Linear Matrix Inequalities (LMIs) in the design parameters. Since LMIs can be efficiently computed, the design of UIOs becomes a feasible task.

Moreover, the design proposed here can be seen as an extension of recent methods for the design of observers for nonlinear systems without unknown inputs based on dissipative properties, as in (Moreno, 2004), that, in turn, generalizes the observer design introduced by (Arcak and Kokotovic, 1999).

This paper is organized as follows. In Section 2 some standard dissipativity concepts and results are recalled, while some preliminaries of unknown input observers and the problem formulation are given in Section 3. In Section 4, the proposed design is explained, and in Section 5, an application example is presented. Finally in Section 6, some conclusions are made.

2. DISSIPATIVITY

Since dissipativity plays a fundamental role in this paper, some standard results for linear time-invariant systems and LTI systems with a nonlinear feedback (absolute stability problem) are recalled (see (Willems, 1972*a*; Willems, 1972*b*; Hill and Moylan, 1980; Khalil, 2002; Moreno, 2004)). Consider the following LTI system

$$\Sigma_L : \begin{cases} \dot{x} = Ax + Bu, \quad x(0) = x_0 \ , \\ y = Cx \ , \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$ and $y \in \mathbb{R}^m$ are the state, the input and the output vectors, respectively. Let us define a quadratic *supply rate*

$$\omega(y,u) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \tag{2}$$

where $Q \in \mathbb{R}^{m\times m}$, $S \in \mathbb{R}^{m\times p}$, $R \in \mathbb{R}^{p\times p}$, and Q, R are symmetric. Based on this supply rate, the dissipativity of (1) can be defined as follows.

Definition 1. System Σ_L is said to be *state strictly dissipative (SSD)* with respect to the supply rate $\omega(y,u)$ – or for short $(Q,S,R) - SSD$ – if there exist a matrix $P = P^T > 0$ and a constant $\epsilon > 0$ such that

$$\begin{bmatrix} PA + A^TP + \epsilon P & PB \\ B^TP & 0 \end{bmatrix} - \begin{bmatrix} C^TQC & C^TS \\ S^TC & R \end{bmatrix} \le 0$$

Now, define the considered (static) nonlinearities.

Definition 2. A time-varying memoryless nonlinearity $\psi : [0,\infty) \times \mathbb{R}^r \to \mathbb{R}^s$, $y^* = \psi(t,u^*)$, piecewise continuous in t and locally Lipschitz in u^*, such that $\psi(t,0) = 0$, is said to be dissipative with respect to the supply rate $\omega(y^*,u^*)$ (2), or for short $(Q,S,R) - D$, if for every $t \ge 0$, and $u^* \in \mathbb{R}^r$

$$\omega(y^*,u^*) = \omega(\psi(t,u^*),u^*) \ge 0$$

Note that the classical sector conditions for square nonlinearities (Khalil, 2002), i.e. $s = r$, can be represented in this form. For example, if ψ is in the sector $[K_1,K_2]$, i.e. $(y^* - K_1u^*)^T(K_2u^* - y^*) \ge 0$, is equivalent to $(Q,S,R) - D$, with $(Q,S,R) = \left(-I, \frac{1}{2}(K_1+K_2), -\frac{1}{2}\left(K_1^TK_2 + K_2^TK_1\right)\right)$; if ψ is in the sector $[K_1,\infty)$, i.e. $(y^* - K_1u^*)^T u^* \ge 0$, is equivalent to $\left(0, \frac{1}{2}I, -\frac{1}{2}\left(K_1 + K_1^T\right)\right) - D$; I is the identity matrix.

Therefore, the following lemma is a generalization of the circle criterion of absolute stability (Khalil, 2002) to nonsquare systems.

Lemma 3. (Hill and Moylan, 1980; Moreno, 2004) Consider the feedback interconnection between Σ_L (1) and ψ (i.e. $u = -y^*$ and $u^* = y$)

$$\begin{aligned} &\dot{x} = Ax + Bu, \quad x(0) = x_0 \ , \\ &y = Cx \ , \\ &u = -\psi(t,y) \ , \end{aligned} \tag{3}$$

If the linear system (1) is $\left(-R, S^T, -Q\right) - SSD$, then the point $x = 0$ of (3) is globally exponentially stable for every (Q,S,R) –D nonlinearity.

Note that the dissipativity of the nonlinearity ψ or its membership to a sector is a form of characterizing it, and to restrict the set of functions by which a stability property as in Lemma 3 is satisfied. This indicates that the use of positive semidefinite supply rates (Q,S,R) in Definition 2 is of no use, since any nonlinearity ψ is dissipative in such sense. Therefore it will be assumed that (Q,S,R) is not positive semidefinite.

3. UNKNOWN INPUT OBSERVERS

The class of nonlinear systems considered for UIO design is

$$\Sigma : \begin{cases} \dot{x} = Ax + G\psi(\sigma) + \varphi(t,y,u) - Bw \ , \\ y = Cx \ , \\ \sigma = Hx \ , \end{cases} \tag{4}$$

with initial conditions $x(0) = x_0$. In (4), $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $w \in \mathbb{R}^q$ and $y \in \mathbb{R}^m$ are the state, the known input, the unknown input and the output vectors, respectively, and $\sigma \in \mathbb{R}^r$ a (not necessarily measured) linear function of the state. The nonlinear function $\varphi(t,y,u)$ is arbitrary and is assumed to be locally Lipschitz in y, continuous in u, and piecewise continuous in t. The s-dimensional vector $\psi(\sigma)$ is assumed to be locally Lipschitz in σ. It will be assumed that the trajectories of Σ exist and are well defined for all times, i.e. there are no finite escape times. Without loss of generality it is assumed that matrices B and C are of full rank.

Note that the class of systems can be enlarged considering that by means of unknown input, state and/or output transformations some systems can be transformed to the particular form (4).

The objective is to design an *unknown input observer (UIO)* for the system Σ (4), that is, a dynamical system that using the information of the known input $u(t)$ and the output $y(t)$ produces an state estimate $\hat{x}(t)$, that converges asymptotically to the actual state $x(t)$ of Σ, i.e. $\lim_{t\to\infty}(\hat{x}(t)-x(t))=0$, in spite of the lack of information on the unknown input w. In this paper, a full-order observer is proposed

$$\Omega:\begin{cases}\dot{\zeta}=A\zeta+G\psi(\hat{\sigma}+N(\hat{y}-y))+\\ \quad +L(\hat{y}-y)+\varphi(t,y,u), \quad \zeta(0)=\zeta_0 ,\\ \hat{y}=C\zeta ,\\ \hat{\sigma}=H\zeta ,\\ \hat{x}=D\zeta+Ey ,\end{cases} \tag{5}$$

where the output injection matrices $L\in\mathbb{R}^{n\times m}$ and $N\in\mathbb{R}^{r\times m}$ as well as the matrices $D\in\mathbb{R}^{n\times n}$ and $E\in\mathbb{R}^{n\times m}$ are the design variables. Note that in Ω the estimated value of the state $\hat{x}$ depends, in general, not only on the observer state ζ but also on the output of the plant y.

The existence of UIO for nonlinear systems depends esentially on an incremental dissipativity property (Rocha-Cózatl and Moreno, 2004), that will be specialized for the aims of this paper, and that can be described in terms of an error system.

3.1 Error Dynamics

Define $e\triangleq\zeta-x$ as the state error, the output error as $\tilde{y}\triangleq\hat{y}-y$, the error in σ as $\tilde{\sigma}\triangleq\hat{\sigma}-\sigma$ and an auxiliary variable $z\triangleq(H+NC)e=\tilde{\sigma}+N\tilde{y}$. Plant Σ and observer Ω form a cascade system, that can be described in new "error" coordinates as follows

$$\Sigma_e^{obs}:\begin{cases}\dot{x}=Ax+G\psi(\sigma)+\varphi(t,y,u)-Bw ,\\ \dot{e}=A_Le+G\nu+Bw ,\\ z=H_Ne ,\\ \tilde{y}=Ce ,\\ \sigma=Hx ,\\ \nu=-\phi(z,\sigma) ,\end{cases} \tag{6}$$

with initial conditions $x(0)=x_0$, $e(0)=e_0$. Here, $\phi(z,\sigma)\triangleq\psi(\sigma)-\psi(\sigma+z)$, $A_L=A+LC$, $H_N=H+NC$. Σ_e^{obs} will be called the error dynamics. Note that $\phi(0,\sigma)=0$ for all σ, and the coupling between plant's state and the error subsystem e is effected through $\sigma=Hx$ in the nonlinearity $\phi(z,\sigma)$. In the linear case, this coupling does not exist, and the error dynamics consists of two decoupled systems.

3.2 Feedback dissipativity of the error dynamics

The following property is fundamental for the existence of an UIO (5) for system (4).

Definition 4. System

$$\Sigma_e:\begin{cases}\dot{x}=Ax+G\psi(\sigma)+\varphi(t,y,u)-Bw,\\ \dot{e}=Ae+G\nu+Bw,\\ \tilde{y}=Ce ,\\ \tilde{\sigma}=He ,\\ \nu=-\phi(\tilde{\sigma},\sigma) ,\end{cases} \tag{7}$$

with initial conditions $x(0)=x_0$, $e(0)=e_0$, is said to be *Linear Partial Output Injection Partial Strictly Dissipative (LPOIPSD)* if there exist:

(i) matrices L and N in (6);
(ii) a continuous differentiable storage function $V(e,x)$, positive definite in e, uniformly in x, i.e. it is satisfied for all $(e,x)\in\mathbb{R}^n\times\mathbb{R}^n$
$$\alpha_1(\|e\|)\le V(e,x)\le\alpha_2(\|e\|) \tag{8}$$
for some class $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$, $\alpha_2(\cdot)$;
(iii) a full row rank matrix $\mathbb{S}\in\mathbb{R}^{q\times m}$;

such that along the trajectories of (6) it is satisfied for all $(e,x)\in\mathbb{R}^n\times\mathbb{R}^n$

$$\dot{V}\le-\alpha_3(\|e\|)+w^T\mathbb{S}\tilde{y} , \tag{9}$$

for a class $\mathcal{K}$ function $\alpha_3(\cdot)$.

Σ_e (7) is said to be partially dissipative because the dissipation inequality has to be satisfied by the part of the state e of Σ_e, whereas the (energetic) behavior of the x subsystem is not relevant. The output injection is called partial and linear since it affects only the e subsystem and not the whole state of the system (x,e) and does it linearly. Note that if system Σ_e is LPOIPSD then Σ_e^{obs} (6) is (partially) $(0,\mathbb{S},0)-$SSD.

Remark 5. This definition is a particular version of that introduced by (Rocha-Cózatl and Moreno, 2004), where more general systems are studied. It is worth noting that in the linear time invariant case this condition is equivalent to the existence of UIO (Moreno, 2001) and it can be designed by means of the Theorem 6, whereas in the nonlinear case it is part of the sufficient conditions (Rocha-Cózatl and Moreno, 2004).

4. DISSIPATIVE DESIGN OF THE UIO

The main result of the paper gives a computable, sufficient condition for the existence of an UIO for the plant Σ (4) and makes the general result of (Rocha-Cózatl and Moreno, 2004) computable.

Theorem 6. Suppose that

(a) The nonlinearity ϕ in (6) is $(Q,S,R)-D$ for some quadratic form $\omega(\phi,z) = \phi^T Q\phi + 2\phi^T Sz + z^T Rz$ for all σ, with $Q \leq 0$.

(b) There exist constant matrices $P = P^T > 0$, L, N, $\mathbb{S}$ and a constant $\epsilon > 0$, such that

$$\begin{bmatrix} PA_L + A_L^T P + \epsilon P & PG & PB \\ G^T P & 0 & 0 \\ B^T P & 0 & 0 \end{bmatrix} - \tag{10}$$
$$- \begin{bmatrix} -H_N^T RH_N & H_N^T S^T & C^T \mathbb{S}^T \\ SH_N & -Q & 0 \\ \mathbb{S}C & 0 & 0 \end{bmatrix} \leq 0 \ .$$

Then there exists an UIO for (4). Moreover, Ω (5) is a full order UIO when L_T, N_T, D and E are given by (17), (18), and (19), respectively. Matrices L_T and N_T replace L and N in (5), respectively.

PROOF. The proof follows the same path as the one in (Rocha-Cózatl and Moreno, 2004).

(i) Dissipativity Property: First it will be shown that satisfaction of (a) and (b) implies that Σ_e (7) is LPOIPSD. For this consider the candidate storage function $V(e,x) = e^T Pe$, that satisfies (8). The derivative of V along the trajectories of (6) can be written as follows

$$\dot{V} = \begin{bmatrix} e \\ \nu \\ w \end{bmatrix}^T \begin{bmatrix} PA_L + A_L^T P & PG & PB \\ G^T P & 0 & 0 \\ B^T P & 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \nu \\ w \end{bmatrix}$$

Since (10) is satisfied, then

$$\dot{V} \leq \begin{bmatrix} e \\ \nu \\ w \end{bmatrix}^T \begin{bmatrix} -H_N^T RH_N - \epsilon P & H_N^T S^T & C^T \mathbb{S}^T \\ SH_N & -Q & 0 \\ \mathbb{S}C & 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \nu \\ w \end{bmatrix}$$
$$= - \begin{bmatrix} \phi \\ z \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} \phi \\ z \end{bmatrix} - \epsilon V(e) + 2\tilde{y}^T \mathbb{S}^T w$$

Now, because of (a)

$$\dot{V} \leq -\epsilon V(e) + 2w^T \mathbb{S}\tilde{y} \ , \tag{11}$$

and (9) in Definition 4 is satisfied.

(ii) Coordinates Transformation: Note that if $w = 0$ or $\mathbb{S}\tilde{y} = 0$ then (11) implies that $\zeta \to x$. However, if $w \neq 0$ then $\zeta \nrightarrow x$ and it is necessary to decouple the effect of the perturbation w on the state estimate $\hat{x}$. This is better done in special coordinates. For this note first that the satisfaction of (10) implies that $\mathbb{S}C = B^T P$, and therefore that

$$\mathbb{S}CB = B^T PB > 0 \ . \tag{12}$$

Since $\det(\mathbb{S}CB) \neq 0$, a matrix $M \in \mathbb{R}^{(n-q)\times n}$ of full row rank exists such that $T = \begin{bmatrix} \mathbb{S}C \\ M \end{bmatrix}$, $\det T \neq 0$ and $MB = 0$. T define a state transformation $(\chi, \xi) = (Tx, Te)$ for (6) with

$$\chi = \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = \begin{bmatrix} \mathbb{S}Cx \\ Mx \end{bmatrix} \ , \ \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} \mathbb{S}Ce \\ Me \end{bmatrix} \ . \tag{13}$$

In these new coordinates, the error dynamics (6) has the form (only the subsystem e is shown)

$$\begin{aligned} \dot{\xi}_1 &= \left(\bar{A}_{11} + \bar{L}_1\bar{C}_1\right)\xi_1 + \left(\bar{A}_{12} + \bar{L}_1\bar{C}_2\right)\xi_2 + \bar{G}_1\nu + \\ &\quad + B^T PBw \\ \dot{\xi}_2 &= \left(\bar{A}_{21} + \bar{L}_2\bar{C}_1\right)\xi_1 + \left(\bar{A}_{22} + \bar{L}_2\bar{C}_2\right)\xi_2 + \bar{G}_2\nu \\ z &= \left(\bar{H} + \bar{N}\bar{C}\right)\xi = \left[\, \bar{H}_1 + \bar{N}\bar{C}_1, \ \bar{H}_2 + \bar{N}\bar{C}_2 \,\right]\xi \ , \\ \tilde{y} &= \bar{C}\xi \ , \qquad \mathbb{S}\tilde{y} = \xi_1 \ , \\ \nu &= -\phi(z,\sigma) \ , \end{aligned} \tag{14}$$

where

$$\bar{A} = TAT^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \bar{B} = TB = \begin{bmatrix} B^T PB \\ 0 \end{bmatrix},$$

$$\bar{G} = TG = \begin{bmatrix} \bar{G}_1 \\ \bar{G}_2 \end{bmatrix}, \bar{L} = TL = \begin{bmatrix} \bar{L}_1 \\ \bar{L}_2 \end{bmatrix},$$

$$\bar{C} = CT^{-1} = \left[\, \bar{C}_1 \ \bar{C}_2 \,\right], \quad \mathbb{S}\bar{C} = \mathbb{S}CT^{-1} = I,$$

$$\bar{H} = HT^{-1} = \left[\, \bar{H}_1 \ \bar{H}_2 \,\right] \ .$$

According to the main result in (Rocha-Cózatl and Moreno, 2004), the satisfaction of conditions (i) and (ii) are sufficient for the existence of an UIO. In the following steps, the UIO will be constructed.

In these coordinates, the storage function $V(e,x) = e^T Pe$ can be written as $V(\xi_1,\xi_2,\psi) = \xi^T \Pi \xi = \xi_1^T \Pi_1 \xi_1 + \xi_2^T \Pi_2 \xi_2 = V_1(\xi_1) + V_2(\xi_2)$, where $\Pi = T^{-T} PT^{-1}$ is a block diagonal matrix. This last fact can be obtained writing $\mathbb{S}C = B^T P$ in the new coordinates, i.e. $\bar{B}^T \Pi = [I, 0]$.

Note that if $\mathbb{S}\tilde{y} = \xi_1 = 0$ then from (11)

$$\dot{V}_2(\xi_2) \leq -\epsilon V_2(\xi_2) \ ,$$

and together with (14) the subsystem

$$\begin{aligned} \dot{\xi}_2 &= \left(\bar{A}_{22} + \bar{L}_2\bar{C}_2\right)\xi_2 + \bar{G}_2\nu \\ z &= \left(\bar{H}_2 + \bar{N}\bar{C}_2\right)\xi_2 \ , \\ \nu &= -\phi(z,\sigma) \ , \end{aligned} \tag{15}$$

has $\xi_2 = 0$ as an equilibrium point that is exponentially stable, uniformly in σ.

(iii) Decoupling of the unknown input: In order to obtain an asymptotic observer, despite of the unknown input, it is necessary to decouple its effect on ξ_2 in (14), since this contains the estimation error of the not measurable part of the states. In order to reach this decoupling, a further output injection of the passive output $\xi_1 = \mathbb{S}\tilde{y}$ will be performed on (14). This leads to

$$\begin{aligned}\dot{\xi}_1 =& \left(\bar{A}_{11}+\bar{L}_1\bar{C}_1+\bar{K}_1\right)\xi_1+\left(\bar{A}_{12}+\bar{L}_1\bar{C}_2\right)\xi_2+\\&+\bar{G}_1\nu+B^TPBw\\ \dot{\xi}_2 =& \left(\bar{A}_{21}+\bar{L}_2\bar{C}_1+\bar{K}_2\right)\xi_1+\left(\bar{A}_{22}+\bar{L}_2\bar{C}_2\right)\xi_2\\&+\bar{G}_2\nu\\ z =& \left(\bar{H}_1+\bar{N}\bar{C}_1+K_N\right)\xi_1+\left(\bar{H}_2+\bar{N}\bar{C}_2\right)\xi_2\ .\end{aligned} \quad (16)$$

Selecting

$$\bar{K}_2=-\left(\bar{A}_{21}+\bar{L}_2\bar{C}_1\right)\ ,\quad K_N=-\left(\bar{H}_1+N\bar{C}_1\right)\ ,$$

and $\bar{K}_1$ such that $A_{11}^*\triangleq\left(\bar{A}_{11}+\bar{L}_1\bar{C}_1+\bar{K}_1\right)$ is Hurwitz, (16) becomes

$$\begin{aligned}\dot{\xi}_1 =& A_{11}^*\xi_1+\left(\bar{A}_{12}+\bar{L}_1\bar{C}_2\right)\xi_2+\bar{G}_1\nu+B^TPBw\ ,\\ \dot{\xi}_2 =& \left(\bar{A}_{22}+\bar{L}_2\bar{C}_2\right)\xi_2+\bar{G}_2\nu\ ,\\ z =& \left(\bar{H}_2+\bar{N}\bar{C}_2\right)\xi_2\ ,\\ \nu =& -\phi\left(z,\sigma\right)\ .\end{aligned}$$

Due to the stability of (15) $\xi_2\rightarrow 0$ exponentially as $t\rightarrow\infty$, and since A_{11}^* is Hurwitz, $\xi_1\rightarrow 0$ when $w\rightarrow 0$, or ξ_1 is bounded when w is bounded.

Therefore,the total output injection L_T in original coordinates is

$$L_T=L+T^{-1}\begin{bmatrix}\bar{K}_1\\ \bar{K}_2\end{bmatrix}\mathbb{S} \quad (17)$$

and the total injection N_T

$$N_T=N+K_N\mathbb{S} \quad (18)$$

The observer is then given by Ω (5), with L_T and N_T replacing L and N, respectively. Moreover, selecting the matrices D and E of the UIO as

$$D=T^{-1}\begin{bmatrix}0\\ M\end{bmatrix},E=T^{-1}\begin{bmatrix}\mathbb{S}\\ 0\end{bmatrix}\ , \quad (19)$$

it follows easily that $\lim\limits_{t\rightarrow\infty}\left(\hat{x}\left(t\right)-x\left(t\right)\right)=0$ exponentially fast, and uniformly in w. ■

Remark 7. Note that (10) implies $Q\leq 0$, and the quadratic form of the nonlinearity $\omega\left(\phi,z\right)$ is not positive semidefinite, avoiding this trivial case.

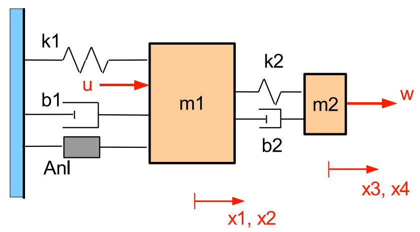
The design of the UIO is based in finding parameters L, N, $\epsilon>0$, $\mathbb{S}$, and P such that the matrix inequality (10) is satisfied, and then the final matrices for the UIO can be calculated from them. In fact, the whole design process can be seen as a set of matrix inequalities and equalities, that can be solved numerically directly.

Although in general inequality (10) is nonlinear in the design parameters, under some conditions it becomes an LMI, for which efficient numerical algorithms are available. Replacing ϵP by ϵI, (10) is a LMI in P, PL, ϵ, $\mathbb{S}$, but not in N, except when $R=0$. If $\phi\in\left[K_1,K_2\right]$ or $\left[K_1,\infty\right]$, it is always possible to use a loop transformation (Khalil, 2002) to render $R=0$. This is the case for example when ψ is Lipschitz or it is in the sector $[0,\infty]$, as for monotonic nonlinearities.

Remark 8. When there is no unknown input ($w=0$), this observer design is also valid and it reduces to one of the cases introduced by (Moreno, 2004).

5. EXAMPLE

Consider the following mechanical system



where x_1 and x_2 are the linear position and velocity of mass m_1, respectively, and x_3 and x_4 the linear position and velocity of mass m_2. A_{nl} is a nonlinear damping device, whose damping force is $F_{A_{nl}}=c_{nl}\,\mathrm{sign}\left(x_2\right)\ln\left(1+\left|x_2\right|\right)$, where $c_{nl}\geq 0$. Moreover, an arbitrary and unknown force w is applied on mass m_2. It is assumed that the state variables $\left(x_1,x_4\right)$ are measured, and an UIO to estimate the state variables $\left(x_2,x_3\right)$ is to be designed.

The state equations of the system are

$$\begin{aligned}\dot{x}=&\begin{bmatrix}0 & 1 & 0 & 0\\ -\dfrac{k_1+k_2}{m_1} & -\dfrac{b_1+b_2}{m_1} & \dfrac{k_2}{m_1} & \dfrac{b_2}{m_1}\\ 0 & 0 & 0 & 1\\ \dfrac{k_2}{m_2} & \dfrac{b_2}{m_2} & -\dfrac{k_2}{m_2} & -\dfrac{b_2}{m_2}\end{bmatrix}x+\\ &+\begin{bmatrix}0\\ \dfrac{1}{m_1}\\ 0\\ 0\end{bmatrix}u+\begin{bmatrix}0\\ -\dfrac{1}{m_1}\\ 0\\ 0\end{bmatrix}\psi\left(\sigma\right)+\begin{bmatrix}0\\ 0\\ 0\\ \dfrac{1}{m_2}\end{bmatrix}w\\ y_1 =& x_3\ ,\qquad y_2=x_4\ ,\\ \psi\left(\sigma\right) =& c_{nl}\,\mathrm{sign}\left(\sigma\right)\ln\left(1+\left|\sigma\right|\right)\ ,\quad \sigma=x_2\ ,\end{aligned} \quad (20)$$

In this case, ϕ in (6) is given by

$$\begin{aligned}\phi\left(z,\sigma\right) =& c_{nl}\,\mathrm{sign}\left(\sigma\right)\ln\left(1+\left|\sigma\right|\right)-\\ &-c_{nl}\,\mathrm{sign}\left(\sigma+z\right)\ln\left(1+\left|\sigma+z\right|\right)\ .\end{aligned}$$

It is easy to check, that this nonlinearity is in the sector $\left[-c_{nl},0\right]$, i.e. it is $\left(-1,-\frac{1}{2}c_{nl},0\right)-$ D, so that hypothesis (a) of Theorem 6 is satisfied with $Q=-1$, $S=-\frac{1}{2}c_{nl}$ and $R=0$.

Since $R=0$, the design inequality (10) is an LMI in the design variables P, PL, ϵ, $\mathbb{S}$, and N if ϵI replaces ϵP, as mentioned before. Using the following values for the parameters: $m_1=5$ [kg], $m_2=1$ [kg], $k_1=30$ [N/m], $k_2=10$ [N/m], $b_1=4$ [Ns/m], $b_2=2$ [Ns/m], $c_{nl}=5$ [N], and with the help of the LMI Toolbox of Matlab the following values are found that satisfy inequality (10):

$$\epsilon = 0.85, \quad N_T = \begin{bmatrix} -0.05 & 0 \end{bmatrix}, \quad \mathbb{S} = \begin{bmatrix} 0, & 1 \end{bmatrix},$$

$$L_T = \begin{bmatrix} -1.7 & 0 \\ 3.5 & -0.4 \\ -1.6 & -1 \\ 0 & -20 \end{bmatrix}, P = \begin{bmatrix} 12 & -0.6 & -8.1 & 0 \\ -0.6 & 3.7 & -1.8 & 0 \\ -8.1 & -1.8 & 13.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Finally, matrices D and E are [3]

$$D = \begin{bmatrix} I & 0_{3\times1} \\ 0_{1\times3} & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0_{3\times1} & 0_{3\times1} \\ 0 & 1 \end{bmatrix}.$$

In Figure 1, a simulation of the designed observer is shown, where $u = 5$ [N] and $w = 4\sin(t)+2$ [N] are used. It can be seen that the unknown input appears at 10 sec. and that the estimation error converges to zero independently of it.

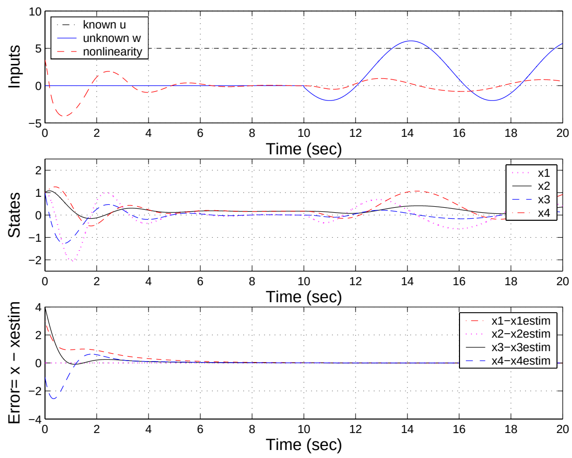


Fig. 1. Simulation of the UIO designed for (20) where (x_1, x_3) in $[m]$, and (x_2, x_4) in $[m/s]$.

6. CONCLUSIONS

A new constructive design of unknown input observers for a class of nonlinear systems has been introduced. The main result is an specialization of the general results of (Rocha-Cózatl and Moreno, 2004) to a class of systems with special structure. The methodology can also be applied to the class of systems that are transformable to this special structure. The characterization of that class of systems and the transformation conditions are important and interesting open questions. One important aspect of the proposed UIO is that the design is reduced to a system of matrix inequalities, that in many circumstances is linear in the design variables, so that the powerful numerical algorithms for solution of LMIs can be used in the design. A further interesting theoretical aspect is the unification of different design methods for the design of nonlinear observers for systems with or without unknown inputs using the dissipativity theory.

[3] where $0_{b\times c}$ is a zero matrix of dimension $b \times c$.

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