DISCRETE-TIME ADAPTIVE CONTROL FOR CONTINUOUS-TIME SYSTEMS USING 2-DELAY LIMITING-ZERO MODEL

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Abstract: A new method for designing a discrete-time adaptive control system for continuous-time system is presented. Using a new discrete-time model of the continuous-time system, called the 2-delay limiting-zero model, this method avoids the problem that the non-minimum phase zeros which arise in the case of rapid sampling of the continuous-time systems having the relative degree greater than one. In this paper, first, the motivation and principle of the 2-delay limiting-zero model of the continuous-time system is discussed. Next, the procedure of constructing the adaptive control system based on the 2-delay limiting-zero model is presented. Moreover, in order to illustrate the effectiveness of the proposed method, computer simulations are carried out for a detailed model of the electro-hydraulic servo system.

Keywords: adaptive control, discrete-time control, continuous time system, 2-delay input model, limiting zero, non-minimum phase system.

1. INTRODUCTION

In many tracking control problems, in which a continuous-time system is controlled by using a digital computer in discrete-time, the model following control is the most clear control strategy and has simple structure in adaptive control. However, unstable zeros (zeros on or outside the unit circle) arose from fast sampling of a continuous-time plant (Astrom et al., 1984; Hara et al., 1987) make the simple MRAC impossible even for above extended design methods.

In this paper, we propose a new discrete-time model, called 2-delay limiting-zero model, which is a combination of the limiting-zero model (Mizuno and Fujii, 1989) and 2-delay input model (Miyasato, 1992) and a design method of model reference adaptive control system based on the proposed model. If a 2-delay limiting-zero model is adopted as the discrete-time model of the continuous-time system, the discrete-time zeros can be assigned by fixed controller and the model reference adaptive controller is designed for the zero assigned discrete-time system, since the unstable zeros of the 2-delay limiting-zero model are known beforehand. Therefore, the model reference control is achieved with simple adaptive controller even for continuous-time system having the relative degree greater than one at a fast sampling

First, a short review of the relation between the continuous-time system and its 1-delay discrete-time form described by z-operator is given. Next, we give the brief description of the 2-delay input model and zero assignment by feed forward compensation (Miyasato, 1992; Mita et al., 1990). Then, we describe the limiting zeros of the 2-delay input model and the motivation of recommending the 2-delay limiting-zero model as a discrete-time model of the continuous-time system. In the main part of this paper, we propose a new design method of the MRACS based on the 2-delay limiting zero model. Moreover, in order to illustrate the effectiveness of the proposed method, computer simulations are carried out for a detailed model of the electro-hydraulic servo system (Masuda et al., 2001) using the simplified version of the proposed method.

2. CONTINUOUS-TIME SYSTEM AND DISCRETE-TIME SYSTEM

Consider a single-input, single-output continuous-time system described by

$$A_c(s)y(t) = B_c(s)u(t-\tau) \tag{1}$$

or

$$y(t) = G(s)u(t-\tau) \tag{2}$$

$$G(s) = B_c(s)/A_c(s) \tag{3}$$

where *G(s)* is a strictly proper rational transfer function and τ is a time delay. *y(t)* and *u(t)* denote the continuous-time system input and output, respectively. $A_c(s)$ and $B_c(s)$ are the polynomials in the differential operator, $s = d/dt$, of order *n* and *m* as follows.

$$A_c(s) = s^n + a_{Cn-1}s^{n-1} + .. + a_{C1}s + a_{C0} \tag{4}$$

$$B_c(s) = b_{Cm}s^m + b_{Cm-1}s^{m-1} + .. + b_{C1}s + b_{C0}\ ,\ b_{Cm} \neq 0,\quad m < n \tag{5}$$

1.1 1-Delay Discrete-Time Model of the Continuous-Time System and its Limiting-Zeros.

Assume that the continuous-time plant interconnected a zero-order hold device is sampled at interval T. Then, the exact discrete-time model of the above continuous-time system is obtained by standard z-transformation as follows.

$$A_d(z^{-1})y(i) = z^{-d}B_d(z^{-1})u(i) \tag{6}$$

where z^{-1} is a shift operator, such as $z^{-1}y(i) = y(i-1)$. $y(i)$ and $u(i)$ denote the discrete-time system input and output at sampling instance i, $A_d(z^{-1})$, $B_d(z^{-1})$ are polynomials in the unit shift operator z^{-1} of order *n* and *n-1* respectively.

$$A_d(z^{-1}) = 1 + a_{D1}z^{-1} + ... + a_{Dn-1}z^{n-1} + a_{Dn}z^{-n} \tag{7}$$

$$B_d(z^{-1}) = b_{D0} + b_{D1}z^{-1} + ... + b_{Dn-1}z^{-(n-1)} \tag{8}$$

The state space representation of the above system is also given by the following form.

$$x(i+1) = A_T x(i) + b_T u(i-d) \tag{9}$$

$$y(i) = c_T^T x(i) \tag{10}$$

where $x(i)$ is a discrete state, $A_T \in R^{n\times n}$, $b_T, c_T \in R^n$ are the system matrix and vectors of the discrete-time model with sampring period *T*. d is a time delay in the discrete-time representation ($d = \mathrm{int}[\tau/T] + 1$). As is well known, when a continuous time system is sampled at sampling period *T*, the continuous-time poles p_i are transformed to $e^{p_i T}$. On the other hand, a relation between the continuous-time zeros and the discrete-time zeros is more complicated. Moreover, from Eqs.(6) - (10), the discrete-time model has excess zeros compared with the continuous-time ones. However, a simple relationship between the zeros of $B_c(s)$ and $B_d(z)$ for the rapid sampling is given by Astrom et al. (1984). The result is as follows.

[Limiting-Zeros]

If $B_c(s)$ has the zeros $z_1,...,z_m$ then the zeros of $B_d(z^{-1})$ converge to the zeros (limiting-zeros) of the following polynomials as *T* tends to 0.

$B_n(z^{-1})$: the zeros of this polynomial are $e^{z_1 T},...,e^{z_m T}$

$B_L^r(z^{-1})$: the zeros of this polynomial only depend on the relative degree of the form (Astrom et al., 1984):

$$B_L^{r'}(z^{-1}) = b_1^r + b_2^r z^{-1} + ... + b_r^r z^{-(r-1)}, r = n - m \tag{11}$$

$$b_p^r = \sum_{j=1}^{p}(-1)^{p-j} j^r (r+1)!/[(r+1-p+j)!(p-j)!] \tag{12}$$

For *r=(n-m)* greater than one, the polynomial $B_L^r(z^{-1})$ has zeros outside (or on) the unit circle. Note that, the zeros of the polynomial Because of the above property of the conventional 1-delay discrete-time model of the continuous-time system, the MRACS are difficult to apply for practical control problems.

1.2 2-Delay Discrete-Time Model of the Continuous-Time System and its Zero Assignment.

In this section, we give the brief description of the 2-delay input discrete-time model of the continuous-time system and zero assignment by 2-delay input control (Miyasato, 1992). By utilizing the 2-delay sampling method, we can treat the single-input, single-output continuous-time system as the 2-inputs, single-output discrete-time system.

Let *(A,b,c,d)* ($A \in R^{n\times n},\ b, c \in R^n$, *d*>0) be a discrete-time model of (1) based on the usual sampling method with the sampling period $T/2$, that is,

$$G_{T/2}(z) = z^{-d}c^T(zI - A)^{-1}b \tag{13}$$

where d is a time delay in the discrete-time representation ($d = \mathrm{int}[\tau/(T/2)]$), and $G_{T/2}(z)$ is the pulse transfer function of the controlled system with sampling period $T/2$. For this controlled system, if the degree *n* and the time delay *d* are known, then the 2-delay sampling representation of the controlled system can be seen as the 2-inputs [$u_1(i), u_2(i)$], single-output [$y(i)$] system described as follows (Mita et al., 1990):

$$A(z^{-1})y(i) = z^{-d}(B_1(z^{-1})u_1(i) + B_2(z^{-1})u_2(i)) \quad (14)$$

$$A(z^{-1}) = 1 + \sum_{k=1}^{n} a_k z^{-k} = z^{-n}\det(zI - A^2) \quad (15)$$

$$B_1(z^{-1}) = \sum_{k=1}^{n} b_{1k} z^{-(k-1)} = z^{-(n-1)} c^T adj(zI - A^2)Ab \quad (16)$$

$$B_2(z^{-1}) = \sum_{k=1}^{n} b_{2k} z^{-(k-1)} = z^{-(n-1)} c^T adj(zI - A^2)b \quad (17)$$

Note that the real input to the continuous-time ystem $u(t)$ is described as follows.

$$\begin{aligned} u(t) &= u_1(iT)\ (iT \le t < iT + T/2) \\ &= u_2(iT)\ (iT + T/2 \le t < (i+1)T) \end{aligned} \quad (18)$$

The next assumption is introduced for the model.

[Assumption 1]

b_{11} or b_{21} is non-zero and $B_1(z^{-1})$ and $B_2(z^{-1})$ are relatively prime.

If the above assumption holds and when the controlled system is known, the equivalent zeros of the controlled system can be arbitrary assigned by the following 2-delay inputs control.

$$u_1(i) = G_1(z^{-1})v(i) \quad (19)$$

$$u_2(i) = G_2(z^{-1})v(i) \quad (20)$$

where

$$G_k(z^{-1}) = g_{k1} + \cdots + g_{kn-1} z^{-(n-2)}\ (k = 1,2) \quad (21)$$

are the polynomial solutions of the following identity (Diophantine identity).

$$G_{10}(z^{-1})B_1(z^{-1}) + G_{20}(z^{-1})B_2(z^{-1}) = B^*(z^{-1}) \quad (22)$$

$$G_1(z^{-1}) \equiv G_{10}(z^{-1}) + B_2(z^{-1})T(z^{-1}) \quad (23)$$

$$G_2(z^{-1}) \equiv G_{20}(z^{-1}) - B_1(z^{-1})T(z^{-1}) \quad (24)$$

where, $B^*(z^{-1})$

$$B^*(z^{-1}) = b_0^* + b_1^* z^{-1} \cdots + b_{n_{b^*}}^* z^{-n_{b^*}} \quad (25)$$

is any Hurwits polynomia and $T(z^{-1})$ is an arbitrary polynomial. From the assumption 1, $G_{k0}(z^{-1})$ $(k = 1,2)$ satisfying (22), always exist. Moreover, by using the 2-delay inputs (19) and (20), all equivalent zeros from the signal $v(i)$ to the output $y(i)$ are assigned to the zeros of the polynomial $B^*(z^{-1})$.

However, in case of adaptive control, all parameters in polynomials $B_1(z^{-1}), B_2(z^{-1})$ should be estimated and the Diophantine identity should also be solved based on the estimated parameters to assign equivalent zeros. In this case, the numerical difficulty may occur when the estimated polynomials has common factors.

If the polynomials $B_1(z^{-1}), B_2(z^{-1})$ of the 2-delay input model have some special properties, the procedure of assigning equivalent zeros may become more easy to implement. From this point of view, we investigate the limiting zeros of the 2-delay input model of the continuous-time system.

1.2 Limiting-Zeros of the 2-Delay Discrete-Time Model of the Continuous-Time System.

Based on the limiting-zeros of the 1-delay discrete-time model of the continuous-time system, we have obtained the following result about limiting-zeros of the 2-delay discrete-time model.

[Limiting-Zeros of 2-delay input model]

If $B_c(s)$ has the zeros $z_1, \ldots, z_m$ then the zeros of $B_1(z^{-1}), B_2(z^{-1})$ converge to the zeros (2-delay limiting-zeros) of the following polynomials as *T* tends to 0 respectively.

$B_{iN}(z^{-1})$ $(i = 1,2)$: the zeros of this *m*th order polynomial are $e^{z_1 T'}, \ldots, e^{z_m T'}$ (T' : function of T)

$B_{iL}^r(z^{-1})$ $(i = 1,2)$: the zeros of this (*r-1*)th order polynomials only depend on the relative degree.

$$B_{1L}^r(z^{-1}) = \frac{1}{r!}\left(\frac{T}{2}\right)^r \sum_{k=0}^{r-1} b_{1k} z^{-k} \quad (26)$$

$$B_{2L}^r(z^{-1}) = \frac{1}{r!}\left(\frac{T}{2}\right)^r \sum_{k=0}^{r-1} b_{2k} z^{-k} \quad (27)$$

$$b_{1k} = \sum_{l=0}^{k} \{(2l+2)^r - (2l+1)^r\} \frac{(-1)^{k-l} r!}{(k-l)!(r+l-k)!} \quad (28)$$

$$b_{2k} = \sum_{l=0}^{k} \{(2l+1)^r - (2l)^r\} \frac{(-1)^{k-l} r!}{(k-l)!(r+l-k)!} \quad (29)$$

The fact that the limiting-zeros of the system only depend the relative degrees of the continuous-time system, is the key property of introducing the 2-delay limiting-zero models.

The above result is very simple and clear, but the convergence speed and the physical characteristics of the limiting-zeros were not sufficiently analysed (Astrom et al., 1984; Hara et al., 1987).

If the convergence speed of the excess zeros to the limiting-zeros is faster than the convergence speed of the other poles and zeros to corresponding discrete-time ones, then $B_1(z^{-1}), B_2(z^{-1})$ can be expressed for the rapid sampling as (Mizuno and Fujii, 1989)

$$B_i(z^{-1}) = B_{iN}(z^{-1}) B_{iL}(z^{-1})\ (i = 1,2) \quad (30)$$

where $B_{iN}(z^{-1})$ has the unknown zeros which converge to $e^{z_1 T'}, \ldots, e^{z_m T'}$ (z_i denote the continuous-time zeros) as the sampling period tents to *0*, and

$B_{iL}(z^{-1})$ has only the known 2-delay limiting-zeros (sometimes unstable). If this conjecture is not true, only the trivial result that all poles and zeros of the model become *1* or *-1*, is obtained. However, the convergence property is confirmed by numerical computation for many different types of continuous-time system and is theoretically proven for 1-delay simple case (Mizuno and Fujii,1987). This is the motivation of proposing the limiting-zero model of the plant.

3. CONTROLLER DESIGN USING 2-DELAY LIMITING-ZERO MODEL

The idea mentioned above leads to the following new representation of the discrete-time model.
The control problem considered here is to determine a suitable control input $u(t)$ such that the discrete-time asymptotic model-following is achieved for any continuous-time system with stable continuous-time zeros.

Assume that the relative degree of the continuous-time system is known and the sampling interval is short compared with the time constant of the plant, then the plant can be described as follows.

$$A(z^{-1})y(i) = z^{-d}(B_{1N}(z^{-1})B_{1L}(z^{-1})u_1(i) + B_{2N}(z^{-1})B_{2L}(z^{-1})u_2(i)) + z^{-d}(B_{1e}(z^{-1})u_1(i) + B_{2e}(z^{-1})u_2(i)) \quad (31)$$

$$B_{ie}(z^{-1}) = B_i(z^{-1}) - B_{iN}(z^{-1})B_{iL}(z^{-1}) \ (i=1,2) \quad (32)$$

where the polynomial $B_{ie}(z^{-1})$ $(i=1,2)$ is a residual polynomial which converges to zero as the sampling period T tends to 0. Based on this property, we will ignore the polynomial $B_{ie}(z^{-1})$ and modelled the controlled system by the approximate model called "2-delay limiting-zero model" of the form:

$$A(z^{-1})y(i) = z^{-d}(B_{1N}(z^{-1})B_{1L}(z^{-1})u_1(i) + B_{2N}(z^{-1})B_{2L}(z^{-1})u_2(i)) \quad (33)$$

If all continuous-time zeros lie in left half of the s-plane, this model has stable zeros $B_{iN}(z^{-1})$ and stable/unstable zeros of $B_{iL}(z^{-1})$.

In almost all cases, the 2-delay limiting-zero model of the system has unstable zeros of $B_{iL}(z^{-1})$, the equivalent zeros of the controlled system should be assigned to stable ones using the following feed forward compensators.

$$B_{1N}(z^{-1})u_1(i) = G_1(z^{-1})v(i) \quad (34)$$

$$B_{2N}(z^{-1})u_2(i) = G_2(z^{-1})v(i) \quad (35)$$

where $G_i(z^{-1})$ $(i=1,2)$ is the solution of the following polynomial identity similar to Eq.(22).

$$G_{10}(z^{-1})B_{1L}(z^{-1}) + G_{20}(z^{-1})B_{2L}(z^{-1}) = B^*(z^{-1}) \quad (36)$$

The above identity can be solved without numerical difficulty, because the polynomial $B_{iL}(z^{-1})$ is known and has simple parameters as shown in Eqs. (26)-(29). If the polynomial $B^*(z^{-1})$ is chosen as a Hurwits one, the model reference control law of the signal $v(i)$ can be synthesized as follows:

$$L(z^{-1})B^*(z^{-1})v(i) = A^*(z^{-1})y_m(i+d) - D(z^{-1})y(i) \quad (37)$$

where $y_m(i)$ is a desired output to be followed and

$$L(z^{-1}) = 1 + \cdots + l_{d-1}z^{-(d-1)} \quad (38)$$

$$D(z^{-1}) = d_0 + \cdots + d_{n-1}z^{-(n-1)} \quad (39)$$

are the solutions of the following Diophantine equation.

$$A^*(z^{-1}) = A(z^{-1})L(z^{-1}) + z^{-d}D(z^{-1}) \quad (40)$$

and $A^*(z^{-1})$

$$A^*(z^{-1}) = 1 + a_1^* z^{-1} + .. + a_{n_a^*}^* z^{-n_{a^*}} \quad (41)$$

is any Hurwits polynomial.

When the parameters of the controlled system are unknown, first, we estimate the unknown parameters then construct the adaptive controller based on the estimated parameters. For simplicity, we describe the controlled system in the following vector form.

[Controlled system]

$$y(i) = \theta^T \xi(i-1) \quad (42)$$

where

$$\theta^T = (a_1, \cdots, a_n, b_{10}, \cdots, b_{1m-1}, b_{20}, \cdots b_{2m-1}) \quad (43)$$

$$\xi(i-1) = (y(i-1), \cdots, y(i-n), B_{1L}(z^{-1})u_1(i-d), \cdots, B_{1L}(z^{-1})u_1(i-d-m+1), B_{2L}(z^{-1})u_2(i-d), \cdots, B_{2L}(z^{-1})u_2(i-d-m+1))^T \quad (44)$$

We consider the adaptive identifier, the adaptive law, and the control law as follows.

[Adaptive identifier]

$$\hat{y}(i) = \hat{\theta}^T(i-1)\xi(i-1) \quad (45)$$

$\hat{\theta}(i-1)$ is a current estimate of θ at the time instant $t = (i-1)T$.

[Adaptive law]

$$\hat{\theta}(i) = \hat{\theta}(i-1) + \frac{P(i-1)\xi(i-1)}{1 + \xi^T(i-1)P(i-1)\xi(i-1)}\varepsilon(i) \quad (46)$$

$$\varepsilon(i) = y(i) - \hat{y}(i) \quad (47)$$

$$P(i) = \frac{1}{\lambda_1(i)}\left(P(i-1) - \frac{\lambda_2(i)P(i-1)\xi(i-1)\xi^T(i-1)P(i-1)}{\lambda_1(i) + \lambda_2(i)\xi^T(i-1)P(i-1)\xi(i-1)}\right) \quad (48)$$

where $P(0) = P(0)^T > 0, 0 < \lambda_1(i) \le 1$, $0 \le \lambda_2(i) < 2$

[Control law]

$$\hat{B}_{1N}(i - d_c, z^{-1})u_1(i) = G_1(z^{-1})v(i) \quad (49)$$

$$\hat{B}_{2N}(i - d_c, z^{-1})u_2(i) = G_2(z^{-1})v(i) \quad (50)$$

$$\hat{L}(i - d_c, z^{-1})B^*(z^{-1})v(i) = A^*(z^{-1})y_m(i + d) - \hat{D}(i - d_c, z^{-1})y(i) \quad (51)$$

where $d_c (\ge 0)$ is a computational delay.
$\hat{L}(i - d, z^{-1})$ and $\hat{D}(i - d, z^{-1})$ are determined such that

$$\hat{A}(i, z^{-1})\hat{L}(i, z^{-1}) + z^{-d}\hat{D}(i, z^{-1}) = A^*(z^{-1}) \quad (52)$$

$$\hat{L}(i, z^{-1}) = 1 + \cdots + \hat{l}_{d-1}(i)z^{-(d-1)} \quad (53)$$

$$\hat{D}(i, z^{-1}) = \hat{d}_0(i) + \cdots + \hat{d}_{n-1}(i)z^{-(n-1)} \quad (54)$$

where $\hat{A}(i, z^{-1})$ and $\hat{B}_i(i, z^{-1})$ $(k = 1,2)$ are

$$\hat{A}(i, z^{-1}) = 1 + \sum_{k=1}^{n} \hat{a}_k(i)z^{-k} \quad (55)$$

$$\hat{B}_{1N}(i, z^{-1}) = \sum_{k=0}^{m-1} \hat{b}_{1k}(i)z^{-k} \quad (56)$$

$$\hat{B}_{2N}(i, z^{-1}) = \sum_{k=0}^{m-1} \hat{b}_{2k}(i)z^{-k} \quad (57)$$

Fig 1 shows the structure of model reference adaptive control system based on 2-delay limiting-zero model

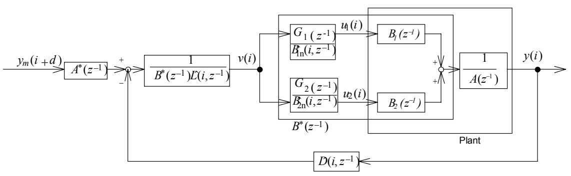

Fig.1 Structure of proposed adaptive control system

4. ROBUSTNESS OF THE CONTROL SYSTEM

Although there is a difference in degree, any model is an approximation of a plant. We argue it advisable to avoid the ARMA model described by z-operator which is ill-suited to the design of discrete-time adaptive control system for continuous-time systems (Fujii and Mizuno, 1981; Goodwin et al., 1986). We recommend the 2-delay limiting-zero model as a plant model. However, the robustness of the adaptive control system designed using the 2-delay limiting-zero model of the plant should be investigated. Such robustness can be evaluated by both theoretical analysis and the control performance of the adaptive controller designed for the model. From the theoretical point of view, there exists the key property that the difference between the exact discrete-time model and the 2-delay limiting-zero model converges to zero as the sampling interval tends to 0. Thus, it concludes that the equilibrium point of the adaptive control system designed by the 2-delay limiting-zero model is stable if the signal in the control system is persistently exiting (P.E.). From the practical point of view, this scheme will be shown to work well for many different types of plants.

5. EVALUATION OF PROPOSED METHOD

In order to investigate the effectiveness of the proposed methods, first, computer simulations are performed for different types of continuous-time plants having the relative degree greater than one. The results are omitted by limitation of space. From the simulation results for simple plants, it is confirmed that the plant can be well modelled and controlled by the 2-delay limiting zero model based controller.

Next, as a practical situation in the application, we apply this method to a detailed non-linear model of an electro-hydraulic servo system as shown in Fig. 2. The control objective is the position control of the mechanical system. The model is originally proposed by the adaptive and learning control research group in the society of instrument and control engineers (SICE) for benchmarking many types of adaptive control techniques (Masuda et al., 2001).

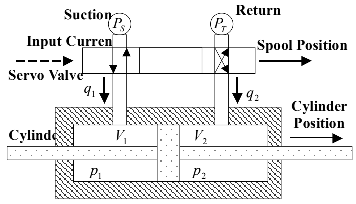

Fig. 2 Schematic diagram of the electro-hydraulic servo system

In this case, it can be assumed that the continuous-time characteristics of this system is roughly described by the following model:

$$y(t) = \frac{K_c}{s^2}u(t) \quad (58)$$

where *y(t)* denotes the position of the cylinder and *u(t)* is the current for servo valve. This model leads to the special form of the 2-delay limiting-zero model as follows.

$$A(z^{-1})y(i) = Kz^{-1}\left(B_{1L}(z^{-1})u_1(i) + B_{2L}(z^{-1})u_2(i)\right) \quad (59)$$

$$A(z^{-1}) = 1 - 2z^{-1} + z^{-2} \quad (60)$$

$$B_{1L}(z^{-1}) = \frac{1}{2}\left(\frac{T}{2}\right)^2 (3 + z^{-1}), B_{2L}(z^{-1}) = \frac{1}{2}\left(\frac{T}{2}\right)^2 (1 + 3z^{-1}) \quad (61)$$

The above model has only one unknown parameter which should be estimated in case of adaptive control. The unknown gain can be estimated as follows.

[Simplified Model]

$$y'(i) = Ku'(i - 1) \quad (62)$$

$$y'(i) = A(z^{-1})y(i) \quad (63)$$

$$u'(i) = B_{1L}(z^{-1})u_1(i) + B_{2L}(z^{-1})u_2(i) \quad (64)$$

[Parameter Estimation]

$$\varepsilon(i) = y'(i) - \hat{K}(i - 1)u'(i - 1) \ (2.11a) \quad (65)$$

$$\hat{K}(i) = \hat{K}(i-1) + \frac{P(i-1)u'(i-1)}{1 + u'(i-1)P(i-1)u'(i-1)}\varepsilon(i) \quad (66)$$

$$P(i) = \frac{1}{\lambda_1(i)}\left(P(i-1) - \frac{\lambda_2(i)P(i-1)u'(i-1)u'(i-1)P(i-1)}{\lambda_1(i) + \lambda_2(i)u'(i-1)P(i-1)u'(i-1)} \right) (67)$$

In the implementation of the adaptive controller, we introduce the extended feed forward compensator which improve the input property of the 2-delay input adaptive control system (Mizuno and Sato, 1998,1999).

Only typical simulation results for different cylinder load are shown. In these simulations, the set points are given by Fig.3.

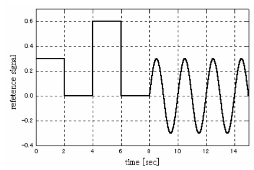

Fig. 3 Set points for control

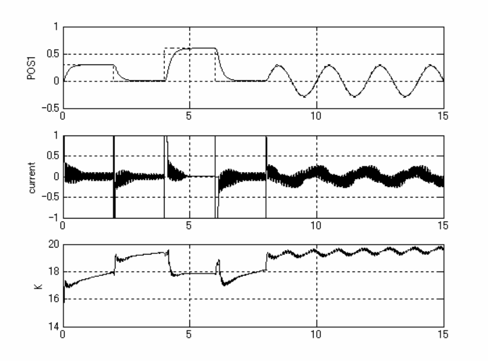

Fig. 4 Simulation result for light load

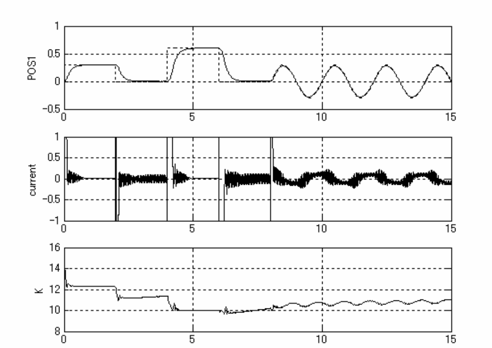

Fig. 5 Simulation result for middle load

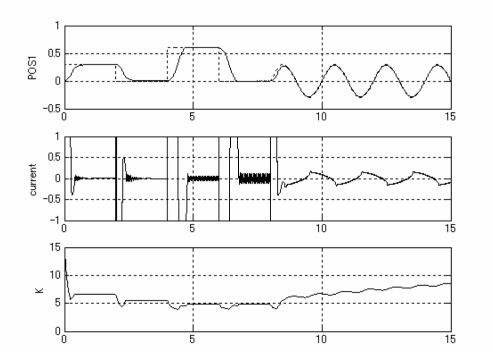

Fig. 6 Simulation result for heavy load

In Fgs. 4-6, the upper trace shows the output, the middle and the lower show input and estimated parameter respectively. From these figures, it can be seen that the proposed adaptive controller works well for different cylinder load by adjusting only one parameter.

6. CONCLUSIONS

We have presented the discrete-time adaptive control system using 2-delay limiting-zero model, especially to handle the continuous-time system having the relative degree greater than one. The effectiveness of the algorithm was confirmed from the simulation studies for simple controlled system and the detailed model of electro-hydraulic servo system. In the later case, the proposed method has a very simple structure with only one estimated parameter. So the adaptive controller is easily implemented with a microprocessor.

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