NONLINEAR H_∞ FOR SPACECRAFT ATTITUDE CONTROL

Ho-Nien Shou

Department of
Aviation & Communication Electronics
Air Force Institute of Technology

Ying-Wen Jan

Mechanical Engineering Department
National Space Program Office

Abstract: This paper presents a nonlinear H_∞ state feedback attitude control design method for spacecraft large angle maneuvers, which is subject to moment-of-inertia (MOI) uncertainty. Moreover, the operation of attitude maneuver is affected by external disturbances. The attitude control design thus employs a nonlinear H_∞ method to achieve stability and robustness so that both MOI uncertainty and external disturbances can be accounted for. In the paper, a solution for robust spacecraft attitude control is conjectured and shown to satisfy the nonlinear H_∞ criterion. This results in a nonlinear H_∞ control law that is capable of robustly stabilizing the maneuver in the presence of external disturbance and spacecraft inertia uncertainty. The simulation results are presented to demonstrate the effectiveness of the proposed design method.

Keywords: moment-of-inertia uncertainty; nonlinear robust control; large angle attitude maneuver; external disturbances.

1. INTRODUCTION

The development of a robust controller for satellite attitude control has become an important engineering issue, especially for large angle maneuver cases. Two major problems encountered during spacecraft attitude control are the disturbances from space environment and the perturbation of its moment-of-inertia (MOI) variation. For example, it is quite often to confront the variation of MOI in a thruster-based control system. The center of gravity will change while the amount of propellant is decreasing. This results in the change of MOI. In this paper, the model of a rigid spacecraft, with disturbance inputs and MOI uncertainty, controlled by three control torques is considered.

The large angle attitude control of spacecraft has received extensive attention in recent decades. One characteristic to this kind of attitude control problem is that nonlinear attitude dynamics is involved, restricting the use of linearized control design methods. Existing nonlinear control design methods for spacecraft attitude control include the use of sliding mode control (Cavallo, *et al*., 1996), mode reference adaptive control (Singh, 1987), quaternion feedback (Joshi, *et al*., 1995; Wie, *et al*., 1985), linear matrix inequality (LMI)(Show, *et al*., 2003), etc. Recently, nonlinear H_∞ control methods have also been proposed (Dalsmo, *et al*., 1997; Kang, 1995; Wu, *et al*., 1999; Yang, *et al*., 2000) to address the attitude control problem. This paper proposes a more general Lyapunov (or Hamilton-Jacobi) function to account for the stability and robustness of the attitude control problems. Unlike the results in (Dalsmo, *et al*., 1997; Wu, *et al*., 1999), the H_∞ controller based on the proposed approach is nonlinear.

2. REVIEW OF NONLINEAR H_∞ CONTROL THEORY

In this section, results in nonlinear H_∞ control are briefly reviewed. Consider a nonlinear system of the form

$$\underline{\dot{x}} = f(\underline{x}) + g(\underline{x})\underline{d} \quad (1a)$$

$$\underline{z} = h(\underline{x}) \quad (1b)$$

where $\underline{x} \in R^n$ is the state vector, $\underline{d}$ is the exogenous disturbance, and $\underline{z}$ is the performance output signal. Assume that $f(\underline{x})$, $g(\underline{x})$, and $h(\underline{x})$ are smooth functions and $\underline{x} = \underline{x}_0$ is the equilibrium point of the system, i.e.,

$$f(\underline{x}_0) = 0 \quad (2a)$$

$$h(\underline{x}_0) = 0 . \quad (2b)$$

The nonlinear system is said to have an L_2 gain less than γ if the following relationship holds

$$\int_0^\infty \underline{z}^T(t)\underline{z}(t)dt < \gamma^2 \int_0^\infty \underline{d}^T(t)\underline{d}(t)dt \quad (2c)$$

for any input $\underline{d} \in L_2[0,\infty)$. The L_2 gain characterizes the relation between the disturbance input energy and performance output energy. A small γ can be interpreted to have a disturbance attenuation property. The following lemma provides a test criterion for the disturbance attenuation property (Isidori, *et al*., 1992; Van der Schaft,1992).

LEMMA 1 *The nonlinear system has an* L_2 *gain less than* γ *if there exists a* C^1 *function* $V: R^n \to R^+$ *with* $V(\underline{x}_0) = 0$ *such that*

$$\left(\frac{\partial V}{\partial \underline{x}}\right)^T f(\underline{x}) + \frac{1}{2\gamma^2}\left(\frac{\partial V}{\partial \underline{x}}\right)^T g(\underline{x})g^T(\underline{x})\left(\frac{\partial V}{\partial \underline{x}}\right) + \frac{1}{2}h^T(\underline{x})h(\underline{x}) < 0$$

where $\left(\frac{\partial V}{\partial \underline{x}}\right)$ *is the partial derivative of* $V(\underline{x})$, *or*

$$\left(\frac{\partial V}{\partial \underline{x}}\right)^T = \begin{bmatrix} \frac{\partial V}{\partial x_1} & \frac{\partial V}{\partial x_2} & \cdots & \frac{\partial V}{\partial x_n} \end{bmatrix}$$

The analysis results can be extended for controller synthesis. Consider the nonlinear control design problem in which the system is described by

$$\begin{aligned} \underline{\dot{x}} &= f(\underline{x}) + g_1(\underline{x})\underline{d} + g_2(\underline{x})\underline{u} \\ \underline{z} &= \begin{bmatrix} h_1(\underline{x}) \\ \rho\underline{u} \end{bmatrix} \end{aligned} \quad (3)$$

where $\underline{u}$ is the control signal and ρ is a weighting scalar for the control signal. It is desired to synthesize a control law, such that the resulting closed-loop system is asymptotically stable and the L_2 gain from $\underline{d}$ to $\underline{z}$ is less than γ. The following celebrated lemma (Isidori, *et al*., 1992) provides a nonlinear H_∞ control law design method.

LEMMA 2 *The closed-loop system has an* L_2 *gain less than* γ *if there exists a positive* C^1 *function with* $V(\underline{x}_0) = 0$ *satisfying the following Hamilton-Jacobi partial differential inequality (HJPDI)*

$$\begin{aligned} H_\gamma &= \left(\frac{\partial V}{\partial \underline{x}}\right)^T f(\underline{x}) \\ &+ \frac{1}{2}\left(\frac{\partial V}{\partial \underline{x}}\right)^T \left(\frac{1}{\gamma^2} g_1(\underline{x})g_1^T(\underline{x}) - \frac{1}{\rho^2} g_2(\underline{x})g_2^T(\underline{x})\right) \\ &\bullet \left(\frac{\partial V}{\partial \underline{x}}\right) + \frac{1}{2}h_1^T(\underline{x})h_1(\underline{x}) < 0 \end{aligned} \quad (4)$$

Furthermore, when the system is zero-state detectable, the closed-loop system is asymptotically stable and indeed a stabilizing feedback control can be constructed

$$\underline{u} = -\frac{1}{\rho^2} g_2^T(\underline{x})\left(\frac{\partial V}{\partial \underline{x}}\right)$$

to satisfy the L_2 *gain requirement.*

It is clear that the construction of the Hamilton-Jacobi function $V(\underline{x})$ constitutes a crucial step in control performance and control law synthesis.

3. SPACECRAFT DYNAMICS AND DESIGN FORMULATION

The equations of motion of the spacecraft are described as follows (Show, *et al*., 2003; Dalsmo, *et al*., 1997; Kang, 1995; Wu, *et al*., 1999)

$$\begin{aligned} J\underline{\dot{\omega}} &= -[\underline{\omega}\times]J\underline{\omega} + \underline{u} + \underline{d} \\ \underline{\dot{\varepsilon}} &= \frac{1}{2}\eta\underline{\omega} + \frac{1}{2}[\underline{\varepsilon}\times]\underline{\omega} \\ \dot{\eta} &= -\frac{1}{2}\underline{\varepsilon}^T\underline{\omega} \end{aligned} \quad (5)$$

where J is the MOI matrix, $\underline{\omega}$ is the angular velocity of the spacecraft, $\underline{u}$ is the control torque , $\underline{d}$ is the environmental disturbance torque, and $\begin{bmatrix} \underline{\varepsilon}^T & \eta \end{bmatrix}^T$ constitutes the quaternion. Note that the dynamical equation has two equilibrium points $\underline{\omega} = 0$, $\underline{\varepsilon} = 0$, and $\eta = \pm 1$. These two equilibrium points, however, correspond to the same attitude as the quaternion is a redundant representation. Indeed, it is known that the quaternion satisfies

$$\underline{\varepsilon}^T\underline{\varepsilon} + \eta^2 = 1$$

In the following, the notation $[\underline{\omega}\times]$ is used to represent the 3 × 3 skew symmetric matrix formed from the vector $\underline{\omega}$. More precisely, let $\underline{\omega} = [\omega_1 \quad \omega_2 \quad \omega_3]^T$, then

$$[\underline{\omega}\times] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

Likewise,

$$[\underline{\varepsilon}\times] = \begin{bmatrix} 0 & -\varepsilon_3 & \varepsilon_2 \\ \varepsilon_3 & 0 & -\varepsilon_1 \\ -\varepsilon_2 & \varepsilon_1 & 0 \end{bmatrix}$$

Let $\underline{x} = [\underline{\omega} \quad \underline{\varepsilon} \quad \eta]^T$, (5) can be written in the matrix form of

$$\underline{\dot{x}} = f(\underline{x}) + g_1(\underline{x})\underline{d} + g_2(\underline{x})\underline{u} \quad (6)$$

where

$$f(\underline{x})=\begin{bmatrix}-J^{-1}[\underline{\omega}\times]J\underline{\omega}\\ \frac{1}{2}\eta\underline{\omega}+\frac{1}{2}[\underline{\varepsilon}\times]\underline{\omega}\\ -\frac{1}{2}\underline{\varepsilon}^T\underline{\omega}\end{bmatrix},\ g_1(\underline{x})=\begin{bmatrix}J^{-1}\\0\\0\end{bmatrix},$$

$$g_2(\underline{x})=\begin{bmatrix}J^{-1}\\0\\0\end{bmatrix}.$$

The true spacecraft inertia is denoted by

$$J=(J_o+\delta_J)$$

where J_o is the measured value of spacecraft inertia, and δ_J is the measurement error. Rewrite (5) to obtain the equations of motion of the spacecraft with uncertainty error as

$$\begin{aligned}(J_o+\delta_J)\dot{\underline{\omega}}&=-[\underline{\omega}\times](J_o+\delta_J)\underline{\omega}+\underline{u}+\underline{d}\\ \dot{\underline{\varepsilon}}&=\frac{1}{2}\eta\underline{\omega}+\frac{1}{2}[\underline{\varepsilon}\times]\underline{\omega}\\ \dot{\eta}&=-\frac{1}{2}\underline{\varepsilon}^T\underline{\omega}\end{aligned}\tag{7}$$

or in the matrix form of

$$\begin{aligned}\dot{\underline{x}}=&\ f(\underline{x})+\delta f(\underline{x})+[g_1(\underline{x})+\delta g_1(\underline{x})]\underline{d}\\ &+[g_2(\underline{x})+\delta g_2(\underline{x})]\underline{u}\end{aligned}\tag{8}$$

The perturbation terms $\delta f(\underline{x})$, $\delta g_1(\underline{x})$, and $\delta g_2(\underline{x})$ are derived as follows:

Assume δ_J is sufficiently small such that the zeroth-order term from the binominal expansion of $(J_o+\delta_J)^{-1}$ is valid. Moreover, assume the scalar Δ_J is the largest variation percentage of MOI, and then we can obtain following approximations:

$$\delta_J\le\Delta_J J_o\tag{9a}$$

$$J_o^{-1}(1-\Delta_J)\le(J_o+\delta_J)^{-1}\le J_o^{-1}(1+\Delta_J)\tag{9b}$$

use the following expressions to denote (9b):

$$(J_o+\delta_J)^{-1}\le J_o^{-1}(1\pm\Delta_J)\tag{10}$$

Then, the uncertainty terms in (8) can be bounded by the following relations:

$$\begin{aligned}\delta f\le\Delta f&=\begin{bmatrix}-J_o^{-1}(1\pm\Delta_J)^2[\underline{\omega}\times]J_o\underline{\omega}+J_o^{-1}[\underline{\omega}\times]J_o\underline{\omega}\\0\\0\end{bmatrix}\\ &\approx\begin{bmatrix}\mp2\Delta_J J_o^{-1}[\underline{\omega}\times]J_o\underline{\omega}\\0\\0\end{bmatrix}\end{aligned}\tag{11a}$$

$$\delta g_1\le\pm\Delta_J g_1\tag{11b}$$

$$\delta g_2\le\pm\Delta_J g_2\tag{11c}$$

In the attitude control design, it is desired that the system is stable and the excursions of angular rate and control input are minimized. This motivates the use of the following performance output signal to be minimized

$$\underline{z}=\begin{bmatrix}h_1(\underline{x})\\ \rho\underline{u}\end{bmatrix}\tag{12a}$$

where ρ is a weighting scalar for the control signal. The function $h_1(\underline{x})$ is assumed to be of the following form

$$h_1(\underline{x})=\begin{bmatrix}\sqrt{\rho_1}\underline{\omega}\\ \sqrt{\rho_2}\underline{\varepsilon}\end{bmatrix}\tag{12b}$$

for some positive scalars ρ_1 and ρ_2. Clearly, it is desired to have $h_1(\underline{x})$ or $\underline{z}$ small so that the three-axis attitude control performance as characterized by the angular rate and attitude error can be kept as small as possible. Also, the control energy is accounted for in the problem formulation.

In summary, the equations (8), (12a) and (12b) can be used to formulate the attitude control problem with following relations:

$$f(\underline{x})=\begin{bmatrix}-J_o^{-1}[\underline{\omega}\times]J_o\underline{\omega}\\ \frac{1}{2}\eta\underline{\omega}+\frac{1}{2}[\underline{\varepsilon}\times]\underline{\omega}\\ -\frac{1}{2}\underline{\varepsilon}^T\underline{\omega}\end{bmatrix},\tag{13a}$$

$$g_1(\underline{x})=\begin{bmatrix}J_o^{-1}\\0\\0\end{bmatrix}\tag{13b}$$

$$g_2(\underline{x})=\begin{bmatrix}J_o^{-1}\\0\\0\end{bmatrix}\tag{13c}$$

$$h_1(\underline{x})=\begin{bmatrix}\sqrt{\rho_1}\underline{\omega}\\ \sqrt{\rho_2}\underline{\varepsilon}\end{bmatrix}\tag{13d}$$

$$\Delta f\approx\begin{bmatrix}\mp2\Delta_J J_o^{-1}[\underline{\omega}\times]J_o\underline{\omega}\\0\\0\end{bmatrix}\tag{13e}$$

$$\Delta g_1=\pm\Delta_J g_1\tag{13f}$$

$$\Delta g_2=\pm\Delta_J g_2\tag{13g}$$

The design objective is to find a control $\underline{u}$ under uncertainty and disturbance such that the system is stable and the L_2 gain from $\underline{d}$ to $\underline{z}$ is less than γ.

4. NONLINEAR H_∞ CONTROLLER SYNTHESIS

Based on the analysis of above section, we will proceed to solve the design problem for the uncertain plant (8). The objective of control design is to find a smooth control law for $\underline{u}$ such that the system is stable and $\|z\|_2/\|d\|_2\le\gamma$. From Lemma 2, it suffices to find a $V(x)\ge0$ satisfying (4).

Consider the following candidate function

$$\begin{aligned}V(\underline{x})=&\ a\underline{\omega}^T J_o\underline{\omega}+2(b_1+b_2\eta)\underline{\varepsilon}^T J_o\underline{\omega}\\ &+2(1-\eta)(c_1+c_2\eta)\end{aligned}\tag{14}$$

for some positive constant a, b_1, b_2, c_1 and c_2. Note that $[\underline{\omega}\quad\underline{\varepsilon}\quad\eta]^T=[0\quad0\quad1]^T\ \Rightarrow\ V(\underline{x}_0)=0$.

It is seen that the equilibrium point $\begin{bmatrix}\underline{\omega} & \underline{\varepsilon} & \eta\end{bmatrix}^T = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$ corresponds to $\underline{x}_0$ in Lemma 1. In what follows, we will show that the candidate function (14) will satisfy the H_∞ criterion described in Lemma 2. Moreover, the condition of $\eta \geq 0$ will be imposed to ensure that the candidate function will always converge to $V(\underline{x}_0) = 0$. Using the identity $2(1-\eta) = \underline{\varepsilon}^T \underline{\varepsilon} + (1-\eta)^2$, the candidate function can be rewritten as

$$V(\underline{x}) = \begin{bmatrix}\underline{\omega}^T & \underline{\varepsilon}^T & \eta - 1\end{bmatrix} \times \begin{bmatrix} aJ_o & (b_1 + b_2\eta) J_o & 0 \\ (b_1 + b_2\eta) J_o & (c_1 + c_2\eta) I & 0 \\ 0 & 0 & (c_1 + c_2\eta) \end{bmatrix} \begin{bmatrix} \underline{\omega} \\ \underline{\varepsilon} \\ \eta - 1 \end{bmatrix}$$

where I denotes the identity matrix with appropriate dimension. The function $V(\underline{x})$ is positive definite when $a > 0$ and

$$(c_1 + c_2\eta) I - \frac{1}{a}(b_1 + b_2\eta)^2 J_o > 0 \tag{15}$$

for all $\eta \geq 0$.

The nonlinear H_∞ control design involves (4). Note that

$$\frac{\partial V}{\partial \underline{x}} = 2 \begin{bmatrix} aJ\underline{\omega} + (b_1 + b_2\eta) J_o \underline{\varepsilon} \\ (b_1 + b_2\eta) J_o \underline{\omega} \\ b_2 \underline{\omega}^T J_o \underline{\varepsilon} - c_1 + c_2(1 - 2\eta) \end{bmatrix}$$

This then gives

$$\left(\frac{\partial V}{\partial \underline{x}}\right)^T f(\underline{x}) = \left(c_1 + c_2(2\eta - 1)\right) \underline{\varepsilon}^T \underline{\omega} + \underline{\omega}^T \left((b_1 + b_2\eta)(\eta I + [\underline{\varepsilon}\times]) J_o - b_2 J_o \underline{\varepsilon}^T \underline{\varepsilon}\right) \underline{\omega} \tag{16}$$

and

$$\left(\frac{\partial V}{\partial \underline{x}}\right)^T \Delta f(\underline{x}) = \underline{\omega}^T \left(\pm 4(b_1 + b_2\eta) \Delta_J [\underline{\varepsilon}\times] J_o\right) \underline{\omega} \tag{17}$$

In the above derivation, the following equality is used

$$\underline{\omega}^T [\underline{\varepsilon}\times] J \underline{\omega} = -\underline{\varepsilon}^T [\underline{\omega}\times] J \underline{\omega} \tag{18}$$

The HJPDI corresponding to (4) is obtained by replacing f, g_1, and g_2 with $f + \Delta f$, $g_1 + \Delta g_1$, and $g_2 + \Delta g_2$, respectively.

$$\begin{aligned} H_\gamma \leq\ & \underline{\omega}^T \{(b_1 + b_2\eta)(\eta I + [\underline{\varepsilon}\times]) J_o - b_2 J_o \underline{\varepsilon}\underline{\varepsilon}^T \\ & + 2a^2\left(\frac{A}{\gamma^2} - \frac{B}{\sigma^2}\right) + \frac{1}{2}\rho_1 \pm 4(b_1 + b_2\eta)\Delta_J [\underline{\varepsilon}\times] J_o\} \underline{\omega} \\ & + \underline{\varepsilon}^T \{2(b_1 + b_2\eta)^2 (\frac{A}{\gamma^2} - \frac{B}{\sigma^2}) + \frac{1}{2}\rho_2\} \underline{\varepsilon} \\ & + \underline{\varepsilon}^T \{c_1 - c_2 + 2c_2\eta + 4a(b_1 + b_2\eta)(\frac{A}{\gamma^2} - \frac{B}{\sigma^2})\} \underline{\omega} \end{aligned} \tag{19}$$

where $A = (1 + \sigma_1)(1 + \frac{\Delta_J^2}{\sigma_1})$ and $B = (1 - \sigma_2)(1 - \frac{\Delta_J^2}{\sigma_2})$.

The following identity has been used to simplify the inequality:

$$\pm 2 g_i \Delta g_i^T \leq \sigma_i g_i g_i^T + \frac{1}{\sigma_i} \Delta g_i \Delta g_i^T \tag{20}$$

where the weighting coefficients σ_i are used to tune Δg_i.

Thus, by selecting c_1 and c_2 such that

$$c_1 - c_2 + 4ab_1\left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) = 0 \tag{21a}$$

and

$$2c_2 + 4ab_2\left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) = 0 \tag{21b}$$

(21a) and (21b) imply that (15) can be written as

$$-2a^2(2b_1 + b_2 + b_2\eta)\left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) I - (b_1 + b_2\eta)^2 J_o > 0 \tag{22}$$

and the function H_γ becomes

$$\begin{aligned} H_\gamma \leq\ & + \underline{\varepsilon}^T \left\{\frac{1}{2}\rho_2 I + 2(b_1 + b_2\eta)^2 \left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) I\right\} \underline{\varepsilon} \\ & + \underline{\omega}^T \Big\{(b_1 + b_2\eta)(\eta I + [\underline{\varepsilon}\times]) J_o - b_2 J_o \underline{\varepsilon}\underline{\varepsilon}^T \\ & \left(\frac{1}{2}\rho_1 + 2a^2\left(\frac{A}{\gamma^2} - \frac{B}{\sigma^2}\right)\right) I + \pm 4(b_1 + b_2\eta)\Delta_J [\varepsilon\times] J_o \Big\} \underline{\omega} \end{aligned} \tag{23}$$

Note that the matrix $\eta I + [\underline{\varepsilon}\times]$ and $[\underline{\varepsilon}\times]$ has a norm less than or equal to 1. Thus,

$$\begin{aligned} & \underline{\omega}^T \{(b_1 + b_2\eta)(\eta I + [\underline{\varepsilon}\times]) J_o - b_2 J_o \underline{\varepsilon}^T \underline{\varepsilon} \\ & \pm 4(b_1 + b_2\eta)\Delta_J [\varepsilon\times] J_o\} \underline{\omega} \\ & \leq (b_1 + b_2 + 4(b_1 + b_2)\Delta_J) \|J_o\| \underline{\omega}^T \underline{\omega} \end{aligned} \tag{24}$$

Substituting this inequality into H_γ, a sufficient condition for the system to have an L_2 gain less than γ can then be established:

$$\begin{aligned} H_\gamma <\ & \underline{\varepsilon}^T \left\{\frac{1}{2}\rho_2 I + 2(b_1 + b_2\eta)^2 \left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) I\right\} \underline{\varepsilon} \\ & + \frac{1}{2}\rho_1 + 2a^2\left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right)\Big\} \underline{\omega} \end{aligned} \tag{25}$$

THEOREM *There exists a controller such that the L_2 gain from $\underline{d}$ to $\underline{z}$ is less than γ if there exist a, b_1, and b_2 such that*

$$-2a^2(2b_1 + b_2 + b_2\eta)\left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) I - (b_1 + b_2\eta)^2 J_o > 0 \tag{26a}$$

$$\frac{1}{2}\rho_1 + 2a^2\left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) + b_1\|J_o\| + b_2\|J_o\| + 4(b_1 + b_2)\Delta_J \|J_o\| < 0 \tag{26b}$$

$$\frac{1}{2}\rho_2 + 2(b_1 + b_2\eta)^2 \left(\frac{A}{\gamma^2} - \frac{B}{\rho^2}\right) < 0 \tag{26c}$$

where $A=(1+\sigma_1)(1+\frac{\Delta_J{}^2}{\sigma_1}), B=(1-\sigma_2)(1-\frac{\Delta_J{}^2}{\sigma_2})$

for all $\eta \geq 0$. *Furthermore, the controller*

$$\underline{u} = -\frac{2}{\rho^2}\left(a\underline{\omega} + b_1\underline{\varepsilon} + b_2\eta\underline{\varepsilon}\right) \tag{27}$$

solves the state feedback nonlinear H_∞ *control problem given by system (7a), (7b) and (8), with* $\underline{x}_0 = [\underline{\omega} \quad \underline{\varepsilon} \quad \eta]^T = [0 \quad 0 \quad 1]^T$.

The tests in the above theorem involve the quaternion variable η. By applying the worst-case analysis, another sufficient condition can be obtained.

COROLLARY *A sufficient condition for the existence of the* H_∞ *controller of the form (27) is the existence of* a, b_1, *and* b_2 *such that*

$$-2a^2(2b_1+b_2)\left(\frac{A}{\gamma^2}-\frac{B}{\rho^2}\right)I-(b_1+b_2)^2 J_o > 0 \tag{28a}$$

$$\frac{1}{2}\rho_1 + 2a^2\left(\frac{A}{\gamma^2}-\frac{B}{\rho^2}\right)+b_1\|J_o\|+b_2\|J_o\| \\ +4(b_1+b_2)\Delta_J\|J_o\|<0 \tag{28b}$$

$$\frac{1}{2}\rho_2 + 2b_1^2\left(\frac{A}{\gamma^2}-\frac{B}{\rho^2}\right)<0 \tag{28c}$$

With some modifications, it can be shown that the proposed method and criteria generalize the results in (Dalsmo, *et al*., 1997). Indeed, suppose we set b_2, c_2 and Δ_J as zeros, the conditions in (Dalsmo, *et al*., 1997) can be recovered using the criteria in the corollary.

5. SIMULATION RESULTS

To substantiate the performance of the controller design, experimental simulations on the ROCSAT-3 spacecraft were carried out, which is motivated by the one given in (Show, *et al*., 2003). During the operation of orbit transfer, the thruster control system of ROCSAT-3 spacecraft is required to perform large angle reorientation maneuvers. Gravity gradient torque is the dominant environmental disturbance at its 450 km parking orbit. Figure 1 depicts this external disturbance (in the body frame) which was generated by simulation as follows. Table 1 contains the inertia matrix and parameters of controller. The initial conditions for the simulation cases are given in Table 2 using the 3-2-1 Euler angles. In this paper, we assume the nominal inertia matrix is perturbed by 20%.

Simulation

To illustrate the robust capability of MOI uncertainty and external disturbance attenuation, two simulation cases with large angle maneuvers were performed. According to the derivation in the previous section, if the inertia uncertainty is sufficiently small, then the sufficient condition for the existence of the nonlinear H_∞ controller is assured. This is demonstrated by deviating all elements of the inertia matrix by 20% from nominal case in the simulation. Figs. 2 and 3 show the trajectories of the components of quaternion. Except the deviations of MOI, each simulation also considers the external torque. The simulation result shows that the nonlinear H_∞ controller results in a fast decay response and has a robust ability to attenuate the effects of external disturbance and MOI uncertainty.

Table 1 Parameters of the satellite system and the controller

Control gain of Simulation:

$\rho = 20, \quad a = 500, \quad b_1 = 200, \quad b_2 = 155$

$$J_o + \delta_J = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} + \begin{bmatrix} \Delta I_{xx} & 0 & 0 \\ 0 & \Delta I_{yy} & 0 \\ 0 & 0 & \Delta I_{zz} \end{bmatrix}$$

I_{xx}=5.5384 kg-m^2, $\left|\frac{\Delta I_{xx}}{I_{xx}}\right| \leq 0.2$

I_{yy}=5.6001 kg-m^2, $\left|\frac{\Delta I_{yy}}{I_{yy}}\right| \leq 0.2$

I_{zz}=4.2382 kg-m^2, $\left|\frac{\Delta I_{zz}}{I_{zz}}\right| \leq 0.2$

Table 2 Initial conditions of three cases

	Simulation Case 1	Simulation Case 2
ε_1	-0.2906	-0.6403
ε_2	-0.6100	-0.0560
ε_3	0.4138	0.7631
η	0.6100	0.0668
ϕ	-84 deg	-44 deg
θ	-30 deg	75 deg
φ	96 deg	137 deg

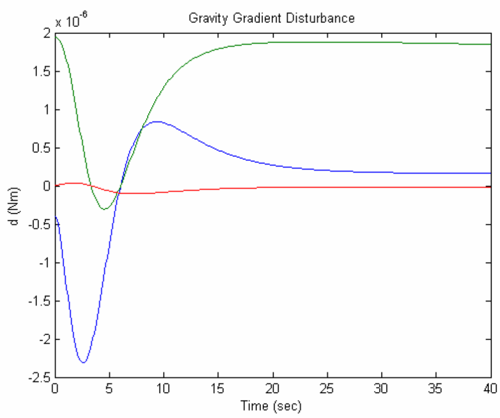

Fig. 1. Gravity gradient disturbance in simulation

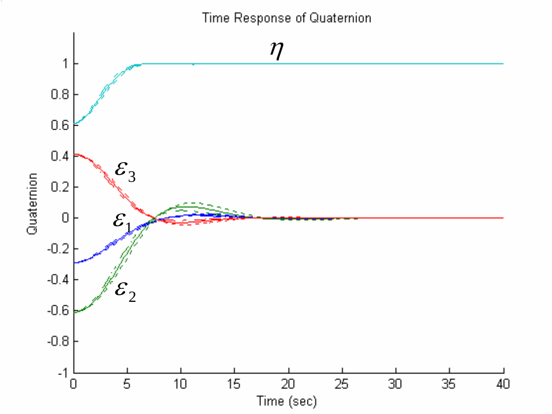

Fig. 2 Time response of four components of quaternion for Simulation(case 1), dashdot: $\delta_J = -0.2J_o$; solid: $\delta_J = 0$; dotted: $\delta_J = +0.2J_o$

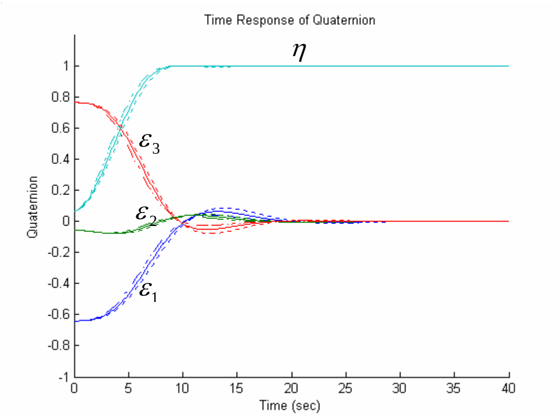

Fig. 3 Time response of four components of quaternion for Simulation (case 2), dashdot: $\delta_J = -0.2J_o$; solid: $\delta_J = 0$; dotted: $\delta_J = +0.2J_o$

6.CONCLUSION

This paper presents a nonlinear H_∞ state-feedback attitude control technique for spacecraft under large angle maneuvers. An additional freedom in the candidate Lyapunov function has identified and explored to solve the nonlinear H_∞ control problem. The sufficient condition for the existence of the nonlinear H_∞ controller has derived for the case of spacecraft inertia uncertainty. It was shown that if the uncertainty is within an appropriate amount of uncertainty, the existence of the nonlinear H_∞ controller is assured. Moreover, we have shown that the nonlinear term in the proposed controller brings quicker decay response when the spacecraft is performing large maneuver. The simulation results achieve the desired stability and robustness of attitude control design for a satellite which is subject to moment-of-inertia uncertainty and external disturbance and hence verify the effectiveness of the proposed method.

REFERENCES

Cavallo, A. and G. De Maria (1996). "Attitude Control for Large Angle Maneuvers", *IEEE Workshop on Variable Structure System, VSS'96*, Tokyo, Japan, pp. 232-236.

Dalsmo, D. and O. Egeland (1997). "State Feedback H_∞-Suboptimal Control of a Rigid Spacecraft", *IEEE Transactions on Automatic Control*. No. 42, Vol. 8, pp. 1186-1189.

Isidori, A. and A. Astolfi (1992). "Disturbance Attenuation and H_∞ Control via Measurement Feedback in Nonlinear Systems", *IEEE Transactions on Automatic Control*, No. 37, Vol. 9, pp.1283-1293.

Joshi, S. M., A. G. Kelkar and J. T.-Y. Wen (1995). "Robust Attitude Stabilization of Spacecraft Using Nonlinear Quaternion Feedback", *IEEE Transactions on Automatic Control*, No. 40, Vol. 10, pp. 1800-1803.

Kang, W. (1995). "Nonlinear H_∞ Control and Its Applications to Rigid Spacecraft", *IEEE Transactions on Automatic Control*, No. 40, Vol. 7, pp. 1281-1285.

Show, L. L., J. C. Juang and Y. W. Jan, (2003). "An LMI-Based Nonlinear Attitude Control Approach", IEEE Transactions on Control Systems Technology, No. 11, Vol. 1, pp. 73-83

Singh, S. N. (1987). "Nonlinear Adaptive Attitude Control of Spacecraft", *IEEE Transactions on Aerospace and Electronic Systems"*, No. 23, Vol. 3, pp. 371-379.

Van der Schaft, A. J. (1992). " $L_2 - \text{gain} \le \gamma$ Analysis of Nonlinear Systems and Nonlinear State Feedback H_∞ Control", *IEEE Transactions on Automatic Control*, No. 37, Vol. 6, pp. 770-784.

Wie, B. and P. M. Barba (1985). "Quaternion Feedback for Spacecraft Large Angle Maneuvers", *Journal of Guidance, Control, and Dynamics*, No. 8, Vol. 3, pp. 360-365.

Wu, C. S., B. S. Chen, and Y. W. Jan (1999). "Unified Design for H_2, H_∞ and Mixed H_2/H_∞ Control of Spacecraft", *Journal of Guidance, Control, and Dynamics*, No. 22, Vol. 6, pp. 884-896.

Yang, C. D. and C. C. Kung (2000)."Nonlinear H_∞ Flight Control of General Six Degree-of-Freedom Motions", *Journal of Guidance, Control, and Dynamics*, No. 23, Vol. 2, pp.278-288