OPTIMAL CONTROL OF A LABORATORY ANTILOCK BRAKE SYSTEM

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Abstract: A general optimal control problem for ABS is formulated and analyzed, with the elimination of excessive slip and reduction of braking distance taken into account. Analytical formulas for singular optimal solutions are derived. These results are applied to a simple laboratory model of ABS. The results of simulations are compared with those obtained by a gain-scheduling approach.

Keywords: ABS, brake control, friction, optimal control, singularities.

1. INTRODUCTION

For the last twenty years intensive development of control systems for car brakes has been observed. The antilock brake system (ABS) fulfils two basic tasks in a car. First, it prevents the wheels from locking, by keeping the slip below the maximum admissible level. Second, it should reduce the braking distance to its minimum possible value. In a typical situation (on dry asphalt) the maximum friction force between the tire and the road occurs at a certain moderate slip, when the wheel is not locked. In the first ABS systems, various on-off controllers were applied (Hattwig 1993, Maier 1995). Later PID and gain-scheduled PID controllers, as well as their robustified versions were introduced. (Johansen *et al*., 2001) proposed a robust gain-scheduled LQ controller where Sontag's procedure is used to stabilize the system. Some strategies based on optimization and off-line trajectory planning are presented in (Johansen, 2001). In a real car it is necessary to estimate the car velocity and parameters of the friction curve in the presence of disturbances and various effects which are difficult to model (Petersen, 2003). To this end, the extended Kalman filter is frequently used.

The laboratory ABS model (LABS) used in this work allows precise identification of friction mechanisms and does not require state estimation. This gives a possibility to determine optimal control basing on a mathematical model and apply it in practice. The paper is organized as follows. Section 2 contains a short description of the laboratory setup and the state equations, also in the scaled version. In the next section the optimal control problem is formulated, optimality conditions are given and singular optimal controls are analyzed. Next, the optimal solution is calculated by the MSE method (Szymkat and Korytowski, 2003). Section 4 presents a comparison with the gain-scheduled LQ controller based on the results of (Johansen *et al*., 2001, 2003). Results of the real life laboratory experiments are discussed in section 5. The paper ends with conclusions.

2. LABORATORY MODEL OF ABS

The laboratory model of ABS (LABS), shown in Fig. 1 consists of two wheels rolling on one another. The upper wheel, mounted on a rocker arm, has a steel rim and a rubber tire. The lower wheel is made of aluminum. The angles of rotation of the wheels are measured by encoders with the resolution of $2\pi/2048 = 0.175°$. The angular velocities are approximated by differential quotients. The sampling period is equal to 2 [ms]. The upper wheel is equipped with a disk brake controlled by a DC motor. Another DC motor, placed on the axle of the lower wheel is used to set the system in motion and accelerate it. During the braking process, the latter motor is switched off. Both motors are steered by PWM signals with frequency 7.0 [kHz].

The relative pulse width of the PWM signal of the upper wheel braking motor is the control variable. The peripheral velocity of the lower wheel can be identified with the speed of the vehicle and the angular velocity of the upper wheel can be identified with the angular velocity of the rotating wheel of the vehicle.

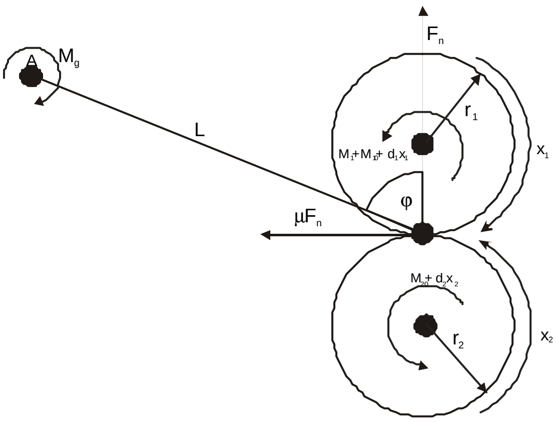


Fig. 1. Diagram of LABS

We use the following notations (see Fig. 1). The angular velocities of the upper and lower wheel [rad/s] are denoted by x_1 and x_2, respectively, $x_3 \equiv M_1$ is the braking moment [Nm] of the upper wheel, r_1 and r_2 are the radii of the upper and lower wheel [m], J_1 and J_2 are the moments of inertia of the upper and lower wheel [kgm^2], d_1 and d_2 are the coefficients of viscous friction in the upper and lower wheel bearing [kgm^2/s], F_n is the force with which the upper wheel presses the lower one [N], $m(l)$ is the coefficient of friction between the wheels, $l = \dfrac{r_2 x_2 - r_1 x_1}{r_2 x_2}$ is the relative difference of peripheral velocities of the wheels, or the wheel slip ($r_2 x_2 \geq r_1 x_1$), M_{10} and M_{20} are the moments of static friction [Nm] of the upper and lower wheel, respectively, M_g is the moment of gravity acting on the rocker arm, L is the distance between the point of contact of the wheels and the axis of the rocker arm [m], j is the angle between the normal at the point of contact of the wheels and the rocker arm [rad], u is the control of the disk brake. The values of model parameters are given in Appendix A.

It is assumed that the friction is proportional to the pressing force F_n with the proportionality coefficient $m(l)$.

2.1 State equations

The dynamics of the system is described by the following state equations

$$\begin{aligned}
\dot{x}_1 &= S(l)(c_{11}x_1 + c_{12}) + c_{13}x_1 + c_{14} + (c_{15}S(l) + c_{16})x_3 \\
\dot{x}_2 &= S(l)(c_{21}x_1 + c_{22}) + c_{23}x_2 + c_{24} + c_{25}S(l)x_3 \\
\dot{x}_3 &= c_{31}(u - x_3),\ 0 \leq u \leq u_{\max} \qquad (1)
\end{aligned}$$

where

$$S(l) = \frac{w_4 l^p}{a + l^p} + w_3 l^3 + w_2 l^2 + w_1 l\ . \qquad (2)$$

The dependence of the friction coefficient on the slip is presented in Fig. 2.

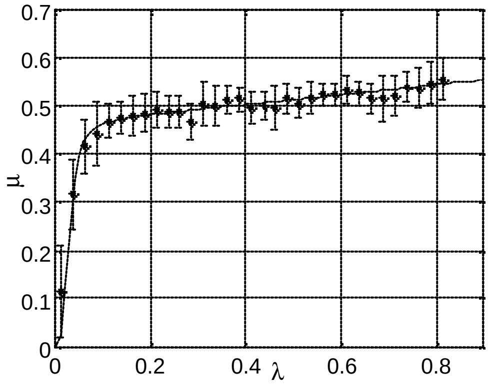


Fig. 2. Friction coefficient as a function of slip. Solid line – approximation, * – measurements (with standard deviations).

Remark: Balance of forces for the system of Fig. 1 leads to the formula $S(l) = \dfrac{m(l)}{L(\sin j - m(l)\cos j)}$.
For the sake of computational simplicity we use the approximation (2). The coefficients are determined by the least squares method.

2 .2 Scaling of variables

The variables are scaled to simplify the equations

$$x_i = \frac{a\,\bar{x}_i}{r_i},\ i = 1, 2, \qquad x_3 = b\,\bar{x}_3 + g\ , \qquad u = k\,\bar{u} + g\ ,$$

$$t = d\,\bar{t}\ ,\ S = w_4 \bar{S}\ .$$

It is assumed $c_{14} + c_{16} g = 0$, $\dfrac{r_1 d}{a} c_{16} b = -1$,

$$c_{13} d = -1,\ \frac{r_2 d}{a} c_{24} = -1,\ \frac{c_{31} d k}{b} = 1.$$

We rewrite the state equations in the new notations, omitting the bars over symbols

$$\begin{aligned}
\dot{x}_1 &= S q_1 - x_1 + (c_{15} S - 1) x_3 \\
\dot{x}_2 &= S q_2 + q_3 + c_{25} S x_3 \\
\dot{x}_3 &= c_{31} x_3 + u \qquad (3) \\
q_1 &= c_{11}x_1 + c_{12}, \quad q_2 = c_{21}x_1 + c_{22}, \quad q_3 = c_{23}x_2 - 1 \\
l &= 1 - x_1 x_2^{-1}, \quad S(l) = \frac{l^p}{a + l^p} + w_3 l^3 + w_2 l^2 + w_1 l
\end{aligned}$$

The constraints on control now take the form

$$\bar{u}_{\min} \leq \bar{u} \leq \bar{u}_{\max}\ , \bar{u}_{\min} = -\frac{g}{k}\ , \bar{u}_{\max} = \frac{u_{\max} - g}{k}\ . \qquad (4)$$

3. OPTIMAL CONTROL

3.1 Control task

The goal of control is to reduce the velocity in time T in such a way that an adequate compromise is ensured between excessive slip, braking distance and accuracy of reaching the target state. These requirements are expressed by the following performance index

$$Q(u,T) = \tfrac{1}{2}\| x(T) - x^f \|_\Omega^2 + \tfrac{1}{2} r \int_0^T (h r_2 x_2 - r_1 x_1)_+^2 \, dt + + r_1 \int_0^T r_2 x_2 dt \qquad (5)$$

$$\| x(T) - x^f \|_\Omega^2 = r_1^2 (x_1(T) - x_1^f)^2 + r_2^2 (x_2(T) - x_2^f)^2$$

The first term in (5) is a penalty for the error in reaching the target, the second penalizes for excessive slip, and may be interpreted as a measure of probability of losing steering qualities by the vehicle. Excessive slip is likely to cause a complete loss of steerability of the vehicle. The third term is proportional to braking distance. The parameters r and r_1 are nonnegative weighting coefficients. The control horizon T can be free or fixed. Notice that the task of slip stabilization can be obtained as a special case after putting $r_1 = 0$. Using the scaled variables introduced in section 2.2, we write the performance index in the form

$$Q(\bar u, \bar T) = \tfrac{1}{2}\| \bar x(\bar T) - \bar x^f \|_\Omega^2 + \tfrac{1}{2} \bar r \int_0^{\bar T} (h \bar x_2 - \bar x_1)_+^2 \, d\bar t + + \bar r_1 \int_0^{\bar T} \bar x_2 d\bar t$$

$$\| \bar x(\bar T) - \bar x^f \|_\Omega^2 = (\bar x_1(\bar T) - \bar x_1^f)^2 + (\bar x_2(\bar T) - \bar x_2^f)^2 .$$

3.2 Optimality conditions

We use Pontryagin's maximum principle. The hamiltonian is as follows

$$H = H_0(x,y) + H_1(x,y) u$$

$$H_0 = y_1 (S q_1 - x_1 + (c_{15} S - 1) x_3) + + y_2 (S q_2 + q_3 + c_{25} S x_3) + y_3 c_{31} x_3 - - \tfrac{1}{2} r (h x_2 - x_1)_+^2 - r_1 x_2$$

$$H_1 = y_3 .$$

The control maximizing the hamiltonian satisfies

$$u(t) = \begin{cases} u_{\max}, & f(t) > 0 \\ u_{\min}, & f(t) < 0 \end{cases}$$

where

$$f(t) = y_3 = H_1|_t .$$

Write the adjoint equations

$$\dot y_1 = -(S_{x_1} q_1 + c_{11} S - 1 + c_{15} S_{x_1} x_3) y_1 - -(S_{x_1} q_2 + c_{21} S + c_{25} S_{x_1} x_3) y_2 - r(hx_2 - x_1)_+$$

$$\dot y_2 = -(S_{x_2} q_1 + c_{15} S_{x_2} x_3) y_1 - -(S_{x_2} q_2 + c_{23} + c_{25} S_{x_2} x_3) y_2 + hr(hx_2 - x_1)_+ + r_1$$

$$\dot y_3 = -(c_{15} S - 1) y_1 - c_{25} S y_2 - c_{31} y_3$$

with terminal conditions

$$y_1(T) = x_1^f - x_1(T), \; y_2(T) = x_2^f - x_2(T)$$

$$y_3(T) = 0 .$$

It can be proved that a singularity of the second order usually occurs in the optimal solution of the considered problem. In the interval of singularity we have $f(t) \equiv 0$, i.e., $y_3(t) \equiv 0$. By differentiating this identity four times we obtain an expression for the optimal singular control as a function of state, $u(t) = u_s(x(t))$. The adjoint variables are eliminated using the relationships $f \equiv \dot f \equiv \ddot f \equiv 0$. To save space we omit the detailed computations which are lengthy and laborious.

3.3 Experiments

Let $h = 0.7$, $r = 1000$, $r_1 = 0$, $T = 0.0064$. The search for optimal control is started in the class of bang-bang functions (Fig. 3). Large density of control switchings indicates the singularity interval and allows the structure of optimal control to be determined.

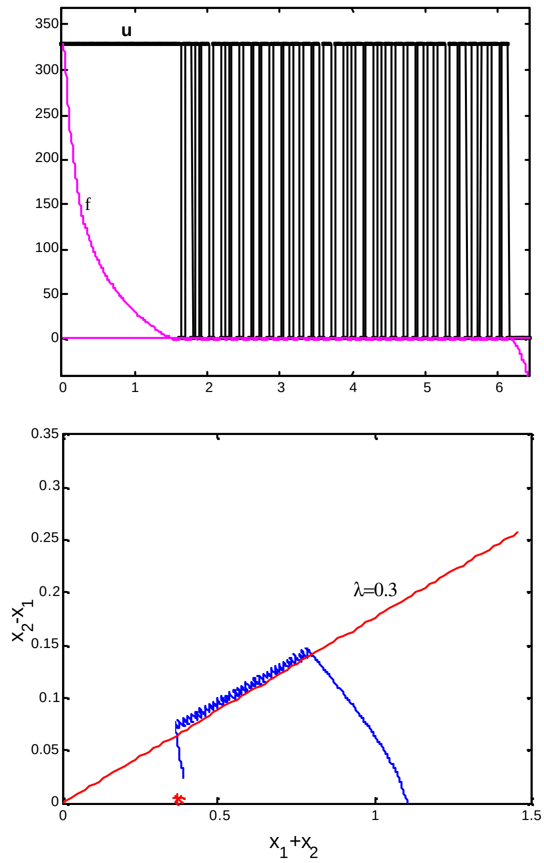


Fig. 3. Bang-bang control and trajectory.

In the next stage, the MSE method was used to calculate the optimal solution (Szymkat and Korytowski, 2003). The results are shown in Fig. 4.

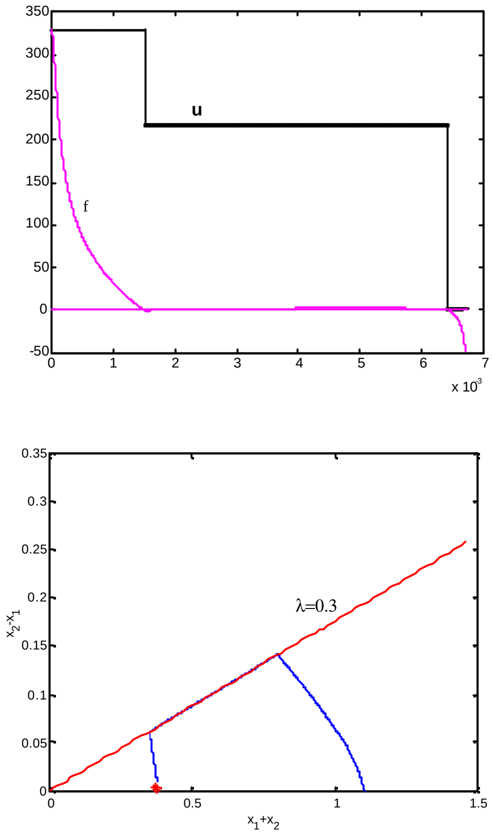


Fig. 4. Optimal control and trajectory.

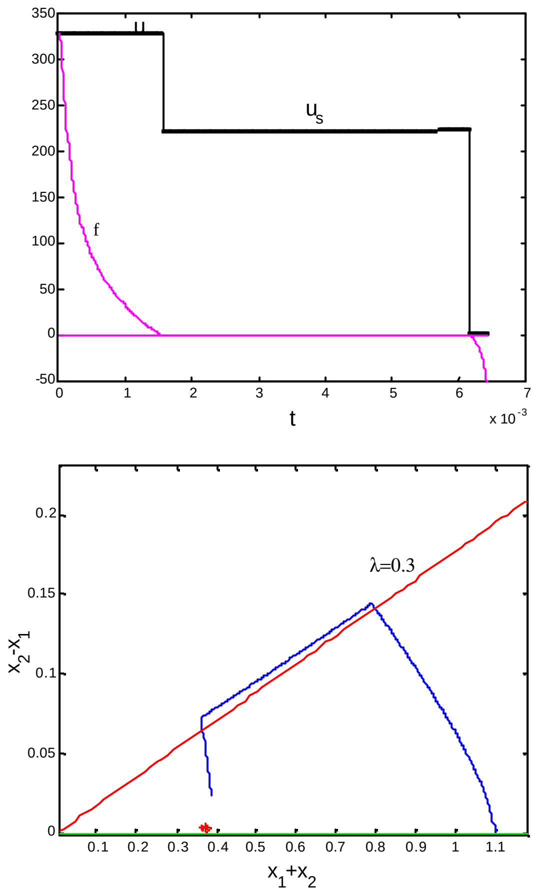


Fig. 5. Optimal control and trajectory.

Let now $h = 0.7$, $r = 1000$, $r_1 = 0$, $T = 0.0064$. As before, a bang-bang approximation of optimal control was used to establish the structure of optimal solution. The results obtained with the use of the explicit representation of singular control are shown in Fig. 5. The next example shows (Fig. 6) the consequences of diminishing the weighting coefficients r 14 times, which means that exceeding the value of slip 0.7 is much less penalized.

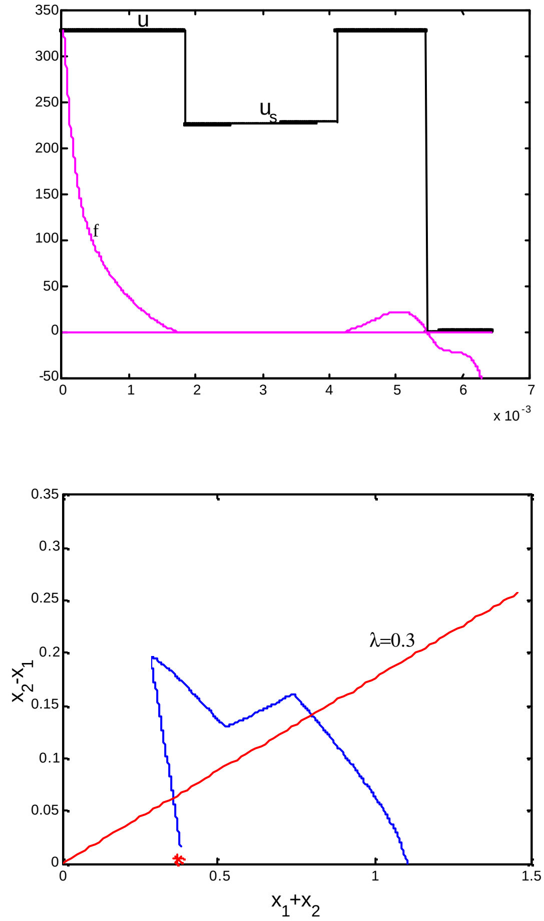
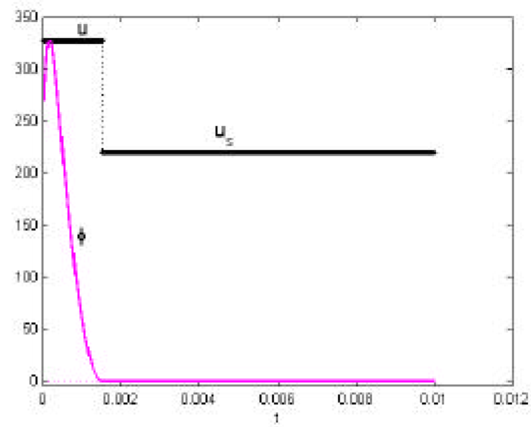


Fig. 6. Optimal control and trajectory.

In the fourth experiment the slip was stabilized at the value 0.3. It was thus assumed $h = 0.7$, $r = 1$ $r_1 = 0$, $T = 0.01$, and the term in the performance index penalizing for missing the target was skipped. The results are shown in Fig. 7.



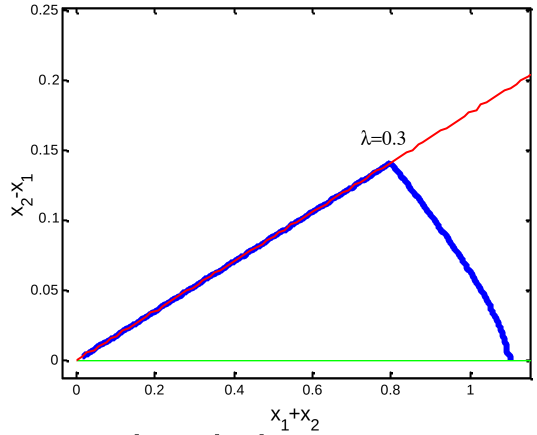


Fig. 7. Optimal control and trajectory.

4. COMPARISON WITH GAIN-SCHEDULED LQ CONTROLLER

This section is devoted to the gain-scheduled LQ (GSLQ) controller of (Johansen *et al.*, 2001, 2003). We consider the model (2.1), neglecting the friction in the bearings and static friction. The state equations take the form

$$\dot{x}_1 = c_{12}S(l) + (c_{15}S(l) + c_{16})x_3$$
$$\dot{x}_2 = c_{22}S(l) + c_{25}S(l)x_3$$
$$\dot{x}_3 = c_{31}(u - x_3)$$

with

$$c_{12} = \frac{M_g r_1}{J_1},\quad c_{15} = \frac{r_1}{J_1},\quad c_{16} = -\frac{1}{J_1}$$
$$c_{22} = -\frac{M_g r_2}{J_2},\quad c_{25} = -\frac{r_2}{J_2}.$$

Substituting $l = \frac{r_2 x_2 - r_1 x_1}{r_2 x_2}$ and treating the velocity x_2 as a disturbance we obtain the equations

$$\dot{l} = f(l, x_2, x_3)$$
$$\dot{x}_3 = c_{31}(u - x_3)$$
$$f = x_2^{-1}\left(((1-l)(c_{22} + c_{25}x_3) - \bar{c}_{12} - \bar{c}_{15}x_3)S(l) - \bar{c}_{16}x_3\right)$$
$$\bar{c}_{12} = \frac{r_1}{r_2}c_{12},\ \bar{c}_{15} = \frac{r_1}{r_2}c_{15},\ \bar{c}_{16} = \frac{r_1}{r_2}c_{16}$$

The model linearized at the equilibrium point $\tilde{x}_0 = (l_0, x_2, x_{30}, u_0)$ has the form

$$\Delta\dot{l} = f_l'(\tilde{x}_0)\Delta l + f_{x_3}'(\tilde{x}_0)\Delta x_3$$
$$\Delta\dot{x}_3 = c_{31}(\Delta u - \Delta x_3)$$
$$\Delta\dot{x}_4 = \Delta l$$
$$\Delta l = l - l_0,\ \Delta u = u - u_0,\ \Delta x_3 = x_3 - x_{30}.$$

This model is non-stationary and its coefficients depend on the velocity x_2, which is taken as a disturbance. In the controller synthesis it will be assumed that x_2 is constant. The third equation is introduced into the model so that the controller has the integrating (astatic) property. The controller synthesis consists of determining a gain matrix $K = [K_1, K_2, K_3]$ which minimizes a quadratic performance index

$$J = \int_0^\infty (\Delta x^{\mathrm{T}} W(x_2)\Delta x + \Delta u^{\mathrm{T}} R\Delta u)\mathrm{d}t$$
$$\Delta x = \mathrm{col}(\Delta l, \Delta x_3, \Delta x_4),\quad \Delta u = -K\Delta x$$
$$W(x_2) = x_2^{3/2}\mathrm{diag}(w_1, w_2, w_3)$$
$$w_1 = 0.1,\ w_2 = 10^{-4},\ w_3 = 15,\ R = 1.$$

It is assumed that the set value of the slip is equal to $l_0 = 0.2$, which corresponds to the braking moment $x_{30} = 5.7156$ [Nm]. The gain matrices are determined by solving an appropriate Riccati equation, for the velocity x_2 in the range from 1 to 180 [rad/s]. The dependence of the controller gains on x_2 is shown in Fig. 8.

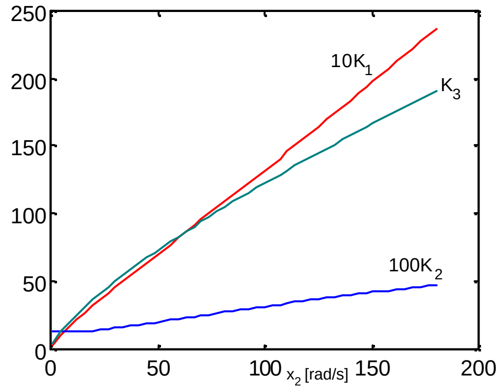


Fig. 8. Controller gains as functions of x_2.

This gain-scheduled LQ controller is confronted with the solution, optimal according to the performance index (5) with the parameters $h = 0.8$, $r = 1$, $r_1 = 0$, $T = 0.01$. Notice that the state trajectory generated by the optimal controller (black line in Fig. 9) exhibits better transient behavior than the GSLQ controller (blue line). The corresponding slip trajectories are shown in Fig. 10.

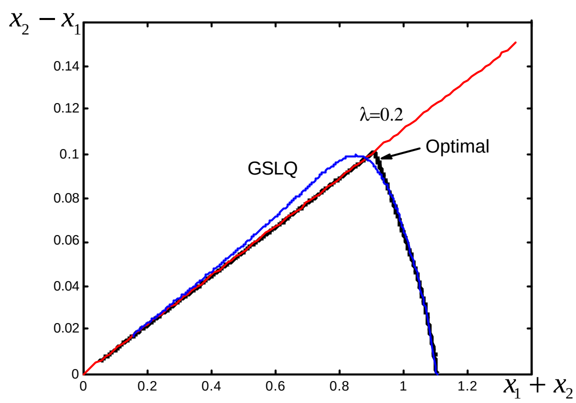


Fig. 9. Comparison of controllers performance. State trajectories.

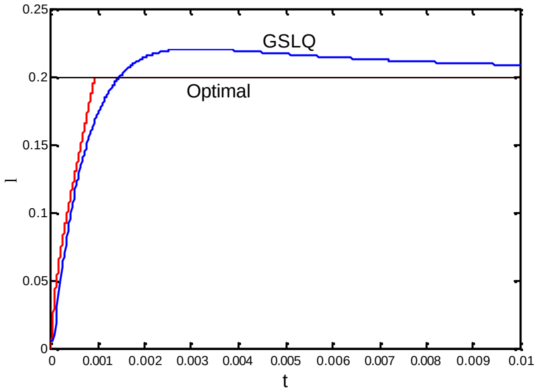


Fig. 10. Comparison of controllers performance. Slip trajectories.

5. EXPERIMENTAL VALIDATION

In the experiment the slip was stabilized at the value 0.3. It was thus assumed $h = 0.7$, $r = 1$, $r_1 = 0$, $T = 1.2$, and the term in the performance index penalizing for missing the target was skipped. The MSE method was used to calculate the optimal solution. Next, the optimal control was applied to the LABS. The results of the open loop experiments are shown in Fig. 11 and 12. A measured time history of the slip and optimal control are shown in Fig. 11. The trajectory of LABS is shown in Fig. 12. It is easy to see that the quality of the slip stabilization is very good.

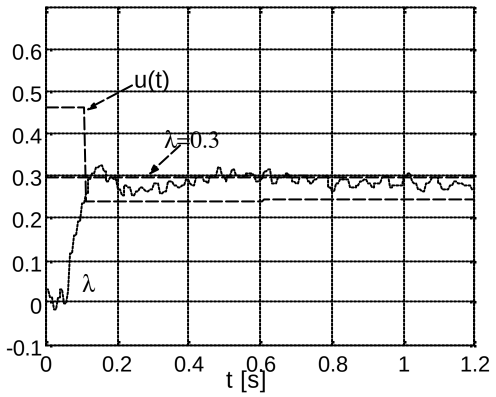


Fig. 11. Experimental slip trajectory and control.

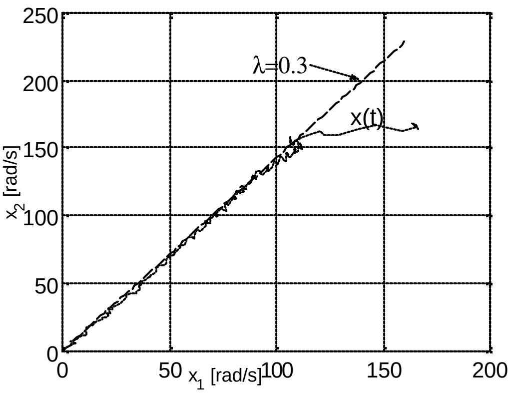


Fig. 12. Experimental trajectory of LABS.

6. CONCLUSIONS

The Laboratory ABS model (LABS) is a simple and convenient tool for experimental verification of different antilock brake control methods. The optimal control setting of the antilock braking problem, with the control time, terminal error, excessive slip and braking distance accounted for in the performance index, provides efficient control algorithms which can be used in practice. This is possible due to the careful identification of LABS. The MSE method of optimal control calculation has proved well-suited for this application. An interesting feature of the optimal solutions in the considered problem is the presence of singularities that may be analytically treated.

Further research should answer the important question how the obtained results can be incorporated into an adaptive real-time control scheme, resulting in a robust, reliable practical solution.

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