EXTENDED GLOBAL TOTAL LEAST SQUARE APPROACH TO MULTIPLE-MODEL IDENTIFICATION

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Abstract: The paper proposes a novel global total least square procedure which has been appropriately extended for multiple-model identification of nonlinear systems. The resulting scheme which is a hybrid iterative procedure, makes repeated use of both the behavioural and classical approaches. Model parameters are optimised in order to minimise the distance between an observed time series and the simulated time series of the resulting optimised behavioural model. The developed procedure is demonstrated when applied to an arbitrary nonlinear system.

Keywords: Algorithms, Estimation, Identification, MIMO, Modelling, Nonlinear systems, Optimisation, Systems theory.

1. INTRODUCTION

Whilst nearly all real-world systems are nonlinear in nature, attempts to design improved system performances through feedback control strategies, fault detection techniques and monitoring of plant efficiency, are usually based on classical linear systems theory. Indeed, within the classical linear systems theoretical framework, issues of system stability and preservation of spectral characteristics are well understood. One approach adopted, when attempting to model nonlinear systems for the purpose of analysis and design, is that of multiple-models, and a good introduction may be found in (Murray-Smith and Johansen, 1997). Recent years, however, has witnessed developments in linear systems theory and the emergence of a wider, all-encompassing, theoretical framework based on time series, known as the behavioural approach, proposed in (Willems, 1986a,1986b,1987). Within this new theoretical framework the classical representation of linear systems co-exists as a special subclass. Traditional input/output models, which are very much central in the classical approaches, are only a special case and may be deduced from the behavioural model. The behavioural approach defines a dynamical system as a family of trajectories, without reference to input/output maps or relations, without reference to state variables, and without reference to behavioural equations. The trajectories are defined by the triple $(\mathbb{T}, \mathbb{W}, \mathbf{B})$, where $\mathbb{T}$ represents the time instants of interest on a finite subset of the time axis, $\mathbb{W}$ represents the signal space (or vector space over a field) in which the time signals, that the system produces, take their values, and $\mathbf{B}$ is a family of $\mathbb{W}$-valued trajectories over the finite time series, which, in general, will be a linear subspace of $\mathbb{W}$. Essentially, the sets $\mathbb{T}$ and $\mathbb{W}$ define the 'size' of the system, whilst $\mathbf{B}$ formalises the 'laws' that govern the system. The notion of model complexity is related to the 'size' of the system denoted $\mathcal{B} \in \mathbf{B}$ and is related to the rank, or number of unique (i.e. linearly independent) trajectories, and the degree, or length of the time series. In practice, the aim is to obtain a reasonably simple model which provides a sufficiently accurate representation of the data.

A distinguishing feature of the behavioural approach is that there is no necessary distinction between the system inputs and outputs, both being regarded as 'external' variables. Such a feature is, indeed, of particular interest when dealing with error-in-variable (EIV) models, as the effect of noise can be understood on the external variables. Therefore, while EIV techniques have been developed within the classical approach (Soderstrom *et al.*, 2002), extension to the behavioural approach seems a particularly natural choice. The global total least squares procedure (GTLS), proposed in (Roorda, 1995), allows estimation of fully parameterised state-space EIV models, with the specificity of treating all external variables equally (i.e. same noise variance). This technique utilises the notion of an isometric state representation (ISR), which, by definition (see (Heij, 1989)), leads to a model for a linear stabilizable system that can be driven forward or in reverse time.

The paper investigates the appropriateness of extending the behavioural approach, combined with the GTLS approach, to encompass a parallel structured weighted multiple-model approach to deal with nonlinear systems. The nature of the problem is such that once an initial GTLS model has been obtained, and refined, it is transformed to an equivalent classical representation, where the total model, together with the scheduling parameters, are repeatedly optimised using a 'global ordinary least squares' (GOLS) scheme. Then, following the optimisation, the model is returned to the behavioural framework and the GTLS optimisation sequentially applied to the linear time-invariant (LTI) sub-models. This cycle is repeated until the distance between the measured and simulated time series has converged to a minima. The theoretical framework for the new approach is derived for the general case of multiple-input multiple-output (MIMO) systems, and demonstrated using a single-input single-output (SISO) nonlinear system.

Inherent in the hybrid approach is the assumption (from GTLS) of measurement noise on the measured signals and, due to its presence within the construction of the scheduling vector which weights the temporal importance of each of the sub-models, it plays a crucial role in the optimisation. In recognition of the potential problems arising due to ISR modelling error, the method uses an approach developed in (Vinsonneau *et al.*, 2004) in which the uncertainty present in a noise residual, associated with an estimated noise free input, is iteratively modified in order to remove, or reduce, the effect of inherent modelling error; arising when linearising a nonlinear system. The algorithm is an initial step towards integrating and extending both the multiple-model approach and the behavioural framework, and constitutes a novel development for handling nonlinear systems.

A description of the GTLS modelling techniques is given in Section 2. Section 3 introduces a multiple-model structure in the classical approach, which minimises a GOLS criterion. This algorithm forms a subset of the multiple-model EIV identification techniques described within the behavioural approach. The algorithm, which minimises the GTLS criterion is defined in Section 4. To provide an insight, the resulting hybrid algorithm is applied to an arbitrary SISO nonlinear system in Section 5, noting that extension to the MIMO case is straightforward.

2. GLOBAL TOTAL LEAST SQUARE

One of the distinguishing characteristics of GTLS when compared to other EIV methods (Soderstrom *et al.*, 2002), is that all the external variables are treated symmetrically. EIV systems in the classical approach are illustrated in Figure 1, where observations are

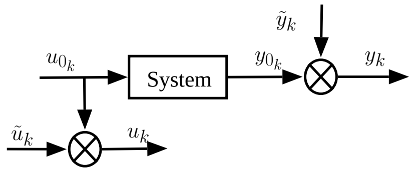

Fig. 1. An error-in-variables dynamic system.

assumed to be corrupted by additive measurement noises $\tilde{u}_k$ and $\tilde{y}_k$ at the input and output, respectively. This results in measured signals:

$$u_k = u_{0_k} + \tilde{u}_k, \quad y_k = y_{0_k} + \tilde{y}_k \tag{1}$$

This section reviews the GTLS technique and the associated terminology, while subsection 2.3 discusses how the use of *a priori* knowledge on the input noise distribution may be utilised to advantage by extending the use of a weighted approach proposed in (Roorda, 1995).

2.1 *Behavioural model: SR and ISR*

The behavioural state space model was originally introduced in (Willems, 1991) where a state-space system is denoted (S). This offers a more general description of systems, noting, for example, that the classical $(i/s/o)$ (input/state/output) state space representations is a special case of S. (Roorda, 1995) adopted the behavioural representation, and termed this a state representation (SR), defined by:

$$x_{k+1} = Ax_k + B\hat{v}_k \tag{2a}$$
$$\hat{w}_k = Cx_k + D\hat{v}_k \tag{2b}$$

where $\hat{w}_k \in \mathbb{R}^q$ denotes the estimated external variables at time instant k, $x_k \in \mathbb{R}^n$ the states, and $\hat{v}_k \in \mathbb{R}^m$ the driving variables.

As defined in (Heij, 1989), and with attention restricted to discrete time systems $\mathbb{T} = \mathbb{Z}$, let $\mathcal{B}_{sr}$ denote realisations of the class of system $\mathbf{B}_{sr}$ with the following form:

$$\mathcal{B}_{sr} = \mathcal{B}_{sr}(M) := \Big\{ (v, x, w) \in (\mathbb{R}^m \times \mathbb{R}^n \times \mathbb{R}^q)^{\mathbb{Z}} ; \begin{pmatrix} \sigma x \\ w \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} \Big\} \tag{3}$$

where σ denotes the time shift operator.

It has been proved in (Willems, 1986a,1986b,1987) that this class coincides with the class of LTI, complete systems. The class of realisations $\mathcal{B}_{sr}$ are said to be minimal if m and n are, individually, as small as possible. Therefore, it is not evident that a minimal realisation exists.

A state representation (A, B, C, D) is said to be isometric (ISR) if, for all $x \in \mathbb{R}^n$, $v \in \mathbb{R}^m$, $w \in \mathbb{R}^q$, and $z \in \mathbb{R}^n$, where $z = Ax + Bv$ and $w = Cx + Dv$, the following relationship holds:

$$|v|^2 + |x|^2 = |w|^2 + |z|^2 \tag{4}$$

A sufficient condition for the ISR to exist is that the system is stabilizable, and, in addition, if the representation is minimal, then A is asymptotically stable (see (Heij, 1989) for proof).

Such representations are of key importance as they may be driven backwards and forwards in time and therefore, facilitate the estimation of an optimal initial state x_0. The representations can be obtained by solving an appropriate Riccati equation.

In this paper, a hybrid scheme is proposed, in which the classical $(i/s/o)$ and the behavioural ISR representations are repeatedly utilised in order to realise the multiple-model GTLS.

2.2 Misfit and cost function

It is necessary to propose a means to asses the model performance, within a class of behavioural models $\mathbf{B}^q$ with q external variables. The misfit $d(w, \mathcal{B})$ of a particular model $\mathcal{B} \in \mathbf{B}^q$ with respect to a time series $w : T \to \mathbb{R}^q$, for observation interval T, is defined as:

$$d(w, \mathcal{B}) := \min_{\hat{w} \in \mathcal{B}_T} \|w - \hat{w}\| \tag{5}$$

where $\mathcal{B}_T$ denotes the restriction of $\mathcal{B}$ to the observation interval T and $\hat{w}$ is the estimate of w.

The GTLS problem can be summarised as follows: assuming a complexity constraint with tolerated size (m, n) with m auxiliary/driving variables (estimated) and n state variables, determine a state representation $M := (A, B, C, D) \in \mathbf{B}^{q,m,n}$, a class of models with q external variables and (m, n) satisfying a complexity constraint, with auxiliary inputs $\hat{v} \in \mathbb{R}^{N\times m}$, where N is the number of observations, that minimises:

$$(\hat{M}, \hat{x}_0) = \arg\min_{M, x_0} \left\{ \|w - \hat{w}(M, x_0)\|_F^2 \right\} \tag{6}$$

where $\|.\|_F$ denotes the Frobenius norm, and $\hat{w}(M, x_0)$ are the simulated external variables of a behavioural state space model, parameterised by M and using x_0 as initial state.

Within the classical approach, the criterion minimised in (6) is equivalent to:

$$J = \sum_{k=1}^{N} \|u_k - \hat{u}_k\|_F^2 + \|y_k - \hat{y}_k\|_F^2 \tag{7}$$

where u_k and y_k are the measured input and output, while the $\hat{u}_k$ and $\hat{y}_k$ are the respective simulated variables at time instant k.

2.3 A priori knowledge on the input noise distribution

In (Roorda, 1995), it was noted that to suppress the approximation of the input, and therefore favour the approximation of the output, the input could be weighted by a scalar α such that

$$\|w - \hat{w}\|_{\alpha,F}^2 := \alpha^2 \|u - \hat{u}\|_F^2 + \|y - \hat{y}\|_F^2 \tag{8}$$

with α sufficiently large.

In this paper, the systems of interest are nonlinear and thus, after linearising, the model aims to estimate an input, free of measurement noise, and an output which tends to reflect an optimal noise free output with 'linearisation' error. Here, the hypothesis is made that some *a priori* knowledge, concerning the input measurement noise variance σ_u, is known. A degree of uncertainty γ can be added, thus allowing the corresponding weight of interest, α, to be estimated using:

$$\hat{\alpha} = \inf_{\alpha} \left\{ \alpha > 0, \left| \sigma_u - \|u - \hat{u}(\alpha)\|_F^2 \right| < \gamma \right\} \tag{9}$$

Such a problem may be readily solved using a line search technique, with a positivity constraint for α.

3. MULTIPLE-MODEL GOLS

This section introduces a new multiple-model state-space representation and minimisation of GOLS is considered.

The discussion focuses on MIMO nonlinear systems with m noise-free inputs and l noisy outputs. The multiple-model approach is a well studied concept (see, for example, (Murray-Smith and Johansen, 1997) and the references therein) to represent nonlinear systems, where each local model aims to represent the system behaviour about a specified operating point. The importance of a particular sub-model at a particular time instant is specified using radial basis functions, which provide smooth transition over the whole operating range.

The problem of minimising the multiple-model GOLS is given in the following problem statement:

Problem 1. (Multiple-model GOLS). The proposed structure consists of s state-space sub-models of parameterisation $\theta := \begin{bmatrix} \theta_1 \dots \theta_s \end{bmatrix}$ where $\theta_i := (\mathrm{A}_i, \mathrm{B}_i, \mathrm{C}_i, \mathrm{D}_i)$ and the transition between sub-models is specified by a scheduling vector composed of normalised radial basis functions which are dependent on r chosen premise variables $\phi \in \mathbb{R}^{N\times r}$, with N being the number of observations. Assuming noise-free inputs $u \in \mathbb{R}^{N\times m}$, determine an estimate of $\hat{\theta}$ the multiple-model parameters θ, the centres $c \in \mathbb{R}^{r\times s}$ and widths $\varpi \in \mathbb{R}^{r\times s}$ of the radial basis functions and the initial state $\hat{x}_0 \in \mathbb{R}^n$, which minimises the difference between the measured outputs and the simulated outputs $\hat{y} \in \mathbb{R}^{N\times l}$:

$$(\hat{\theta}, \hat{c}, \hat{\varpi}, \hat{x}_0) = \arg \min_{\theta,c,\varpi,x_0} \left\{ \|y - \hat{y}(\theta, c, \varpi, x_0)\|_F^2 \right\} \tag{10}$$

The problem dealt with here is formulated and solved as two sub-problems: namely (1) identification of the parameters for each of the sub-models; and (2) finding the scheduling parameters of the radial basis functions, which will minimise the modelling error. Both sub-problems are interdependent and an iterative refinement of each set of parameters is necessary.

3.1 Multiple-model structure description

Figure 2 shows the structure of the model with s sub-models to be parameterised.

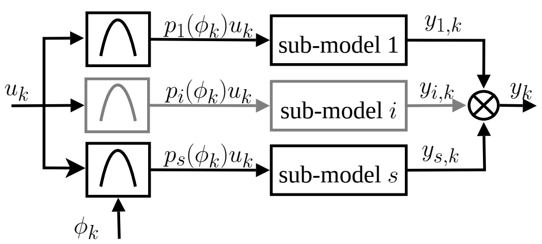

Fig. 2. Structure of the multiple model.

At each time step, the sub-models inputs $\hat{u}_{i,k}$ are expressed as a weighted equivalent of the inputs u_k with weights $p_{i,k} = p_i(\phi_k)$ with $\sum_i p_{i,k} = 1$ for every k, while the sub-models outputs $\hat{y}_{i,k}$ are estimated using the following classical state-space representation:

$$x_{i,k+1} = \mathrm{A}_i x_{i,k} + \mathrm{B}_i p_{i,k}(\phi_k, c, \varpi) u_k \tag{11a}$$
$$\hat{y}_{i,k} = \mathrm{C}_i x_{i,k} + \mathrm{D}_i p_{i,k}(\phi_k, c, \varpi) u_k \tag{11b}$$

The multiple-model outputs are given as the sum of the sub-models outputs:

$$\hat{y}_k(\theta, c, \varpi) = \sum_{i=1}^{s} \hat{y}_{i,k}(\theta_i, c, \varpi) \tag{12}$$

for $k = 1, \dots, N$, with N the number of observations.

The ‘weighting’ or ‘scheduling’ vectors p_i are defined as unknown functions of the ‘premise’ variables $\phi_k \in \mathbb{R}^r$. Typically, it will depend on the input and the state, that is:

$$\phi_k = \psi(x_k, u_k) \tag{13}$$

Let the ith radial basis function be equal to:

$$r_i(\phi_k; c_i, \varpi_i) = \exp(-(\phi_k - c_i)^T \mathrm{diag}(\varpi_i)^2 (\phi_k - c_i)) \tag{14}$$

where c_i is the centre and ϖ_i is the width of the ith radial basis function. The weights p_i are obtained from the radial basis function after normalisation:

$$p_i(\phi_k; c_i, \varpi_i) = \frac{r_i(\phi_k; c_i, \varpi_i)}{\sum_{j=1}^{s} r_j(\phi_k; c_j, \varpi_j)} \tag{15}$$

3.2 Initialisation

An initial model is estimated, which solves the prediction error minimisation (PEM) problem for a full parameterised state-space model. Here, the N4SID subspace-based method is used (see (Overschee and Moor, 1996) for more details), for the example in Section 5, but it has been noted in (McKelvey, 1995) that such methods may not perform well on data with non-zero initial conditions. In such a case, it might be preferable to use another type of PEM method.

The initial model is then replicated in each of the sub-models. Due to the normalisation of the scheduling vector p_i in (15), the sum of the sub-models inputs is equal to the initial model inputs, and given that the sub-models are linear, the sum of the resulting outputs is equal to the outputs of the initial single model.

3.3 Parameter optimisation for sub-models

Having an initial parameterisation for a multiple model, an optimisation can then be performed to minimise the output error (OE) depending on the parameters Θ:

$$\Theta := \left[\, vec(\theta)^T \;\; vec(c)^T \;\; vec(\varpi)^T \,\right]^T \tag{16}$$

where $vec(\cdot)$ denotes the vectorisation operator that forms a vector from a matrix by stacking the columns on top of each other. The induced cost function is defined as:

$$\begin{aligned} J_N &:= \sum_{k=1}^{N} \left\| y_k - \sum_{i=1}^{s} \hat{y}_{i,k}(\Theta) \right\|_F^2 \\ &= E^T(\Theta) E(\Theta) \end{aligned} \tag{17}$$

where

$$E(\Theta) = \left[\, E_1^T(\Theta) \;\dots\; E_N^T(\Theta) \,\right]^T \tag{18}$$

and

$$E_j^T(\Theta) = y_j - \sum_{i=1}^{s} \hat{y}_{i,j}(\Theta), \quad j = 1, \dots, N \tag{19}$$

The minimisation of J_N is a nonlinear, non-convex problem, because of the complicated dependence of J_N on the parameters θ, c, and ϖ. Therefore, this optimisation problem is reformulated into two sub-problems: the optimisation of the local models with a fixed scheduling vector; and the optimisation of the parameters of the scheduling vector with fixed sub-model parameters. This type of problem is commonly solved using the Levenberg-Marquardt method (see (Moré, 1978)).

4. MULTIPLE-MODEL GTLS

This section focuses on the multiple-model GTLS, where the system input is considered noisy. To improve the model estimation, this approach aims to estimate a multiple-model EIV. In general, the system is a MIMO nonlinear system with m noisy inputs and l noisy outputs. The multiple-model structure is identical to that in Figure 2.

4.1 Multiple-model GTLS definition

The extended GTLS approach, which reflects the multiple-model structure put forward in this paper, is given in the following problem statement:

Problem 2. (Multiple-model GTLS). The proposed structure is s state-space sub-models of parameterisation $M_{mm} := \left[M_{mm_1} \ \ldots \ M_{mm_s} \right] \in \mathbf{B}^{q,m,n,s}$ where $M_{mm_i} := (A_i, B_i, C_i, D_i)$, where the transition between sub-models is specified by a scheduling vector (15) composed of normalised radial basis functions (14) which are dependent on r chosen premise variables $\phi \in \mathbb{R}^{N \times r}$ introduced in (13), where N is the number of observations. Within a complexity constraint (m, n, s) with m auxiliary/driving variables (estimated), n state variables, the external variables $w = \left[u \ y \right] \in \mathbb{R}^{N \times q}$, s sub-models, and assuming *a priori* knowledge on the input noise variances $\sigma_u \in \mathbb{R}^m$, within uncertainty boundary $\gamma \in \mathbb{R}^m$, determine the multiple-model parameters M_{mm}, the centres $c \in \mathbb{R}^{r \times s}$ and widths $\varpi \in \mathbb{R}^{r \times s}$ of the radial basis functions and the initial state $\hat{x}_0 \in \mathbb{R}^n$, with auxiliary inputs $\hat{v} \in \mathbb{R}^{m \times N}$, such that:

$$(\hat{M}_{mm}, \hat{x}_0, \hat{c}, \hat{\varpi}) = \arg \min_{M, x_0} \left\{ \| w - \hat{w}(M_{mm}, x_0, c, \varpi, \sigma_u, \gamma) \|_F^2 \right\}$$

4.2 Algorithm

Initialisation: An initial model, obtained using the modified canonical correlation analysis (see (Roorda, 1995)) and iteratively improved using GTLS techniques, provides estimated noise free external variables $\hat{w}^{(0)} := \left[\hat{u}^{(0)} \ \hat{y}^{(0)} \right] \in \mathbb{R}^{N \times q}$ under the restriction specified in (9). In addition, the initial scheduling vector parameters $\left(c^{(0)}, \varpi^{(0)} \right)$ are chosen such that the operating points of the local models span the operating range under consideration. An iterative refinement is then applied until the sequence of 'global' misfit:

$$d(w, \mathcal{B})^{(j)} = \left\| w - \hat{w}^{(j)} \right\|_F^2 \qquad (20)$$

where j denotes the j^{th} iteration, converges. In reality, the algorithm is stopped when :

$$d(w, \mathcal{B})^{(j+1)} - d(w, \mathcal{B})^{(j)} < \nu \qquad (21)$$

where ν is a prescribed threshold level.

Step 1: *Solve the multiple-model GOLS problem*
The GOLS problem is solved using the noise-free estimated inputs $\hat{u}^{(j)}$ and the measured outputs y, giving model parameters $\theta^{(j)}$, as well as the estimated outputs of each sub-models $y_{mm_i}^{(j)}$. Each of these s sub-models are then transformed from an $i/s/o$ representation to an ISR representation $M_{mm_i}^{(j)}$, contained within the multiple model parameters $M_{mm}^{(j)}$.

Step 2: *Solve the GTLS problem for each sub-model*
In order to solve the GTLS problem for each sub-model, it is required to generate the noisy external variables $w_{mm_i}^{(j)}$ corresponding to each sub-model. As the inputs of the sub-models are weighted quantities in terms of the measured inputs, the resulting 'external' variables are simply the weighted measured inputs:

$$w_{mm_{i,k}}^{(j)} = \left[p_{i,k} \ g_{i,k} \right] * w_k \qquad (22)$$

where $g_{i,k}$ denotes the contribution of each of the sub-models to the outputs and the result of the operator $*$ is a vector whose entries are obtained by component-wise multiplication. This can be reflected by the ratio:

$$g_{i,k} := \frac{\hat{y}_{i,k}}{\hat{y}_k} \quad \text{for} \quad \hat{y}_k \neq 0 \qquad (23)$$

where $\hat{y}_{i,k}$ and $\hat{y}_k$ are defined, respectively, in (11) and (12). If $\hat{y}_k = 0$, then $g_{i,k} := 0$. The GTLS problem of (6) is then solved for each sub-model under a similar constraint to the one shown in (9), namely:

$$\hat{\alpha}_i = \inf_{\alpha_i} \left\{ \alpha_i > 0, \left| \bar{p}_i \sigma_u - \| p_i u - \hat{u}_i(\alpha_i) \|_F^2 \right| < \gamma \right\} \qquad (24)$$

where $\bar{p}_i$ is the mean value of $p_{i,k}$ for $k = 1, \ldots, N$.

Step 3: *Estimated model variables*
The resulting estimated noise-free external variables $\hat{w}_{mm}^{(j+1)}$ of the sub-models can then be used to re-estimate the corresponding noise-free inputs $\hat{w}_s^{(j+1)}(1 : m)$, which in turn, taking the mean value, gives, when p_i is non-zero:

$$\hat{w}_k^{(j+1)}(1 : m) = \frac{1}{s} \sum_{i=1}^{s} \frac{\hat{w}_{mm_{i,k}}^{(j+1)}(1 : m)}{p_i(\phi_k, c^{(j)}, \varpi^{(j)})} \qquad (25)$$

The corresponding noise-free outputs $\hat{w}_s^{(j+1)}(m+1 : q)$ is given by the sum of those of the sub-models:

$$\hat{w}_k^{(j+1)}(m+1 : q) = \sum_{i=1}^{s} \hat{w}_{mm_{i,k}}^{(j+1)}(m+1 : q) \qquad (26)$$

Each of the above steps are carried out iteratively until (21) is verified.

5. NONLINEAR SISO EXAMPLE

The extended multiple-model GTLS method, is demonstrated using the following nonlinear SISO system:

$$\begin{aligned} y_{k+1} = {} & 1.5 y_k - 0.7 y_{k-1} + 0.08 u_{k-1} + 0.04 u_{k-2} \\ & + 0.02 y_{k-1} u_{u-1} - 0.01 y_{t-1} u_{t-2} + 0.08 u_{t-1}^2 \end{aligned}$$

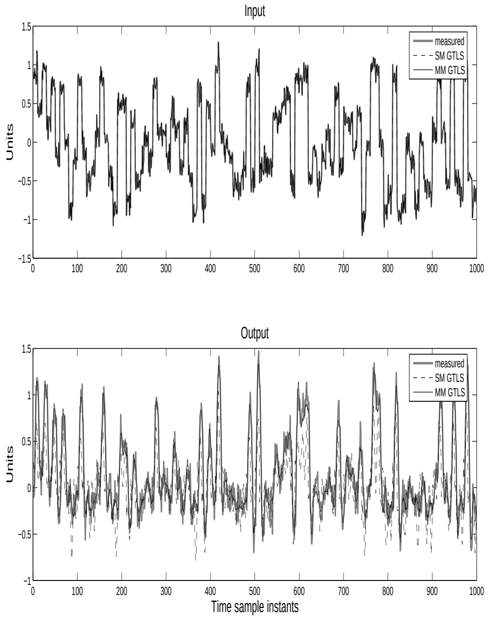

Fig. 3. External variables and their respective estimations using a single model (SM) and a multiple model (MM) for one particular run.

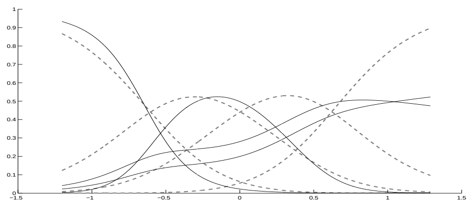

Fig. 4. Evolution of the radial basis functions r_i after optimisation with respect to c and ϖ

where initially $y_0 = y_1 = y_2 = 0$ and the external variables used for the modelling are the input and the output with additive noise sequences (here, $\mathcal{N}(0, 0.1)$) being independent and identically distributed, as shown in Figure 1. A multiple-model GTLS representation of the system is estimated using $s = 4$ sub-models and the corresponding estimated external variables are shown in Figure 3. For 100 runs, the mean of the misfits for the single model GTLS is 10.9 while the proposed multiple-model GTLS achieves a mean misfit of 4.47. The evolution of the radial basis functions r_i which form the scheduling vector is given in Figure 4, for one particular case. The dashed line set corresponds to the initial radial basis functions, chosen to span the operating range, while the solid line set shows the functions following optimisation with respect to c and ϖ.

6. CONCLUSION

An extension of the behavioural modelling approach to encompass a class of nonlinear systems, which exploits a weighted multiple-model representation has been proposed. Each sub-model, which retains the linear time invariance property, is repeatedly estimated using a new hybrid algorithm comprising a multiple-model global total least square procedure and a multiple-model global ordinary least square procedure which are sequentially executed. The classical multiple-model global ordinary least square technique is necessitated due to the requirement to re-construct the external variables for each successive implementation of the multiple-model global total least square method. Whilst the approach has been demonstrated using an arbitrary SISO nonlinear system, it may be readily applied to the MIMO case.

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