FLEXIBLE AEROSPACE VEHICLES SIMULATION AND NONLINEAR CONTROL SYNTHESIS

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Abstract: Possible approaches to the mathematical description of different types of flexible flying vehicles (FFV), and also the principles of software development for control system synthesis, dynamic properties research and FFV motion simulation are observed. The peculiarities of the program package specially created for such purposes are described. It includes the library of program modules designed on the base of mathematical models of FFV units and control system, and also such significant physical effects as flexibility, liquid oscillations, time lag of engines, local aerodynamic effects, etc. The adequate choice of the necessary program modules allows to investigate the dynamic properties of existing and prospective FFV. *Copyright © 2005 IFAC*

Keywords: flight control, flexible vehicle, oscillations damping, simulation, bending, sloshing, local angle of attack, control law.

1. INTRODUCTION

Increasing requirements to a maneuverability of air and space vehicles at minimum structure weight and great speed of motion makes investigation of FFV elastic properties very significant for providing the mission success[1]. Paying attention to these effects has a great importance at control of space stations and space probes, airplanes and other mobile objects liable to the considerable dynamic loads due to engines activity and environment resistance. The presence of flexibility determines a capability of oscillations appearance in control system at different resonance frequencies. Many cases when flexibility of the control plant was the reason for control system instability are known. Sometimes flexibility results in the arising of oscillations and finally in the structural failure. Creation of effective control units is embarrassed with the complexity of obtaining the trustworthy information about the elastic properties of vehicle, the considerable dependence of eigen frequencies on the varying weight, velocity and drag. Last two parameters largely depend on a flight path, which is often unknown beforehand. Complexity of calculation of the local aerodynamic loads applied to a surface of object makes the problem of control system design difficult in accomplishment.

Many vehicles considerably change mass and aerodynamic characteristics during the flight. From the point of view of control theory such vehicles are the typical non-linear and non-steady plants. The aim of designer is to create the light construction. For these reason such objects are deformed in flight, and their elastic properties appear. Elastic longitudinal and lateral oscillations of the complex form arise, which frequencies are changing during the flight. Elastic oscillations are usually described by differential partial equations or ordinary differential equations of the large dimension. Deformation of a body results in appearance of the local attack angles and slide angles. As a result of it the local forces and moments of forces are appearance that are synchronized with the changes of local angles of attack and slide. The local forces and moments are the reasons of amplification or attenuation of elastic oscillations. At excessive development of elastic oscillations the structural failure may take place.

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Oscillations of fuel and oxidizing agent in tanks result in origination of forces and moments of forces regarding all three axes of the vehicle. Eigenfrequencies and liquid oscillation frequencies depend on the shape of tanks and their arrangement at the vehicle, level of tanks occupation by liquid (Brodsky, Nebylov and Panferov, 2004; Brodsky et al, 2004). Development of liquid oscillations in tanks depends on motion of object and in turn influences on object motion, in particular on elastic component of oscillations. For this reason it is necessary to include the model of liquid oscillations in the structure of generalized model of vehicle motion. Technical complexity of the state-of-the-art vehicles results in the necessity to separate the vehicle designing process on some stages. The perspective approach to the solution of an elastic object control problem consists in arrangement of several sensors in different points of object and in creation the integrated control systems. Accuracy of elastic oscillations parameters estimation and, therefore, control efficiency depends on selection of sensors location points.

At the first stage of design a vehicle is considered as a rigid body of variable mass. At this stage the questions of vehicle rational aerodynamic configuration and required efficiency of control system actuators are solved. The possible methods and means of the vehicle motion stabilization as a rigid body are considered. Usually, at such research the theoretical methods of aerodynamics, flight dynamics, automatic control, and also specialized and universal programs are used. If the vehicle is unstable, at this design stage the synthesis of the elementary control system implements, ensuring a steady motion on a desirable path. During such simulation the parameters of a reference path, concerning which the linearization of equations implements automatically, are calculated. It is supposed that features of elastic object, unaccounted at this design stage, do not result in large deviations from a reference path and the possibility of model linearization is saved.

At a following design stage the flexibility of vehicle and oscillation of liquid in tanks are taken into account. The local forces and moments of forces as functions from time and coordinate along a centerline of vehicle are computed. Analytical and semi-graphical methods of design yield at this stage only approximated outcomes. For research of elastic systems the special programs exist, for example, ANSYS, NASTRAN, COVENTOR, FEMLAB, Structural Dynamics Toolbox for use with MATLAB, etc. In these programs the finite element method is used, which has been well recommended at calculation concerning the simple designs. For dynamic processes investigation and also for simulation of elastic oscillations of the flying vehicles, which consist of hundreds and thousand units of complex form, such approach is unsuitable. These programs do not allow solving the full complex of problems, which appear at designing of actual vehicle. The indicated reasons determine the necessity of design of the specialized program for simulation of motion for elastic objects of the complex form, the analysis of their dynamic properties and control system design.

In the present paper other approach to modeling, simulation and control system design for elastic objects is discussed. It is known, that the elastic object is described by partial derivative equations. The theory of such control plants is complex, bulky and analytically insufficiently designed presently. There are numerical methods of calculation of the arbitrary quantity of harmonics of elastic vibrations and replacements of partial derivative equations by ordinary differential equations of high dimension. For automation of such mathematical model of elastic flying vehicle formation, synthesis of a control law, analysis and simulation of controlled flight, and also representation of outcomes of simulation in the two-dimensional and three-dimensional space the authors have designed the concept of a specialized software package.

2. STRUCTURE OF CONTROL SYSTEM AND MATHEMATICAL MODELS OF UNITS

The block diagram of control system of a flexible vehicle is shown in Fig. 1.

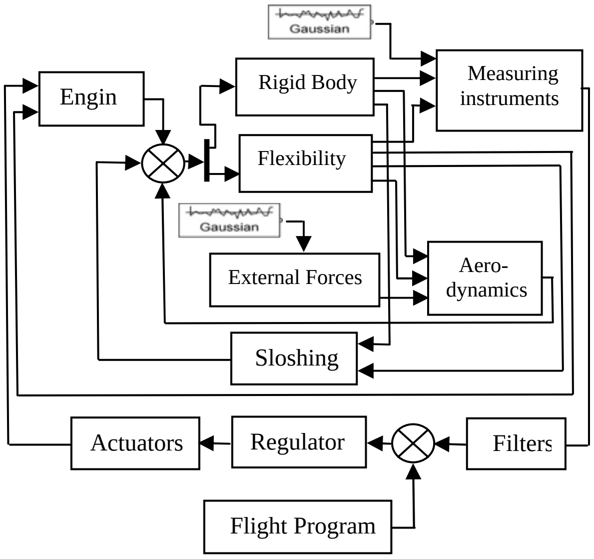

Fig.1. The simplified block diagram of control system for flexible vehicle

2.1 *Solid body dynamics model*

Solid dynamics is described by differential equation system in the reference frame connected with the solid body

$$\frac{d\mathbf{V}}{dt} = \frac{\mathbf{F}}{m} - \Omega \times \mathbf{V}, \tag{1}$$

$$\frac{d\Omega}{dt} = \mathbf{I}^{-1}\left(\mathbf{M} - \Omega \times (\mathbf{I} \cdot \Omega)\right), \tag{2}$$

where $\mathbf{V}$ is vector of a linear velocity of object in an inertial reference frame; Ω is an angular velocity vector of object; $\mathbf{F}$ is vector of an external forces; $\mathbf{M}$ is vector of external forces moments; $\mathbf{I}$ is tensor of moments of inertia.

2.2 *Flexibility*

The model of flexible displacements of elastic line from the longitudinal neutral axis is given by a system of differential equations

$$\Delta \mathbf{M}\ddot{\mathbf{q}} + \Delta \Xi \dot{\mathbf{q}} + \mathbf{q} = \Delta \mathbf{f}\,, \tag{3}$$

where $\mathbf{q}(t)$ is displacement of an elastic line in a normal direction to the longitudinal axis; Δ is a symmetrical stiffness matrix; $\mathbf{M}$ is a diagonal mass matrix; Ξ is a symmetrical structural damping matrix; $\mathbf{f}$ is a distributed load. More detail description of model of flexibility is referred in (Brodsky et al, 2004).

2.3 *Aerodynamics*

The initial data for calculation of the distributed aerodynamic coefficients $C_n(x)$ for the forces, which are applied to a vehicle surface, are theoretical curves for particular vehicle or data from an experiment. The distributed forces produce the local loads participating in formation of elastic oscillations. Total (integral) loads result forces and moments of forces determining the motion of vehicle center of gravity. Local and integral aerodynamic coefficients are connected by the equations

$$C_n = \sum_i C_{n_i} = \int_0^l c_n(x)dx\,, \tag{4}$$

$$C_m(x_{cg}) = \sum_i C_{n_i}(x_i - x_{cg}) = \int_0^l c_n(x)(x - x_{cg})dx\,, \tag{5}$$

$$C_{mq}(x_{cg}) = \sum_i C_{n_i}(x_i - x_{cg})^2 = \int_0^l c_n(x)(x - x_{cg})^2 dx\,, \tag{6}$$

where x_{cg} is a coordinate of object center of gravity.

Local aerodynamical forces in a longitudinal plane of vehicle for i-th segment of a surface can be calculated in accordance with a local angle of attack a_i^* for a arbitrary moment of time (Kuzovkov, 1976) by the formulae

$$a_i^* = a + \frac{x_{cg} - x_i}{V_i}\dot{\vartheta} - \frac{\dot{q}_i}{V_i} + \frac{\partial q_i}{\partial x_i}\,, \tag{7}$$

where a is an angle of attack for a solid body; ϑ is pitch; V_i is local air velocity; $\dot{q}_i$ is velocity of shape of elastic line in current time; $\frac{\partial q_i}{\partial x_i}$ is slope of elastic line in the current time.

2.4 *Models of local external actions*

All distributed and local loads are stochastic processes in wide band of frequencies. The in-depth physical description of these processes is very complex. Process of formation, development and vortex break-down depends rather complex also on the shape of object, its angular orientation, elastic deformations of a body, a velocity and an altitude of flight. The main influence on elastic oscillations is made by low-frequency external forces in frequency band from zero up to frequencies of second and third harmonics. The main synchronizing factor of a vortex break-down and appearance of external forces is atmosphere turbulence.

Even at a constant value of an object velocity in the atmosphere the vortexes are formed. These vortexes generate complex and time-varying pattern of local loads distribution on a surface of object. At high speed of flight in separate parts of vehicle there are local peaks of pressure. For their modeling it is important to define the zones of applying the large local loads and their varying in time. Usually these zones are arranged close to the transitions from conical surface to cylindrical one or in places of joints of surfaces of different more complex forms. Designers of vehicles usually aim to avoid such roughness, but it is not possible to remove them completely. Models for the description of the most typical factors, leading to local loads, are discribed here. Such factors are: a constant lateral wind with taking into account vortex generation, vortex generation in the field of joining of the nose cone and the cylinder, vortex generation in places of joints of separable stages, effect of vortexes in the area of stabilizing fin.

As an example the scheme of formation of the equivalent disturbance W_e is shown in Fig.2 at action of a steady wind in view of vortex generation. The following notations are used: σ_v is a mean-square deviation of a velocity of vortexes in a localized zone of object surface; T_v is a time constant of process of vortex generation; $\xi_1(t), \xi_2(t)$ are input white noises for the shaping filters.

In the blocks Subsystem and Subsystem1 the forming filters are located.

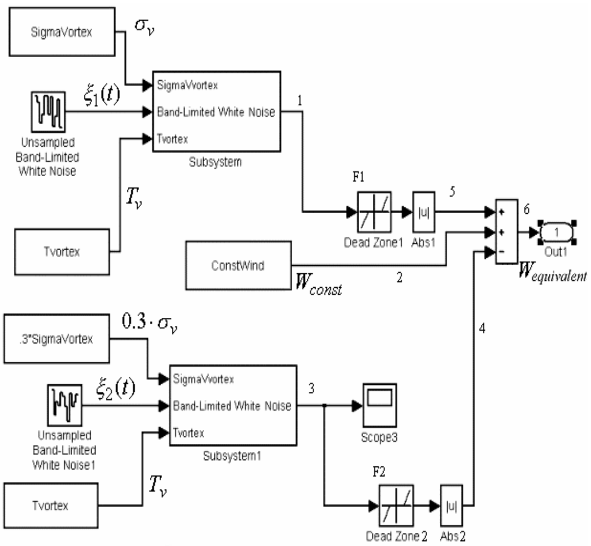

Fig.2. Formation of the equivalent disturbance at action of a steady wind in view of vortex generation

The resulting equation, on the base of which the equivalent disturbance formation is performed at a steady wind action is following:

$$W_{экв.} = \mathrm{abs}\left(F1\left(\sigma_v \sqrt{\frac{2}{T_v}} \cdot \frac{T_v}{T_v s+1} \cdot \xi_1(t) \right) \right) + W_{const} -$$

$$\mathrm{abs}\left(F2\left(0.3\sigma_v \sqrt{\frac{2}{T_v}} \cdot \frac{T_v}{T_v s+1} \cdot \xi_2(t) \right) \right). \qquad (8)$$

Parameters of insensitive zones of the functions F1 and F2 (Fig.2) definite the frequency of a vortex abruption and connected with it appearance of pressure increasing or decreasing at the local zone. The first summand definite the action of vortex abruption, which causes the impulse power to the direction of constant wind W_{const}, and the third summand - against this wind. The value W_e is used for the correction of local attack angle, which is multiplied on local aerodynamic coefficient for local aerodynamic force calculation. The velocity of object motion concerning the air causes vortex glide along the object surface from the nose up to stabilizer. This effect of vortex gliding is modeled by vector scheme of delay.

Large local loads appear near the places of particular constructive elements connection. Usually these places correspond to the bound of conic and cylindrical surface and places of connection of different diameter cylinders. Vortex streams are formed a bit further of the places of connection of the details in the direction of the stream. Vortex intensity and its abruption frequency mainly depend from flight conditions. The equation, which allows taking into account the results of calculations and the results of the experiment, are

$$W_{heterog} = \sigma_t \sqrt{\frac{2}{T_{L\_V}}} \cdot \frac{T_{L\_V}}{T_{L\_V} s+1} \cdot \xi_1(t) \ +$$

$$\mathrm{Sign}\left(F\left(\sqrt{\frac{2}{T_2}} \cdot \frac{T_2}{T_2 s+1} \cdot \xi_2(t) \right) \right) \times$$

$$\sigma_{Ampl} \sqrt{\frac{2}{T_{Ampl}}} \cdot \frac{T_{Ampl}}{T_{Ampl} s+1} \cdot \xi_3(t) \ , \qquad (9)$$

where $T_{L\_V} = \frac{L}{V}$; L is the turbulence scale (100-1000m); V is a flight velocity; T_2 is a vortex abruption time constant (while modeling usually it is taken as $T_2 = T_{L\_V}$); F is a nonlinear function, determining the vortex abruption frequency and connected with pressure increasing or decreasing at a local zone; T_{Ampl} is a time constant of local load of impulse component amplitude changing; σ_{Ampl} is a standard deviation of local load of impulse amplitude component.

At the surface of stabilizer the local loads usually have high intensity and may cause large flexible deformations. The equation for the description of local disturbances on the stabilizer is:

$$W_{stab} = \mathrm{sign}\left(F\left(\sqrt{\frac{2}{T_2}} \cdot \frac{T_2}{T_2 s+1} \cdot \xi_2(t) \right) \right) \times$$

$$\sigma_{Ampl} \sqrt{\frac{2}{T_{Ampl}}} \cdot \frac{T_{Ampl}}{T_{Ampl} s+1} \cdot \xi_3(t) \, . \qquad (10)$$

For model example the graph of varying in time the normalized value W_{stab} is shown in Fig.4.

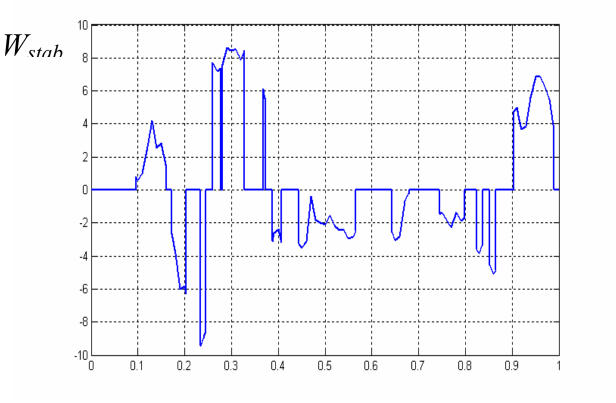

Fig.4. Graph of normalized value W_{stab} varying in time

2.5 *Liquid oscillation in tanks*

It is known, that the dynamical effects of liquid oscillation may be roughly approximated by changing the liquid mass to the solid mass harmonic oscillator. Oscillator parameters depend on tank diameter, liquid level, etc. Oscillated liquid produces the forces and moments of force, changes inertial moment of object, causes the oscillations of vehicle, its center of mass location displacement, etc. (Mishin, 1990).

2.6 *Sensors*

Flexible object body oscillations act on gyroscope gauges of angles and angle vehicles indications and on accelerometers also. Mathematical models of such devices are well known (Brodsky et al., 2004; Nebylov and Wilson, 2002). At researching the flexible objects it is important to take into account the sensors location. Optimization of points of sensors location is available only by researching the dynamical properties of the plant and the control system.

2.7 *Regulators and filters*

For application the various and well tested analysis and synthesis algorithms of automatic control linear systems in time and frequency domains, it is necessary to provide entering and adjusting of regulators and filters models as transfer functions and as differential equations. For flying vehicle control system research with quickly changing characteristics of stability and controllability the special research is necessary to provide the possibility of unsteady regulators and filters use.

2.8 *Actuators*

The electro-hydraulic steering actuators are often used at the up-to-date vehicles. At the stage of initial research the simple description of steering actuators as dynamical units of first and second order are used. Considering vehicle flexibility it is necessary to pay attention to the opportunity of aero-hydro flexible oscillations appearance (Caldwell, 2000).

2.9 *Flight program*

Modern high-speed vehicles generally make complex maneuvers that not always can be predicted beforehand. The modeling program should provide an opportunity for simple appoint the program of flight both in vertical and in horizontal plane.

2.10 *Engine*

If turned engines are used for control of vehicle motion, it is necessary to consider two components of force and moment of forces: static and dynamic. The first component is defined by angles of turn of the engine concerning the vehicle body δ_y, δ_z. For the projections of forces F and the moments of forces M on the axis of object it is possible to write down

$$F_{xT} = T\cos\delta_y\cos\delta_z \approx T;\ F_{yT} = T\sin\delta_z \approx T\delta_z;$$
$$F_{zT} = T\sin\delta_y \approx T\delta_y;\quad \mathrm{M}_{\mathrm{xT}} = 0;$$
$$M_{yT} = Tl_{CG}\sin\delta_y \approx Tl_{CG}\delta_y; M_{zT} = Tl_{CG}\sin\delta_z \approx Tl_{CG}\sin\delta_z,$$

where T is an engine thrust, l_{CG} is a distance from the hinge of the engine up to the vehicle center of mass.

The dynamic component of control force and the moment of forces is defined by angular acceleration of the engine and linear acceleration of its center of mass. Summarizing these components, it is possible to write down for axes y and z of the object:

$$M_d = m_d l_h l_{CG}\ddot{\delta} + J_d\ddot{\delta};\ F_d = (m_d l_h l_{CG}\ddot{\delta} + J_d\ddot{\delta})/l_{CG}, \quad (11)$$

where m_d is engine mass, J_d is engine inertial moment; J_h is the distance from engine mass center up to the hinge.

3. PARTICULARITIES OF THE PROGRAM DESIGN AND OPERATION

The core of the program is built in Matlab with Simulink use. Such approach allows to use the well approved modules for analysis, synthesis, transforming math model etc., included in Control System Toolbox, Symbolic Math Toolbox and another Matlab abilities. The program interface has been designed in two variants. In the first one the means of interface developing Matlab GUI are used, in the second one language C++, accommodates more abilities and more easy-to-users, is used. For simulation accessibility of various class vehicles, the program is provided with wide libraries of ready program blocks: vehicle models, engines, steering actuators, standard regulators and filters, sensors, tank liquid oscillation models, local external actions models. Library can be amplified by making principal new models or by parameters correction of existing model.

On the first stage of work with the program it is necessary to enter the constructive parameters of vehicle, data about mass variance and moments of inertia in process of flight, location and filling of tanks with fuel and oxidant, aerodynamic parameters, flexible parameters, initial position of vehicle, etc. Program interface allows changing numerous parameters on the screen (by mouse) and changing digital data directly in databases, represented in interface by tables. In the last case all changes are imaging on the vehicle picture. For accessibility the program is provided with the test example, which allows varying of all parameters and saving them at the hard disk.

By the initial data the program automatically assembled nonlinear vehicle spatial model. As the stability and controllability characteristics at various velocities and altitudes of flight are different, it is necessary to definite the flight path and velocity profile beforehand. Further for relatively simple model of nonlinear vehicle without taking into account the flexible oscillations and maximum simplification of basic elements models of control system, aim of which is providing of stability motion over the trajectory. After previous simulation and program control signals correction, if necessary, the space state vector is saved in the database for all flight time, as the base trajectory for further complex model linearization. Liquid oscillations in the tanks, etc. assumed not to come to large base trajectory diversion for rightly designed system. The program allows control system complexity managing, that gives opportunity to analyze the action of each factor on the control system dynamic characteristics.

The program enables:

- to find a linear mathematical model of any fragment of a control system for any instant of flight in a state space and as transfer functions for the given input vectors and output actions, and also for an open loop system;
- to simplify a mathematical model and to check simplification by comparison of time or frequency characteristics;
- to make a structural and parametric control system synthesis in frequency domain for any instant of flight and to check results by frequency and time responses, and also by roots of a characteristic equation disposition;
- to make a synthesis of the regulator in time domain, including Kalman filter and optimal regulator, to make a reduction of the regulator and also to analyze the quality of control;

- to estimate the quality of a control system with varying in time parameters by digital simulation with taking into account the act of external disturbances and measurement errors maximally approximated the actual ones;

- to build schedules for change of any parameter in process or after simulation, to display animations of a bending modes of an elastic axis of the flexible vehicle;

- to display the three-dimensional animation of flight with a capability to exaggerate bending and oscillations of a liquid in tanks for the greater obviousness.

The program consists of the separate modules, which adding and change allows to use it for simulation the broad range of flexible objects.

As an example the fragments of animation of the first and the second derivatives of center line displacement are shown in Fig.5 and Fig.6 for one instant during elastic oscillations process. The axis of abscissa *x* corresponds to the coordinate of vehicle centerline, and at the axis of ordinate the corresponding derivative in the normalized scale is shown. Circles mark points of conditional division of the vehicle on the mass points at the discrete description of object.

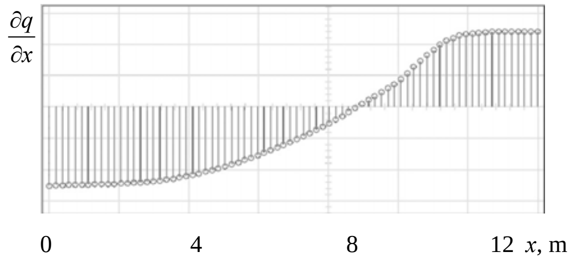

Fig.5. Derivative of displacement of a center line during elastic oscillations

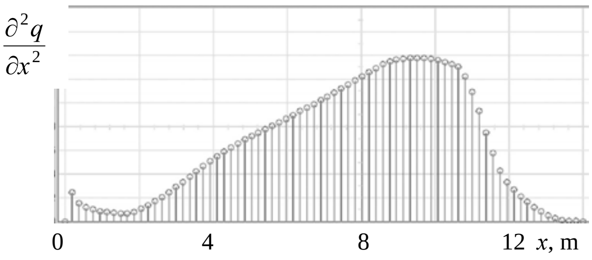

Fig.6. Second derivative of displacement of a center line during elastic oscillations

4. CONCLUSION

The software for simulation of flexible essentially non-steady vehicle motion, synthesis of control systems for such a vehicle, research of dynamic properties by different methods in time and frequency domains, is developed and described in this paper. The basis of the program is the structure which allows analyzing the dynamic responses, simulation and visual information representation of the complex dynamic systems. The program is equipped with the library of program modules. These modules are designed on the basis of mathematical models of vehicle elements and control system, and also on the base of significant physical effects such as elasticity, oscillations of liquid, time lag in engines, local aerodynamic effects, etc. Selection of the certain program modules allows to investigate the dynamic properties, stability and controllability for different existed and prospective vehicles and other transport means. The module structure of the program allows to enlarge area of its application at the expense of adding the separate modules written in the language C++, Matlab, Femlab, Simulink, etc. The program interface allows changing parameters of vehicle in a broad band, simulation of motion of the arbitrary kind and output of the results of simulation and analysis in digital or graphic representation.

The work was fulfilled during the last three years in the International Institute for Advanced Aerospace Technologies of the State University of Aerospace Instrumentation. The authors are interested in extension the field of possible application of the program package and also the theory and algorithms for the flexible objects investigation.

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