CONTROL OF SINGULARLY PERTURBED SYSTEMS UNDER ACTUATOR SATURATION

I. Lizarraga[1], S. Tarbouriech[2], G. Garcia[2]

[1] Departamento de Electricidad y Electrónica, Universidad del País Vasco
Apdo. 644 – Bilbao 48080 (Spain). E-mail: ibone@we.lc.ehu.es
[2] L.A.A.S.-C.N.R.S
7, Avenue du Colonel Roche – 31077 Toulouse Cedex 4 (France)
E-mail: tarbour@laas.fr, garcia@laas.fr

Abstract: This paper addresses the stabilization problem of singularly perturbed systems subject to actuator saturation. A solution in terms of linear matrix inequalities is proposed. The saturation is represented by means of a modified sector nonlinearity. The numerical problems due to the ill conditioning of the system model are avoided using a non-decoupling strategy. Some numerical results are also included for testing the proposed methods.

Keywords: Singular perturbation method, Actuator saturation, Linear Matrix Inequalities

1. INTRODUCTION

Added to the dominant slow dynamics, many physical systems exhibit a high frequency dynamics that is usually neglected for modeling and control purposes. For some systems, neglecting this dynamics can cause bad performance or even instability in closed-loop. Very often, this kind of systems can be described using the singular perturbation theory. In the standard form of singular perturbation theory, the time scale separation of the system dynamics is explicitly represented by a small perturbation parameter ε. The presence of two very different time-scales is responsible for the numerical problems that are characteristic of singularly perturbed systems. For controlling these systems a decoupled strategy is usually applied (Kokotovic, *et al.*, 1999). In this strategy the system is decomposed into a slow and a fast subsystem (in which the ε parameter has been removed) and a controller is designed for each subsystem. The control action for the overall system is obtained by combining the slow and fast controllers. In this way, numerical problems are avoided and the control problem is simplified. However, the decoupled strategy can not be straightforwardly applied to a system with actuator saturation. Indeed, due to the saturation nonlinearity, the system can not be decomposed in closed loop. In this paper a new strategy is proposed for stabilizing singularly perturbed systems with actuator saturation. The new strategy avoids the numerical problems associated with the presence of the small perturbation parameter ε without explicitly decomposing the system and the control law into slow and fast terms.

Previous works concerning the problem of singularly perturbed system with saturation are (Liu 2001) and (Garcia and Tarbouriech, 2003). In (Liu 2001) system decomposition is possible because only slow variables are used for feedback, whereas the fast dynamics is assumed to be stable. In (Garcia and Tarbouriech, 2003) no restrictions on the stability of the system are imposed but the control value is maintained below the saturation level (bounded control). The controller is then obtained solving a set of LMIs (Linear Matrix Inequalities), one of them ensuring that the control law never saturates. Thus, the system is forced to behave linearly and the decoupled control is possible. The drawback of this approach is that saturation avoidance does not allow to use all the capacity of the actuator.

In this work a different approach is presented. As in (Garcia and Tarbouriech, 2003), a set of LMIs is also obtained, although no decomposition of the system is performed. This will make possible to solve the linear and bounded control problems as well as the general saturation case. Since the system may be unstable in open-loop, global results cannot be achieved due to the saturation. Therefore only local results are developed here. The saturation has been represented by means of a locally sector-bounded nonlinearity and a quadratic Lyapunov function has been used for the stability analysis (Gomes da Silva and Tarbouriech, 2003). Contrarily to other approaches (Gomes da Silva, *et al.*, 2002; Kapila, *et al.*, 2001; Pare, *et al.*, 2002), where classical sector conditions are considered, the modified sector condition allows us to obtain directly LMI conditions for controller synthesis.

The paper is organized as follows. The problem of designing a stabilizing state-feedback controller for a singularly perturbed system with actuator saturation is formally stated in Section 2. In Section 3 the new non-decoupling control strategy for linear singularly perturbed systems is introduced. In Section 4, the general solution for the control design with saturation is proposed. Finally, numerical results are shown in Section 5.

2. PROBLEM STATEMENT

Let us consider the following singularly perturbed system:

$$\begin{pmatrix} \dot{x} \\ \varepsilon \dot{z} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u \qquad (1)$$

where $\varepsilon > 0$, $x \in \mathbb{R}^n$, $z \in \mathbb{R}^m$ and $u \in \mathbb{R}^r$. The augmented state $\eta = \begin{pmatrix} x & z \end{pmatrix}' \in \mathbb{R}^{n+m}$ is the state, u is the control. A_{11}, A_{12}, A_{21}, A_{22}, B_1 and B_2 are constant matrices of appropriate dimensions.

The control vector *u(t)* takes values in the compact set $U \subset \mathbb{R}^r$:

$$U = \left\{ u \in \mathbb{R}^r; -u_{0(i)} \le u_{(i)} \le u_{0(i)}, u_{0(i)} > 0, i = 1, \ldots, r \right\} \qquad (2)$$

For any vector $v \in \mathbb{R}^r$, the saturation function $sat(v)$, is defined according to (2):

$$sat\left(v_{(i)}\right) = sign\left(v_{(i)}\right) \min\left(u_{0(i)}, \left|v_{(i)}\right|\right)$$

System matrices A_ε and B_ε issued from (1) can be defined by:

$$A_\varepsilon \triangleq \begin{pmatrix} A_{11} & A_{12} \\ \dfrac{A_{21}}{\varepsilon} & \dfrac{A_{22}}{\varepsilon} \end{pmatrix}, B_\varepsilon \triangleq \begin{pmatrix} B_1 \\ \dfrac{B_2}{\varepsilon} \end{pmatrix} \qquad (3)$$

The main problem addressed in this paper can be stated as follows:

Problem 1 *Find a control gain* $K \in \mathbb{R}^{r \times (n+m)}$ *and a set of initial conditions* I_0 *such that the system (1) controlled with* $u(t) = sat\left(K\eta(t)\right)$ *is asymptotically stable for any* $\eta(0) \in I_0$.

For the singularly perturbed systems, due to the small value of the perturbation parameter ε, some numerical problems arise when solving Problem 1. A solution to this drawback consists of solving well-behaved ε-independent small-order problems. The design is separated into slow and fast designs and the two controllers are then combined to build a composite control which stabilizes the original system for sufficiently small ε. In contrast, in this work a solution that does not require decomposing neither the system nor the control design problem is proposed. As in the previous approach, the controller is valid for small values of ε and this parameter does not appear in the design conditions (thus avoiding the numerical stiffness).

If no particular assumption about the open-loop system stability is considered, the saturation prevents from obtaining global stabilization conditions. Only local stability can be achieved. Since the exact characterization of the stability domain is a very complex task, a practical design objective can be obtaining an approximation with the maximal size.

3. STABILIZATION OF LINEAR SINGULARLY PERTURBED SYSTEMS

First the linear case is considered. We suppose that the control is not constrained.

The following sets are defined:

$$\mathcal{W}_\varepsilon = \left\{ W(\varepsilon) = \begin{pmatrix} W_1(\varepsilon) & W_2(\varepsilon) \\ W_2(\varepsilon)' & W_3(\varepsilon) \end{pmatrix} > 0; \begin{matrix} W_1(\varepsilon) \in \mathbb{R}^{n \times n} \\ W_3(\varepsilon) \in \mathbb{R}^{m \times m} \end{matrix} \right\} \qquad (4)$$

$$\mathcal{S}_\varepsilon = \left\{ S(\varepsilon) = (S_1(\varepsilon), S_2(\varepsilon)); S_1(\varepsilon) \in \mathbb{R}^{r \times n}, S_2(\varepsilon) \in \mathbb{R}^{r \times m} \right\} \qquad (5)$$

$$\mathcal{C}_\varepsilon = \left\{ (W(\varepsilon), S(\varepsilon)); \quad W(\varepsilon) \in \mathcal{W}_\varepsilon, S(\varepsilon) \in \mathcal{S}_\varepsilon \right.$$
$$\left. \text{and } A_\varepsilon W(\varepsilon) + W(\varepsilon) A_\varepsilon' + B_\varepsilon S(\varepsilon) + S(\varepsilon)' B_\varepsilon' < 0 \right\} \qquad (6)$$

The following expressions are proposed as power series expansions of *W(* ε *)* and *S(* ε *)*:

$$W(\varepsilon)=\begin{pmatrix} W_1^0 & W_2^0 \\ W_2^{0\prime} & \varepsilon^{-1}W_3^0 \end{pmatrix}+\sum_{i=1}^{\infty}(\varepsilon)^i\begin{pmatrix} W_1^i & W_2^i \\ W_2^{i\prime} & \varepsilon^{-1}W_3^i \end{pmatrix}$$
$$S(\varepsilon)=\begin{pmatrix} S_1^0 & \varepsilon^{-1}S_2^0 \end{pmatrix}+\sum_{i=1}^{\infty}(\varepsilon)^i\begin{pmatrix} S_1^i & \varepsilon^{-1}S_2^i \end{pmatrix} \tag{7}$$

Introducing the notation:

$$W^0 \triangleq \begin{pmatrix} W_1^0 & W_2^0 \\ W_2^{0\prime} & \varepsilon^{-1}W_3^0 \end{pmatrix},\quad S^0 \triangleq \begin{pmatrix} S_1^0 & \varepsilon^{-1}S_2^0 \end{pmatrix}$$

the following result, deduced from the application of Theorem in (Bernussou, *et al*., 1989) to system (1), is obtained:

Lemma 1 *System (1) is stabilizable by a control law* $u(t)=K(\varepsilon)\eta(t)$ *if and only if* $\mathcal{C}_\varepsilon \neq \emptyset$.
A control gain is given by $K(\varepsilon)=S(\varepsilon)W(\varepsilon)^{-1}$ *and* $\lim_{\varepsilon\to 0} K(\varepsilon)=S^0(W^0)^{-1}$.

Using (7) and (3), condition in (6) for $\varepsilon \to 0$ can be written:

$$\begin{pmatrix} H_1 & \varepsilon^{-1}\tilde{H}_2' \\ \varepsilon^{-1}\tilde{H}_2 & \varepsilon^{-2}\tilde{H}_3 \end{pmatrix}<0 \tag{8}$$

with

$$H_1 = A_{11}W_1^0 + W_1^0A_{11}' + A_{12}W_2^{0\prime} + W_2^0A_{12}' + B_1S_1^0 + S_1^{0\prime}B_1'$$
$$\tilde{H}_2 = A_{21}W_1^0 + A_{22}W_2^{0\prime} + W_3^0A_{12}' + B_2S_1^0 + S_2^{0\prime}B_1'$$
$$\tilde{H}_3 = A_{22}W_3^0 + W_3^0A_{22}' + B_2S_2^0 + S_2^{0\prime}B_2'$$

Condition (8) is equivalent to:

$$\begin{pmatrix} H_1 & \tilde{H}_2' \\ \tilde{H}_2 & \tilde{H}_3 \end{pmatrix}<0 \tag{9}$$

Lemma 1 gives the following result:

Theorem 1 *Suppose that there exist symmetric positive definite matrices* $W_1^0 \in \mathbb{R}^{n\times n}$ *and* $W_3^0 \in \mathbb{R}^{m\times m}$*, and matrices* $W_2^0 \in \mathbb{R}^{n\times m}$, $S_1^0 \in \mathbb{R}^{r\times n}$ *and* $S_2^0 \in \mathbb{R}^{r\times m}$*, that satisfy condition (9), then there exists an upper bound* $\hat{\varepsilon}>0$ *such that for all* $\varepsilon \in (0,\hat{\varepsilon}[$ *the following properties hold:*

i) The matrices W^0 *and* S^0 *from (7) satisfy* $(W^0,S^0)\in\mathcal{C}_\varepsilon$.

ii) The control law:

$$u=K\eta=[S_1^0 \quad S_2^0]\begin{bmatrix} W_1^0 & 0 \\ W_2^{0\prime} & W_3^0 \end{bmatrix}^{-1}\eta$$

stabilizes system (1).

Proof. The result follows directly from the application of Lemma 1 as $\varepsilon \to 0$. ◊

Remark. Condition (9) is linear in the controller variables and does not depend on the perturbation parameter ε. Therefore, numerical problems during the control design are avoided and a solution can be obtained using LMI solvers.

4. STABILIZATION RESULT FOR LINEAR SINGULARLY PERTURBED SYSTEMS WITH SATURATION

To solve Problem 1 the saturation is expressed in terms of a sector nonlinearity and a new sector condition, introduced in (Gomes da Silva and Tarbouriech, 2003), is considered.

4.1 Sector condition

Let us define:

$$\psi(K\eta)=sat(K\eta)-K\eta \tag{10}$$

This function $\psi(K\eta)$ is a decentralized sector nonlinearity that satisfies the following sector condition:

$$\psi(K\eta)'\left[\psi(K\eta)+E\eta\right]\leq 0 \tag{11}$$

with $E\in\mathbb{R}^{r\times(n+m)}$, for any vector η belonging to the polyhedral set *D(K-E, U*$_0$*)* defined by:

$$D(K-E,U_0)=\left\{\eta\in\mathbb{R}^{n+m}; -u_{0(i)} \leq (K-E)_{(i)}\eta \leq u_{0(i)} \quad i=1,\ldots,r\right\}$$

Introducing (10) and using system matrices in (3), system (1) can be expressed in closed-loop as:

$$\dot{\eta}=(A_\varepsilon+B_\varepsilon K)\eta+B\psi(K\eta) \tag{12}$$

4.2 Stabilization problem

The objective is to solve Problem 1. For that, the sector nonlinearity (10) and the sector condition (11) are considered, so the system (1) is expressed as in (12).

The control gain is expressed according to Lemma 1 and the same structure as in (7) is proposed for *W(*ε*)* and *S(*ε*)*.

The stability analysis is performed using quadratic Lyapunov functions. A candidate function can be written as:

$$V(\eta) = \eta' W(\varepsilon)^{-1}\eta \quad W(\varepsilon) = W(\varepsilon)' > 0 \tag{13}$$

System (12) will be asymptotically stable for any initial condition in a set I_0 if

$$\dot{V}(\eta) < 0, \quad \forall \eta \in I_0, \eta \neq 0 \tag{14}$$

To prove condition (14), sector condition (11) will be used, so for matrix E the following expression in terms of $W(\varepsilon)$ is introduced:

$$E = Y(\varepsilon)W(\varepsilon)^{-1}, \quad Y(\varepsilon) \in \mathbb{R}^{r\times(n+m)} \tag{15}$$

Power series expansion of $W(\varepsilon)$ is given in (7) and for $Y(\varepsilon)$ the following expansion is proposed

$$Y(\varepsilon) = \begin{pmatrix} Y_1^0 & \varepsilon^{-1}Y_2^0 \end{pmatrix} + \sum_{i=1}^{\infty} (\varepsilon)^i \begin{pmatrix} Y_1^i & \varepsilon^{-1}Y_2^i \end{pmatrix}$$

Taking the limit for $\varepsilon \to 0$:

$$\lim_{\varepsilon\to 0} E = \begin{bmatrix} Y_1^0 & Y_2^0 \end{bmatrix} \begin{bmatrix} W_1^0 & 0 \\ W_2^{0\prime} & W_3^0 \end{bmatrix}^{-1} \tag{16}$$

The stability domain I_0 will be included inside the polyhedral set $D(K\text{-}E, U_0)$.

The following result solves Problem 1.

Proposition 1 *If there exist symmetric positive definite matrices:* $W_1^0 \in \mathbb{R}^{n\times n}$, $W_3^0 \in \mathbb{R}^{m\times m}$, *matrices* $W_2^0 \in \mathbb{R}^{n\times m}$, $S_1^0 \in \mathbb{R}^{r\times n}$, $S_2^0 \in \mathbb{R}^{r\times m}$, $Y_1^0 \in \mathbb{R}^{r\times n}$ *and* $Y_2^0 \in \mathbb{R}^{r\times m}$, *and a diagonal positive definite matrix* $T \in \mathbb{R}^{r\times r}$ *such that*

i)
$$\left[\begin{array}{c|c} \begin{matrix} H_1 & \tilde{H}_2' \\ \tilde{H}_2 & \tilde{H}_3 \end{matrix} & \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} T - \begin{bmatrix} Y_1^{0\prime} \\ Y_2^{0\prime} \end{bmatrix} \\ \hline T[B_1' \ B_2'] - [Y_1^0 \ Y_2^0] & -2T \end{array}\right] < 0$$

ii)
$$\left[\begin{array}{c|c} \begin{matrix} W_1^0 & 0 \\ 0 & W_3^0 \end{matrix} & \begin{bmatrix} S_{1(i)}^{0\ \prime} \\ S_{2(i)}^{0\ \prime} \end{bmatrix} - \begin{bmatrix} Y_{1(i)}^{0\ \prime} \\ Y_{2(i)}^{0\ \prime} \end{bmatrix} \\ \hline \begin{bmatrix} S_{1(i)}^0 & S_{2(i)}^0 \end{bmatrix} - \begin{bmatrix} Y_{1(i)}^0 & Y_{2(i)}^0 \end{bmatrix} & u_{0(i)}^2 \end{array}\right] \geq 0, i = 1\ldots r$$

Then for $\varepsilon \in (0, \hat{\varepsilon}[$ *, the control law*

$$u = K\eta = [S_1^0 \quad S_2^0] \begin{bmatrix} W_1^0 & 0 \\ W_2^{0\prime} & W_3^0 \end{bmatrix}^{-1} \eta$$

and the ellipsoid

$$I_0 = \left\{ \eta \in \mathbb{R}^{n+m}; \quad \eta' P_e \eta \leq 1 \right\}$$

$$P_e = \begin{pmatrix} W_1^0 & W_2^0 \\ W_2^{0\prime} & W_3^0 + W_2^{0\prime}(W_1^0)^{-1}W_2^0 \end{pmatrix}^{-1} \tag{17}$$

solve problem 1.

Proof. Using Schur complement and multiplying on the left and on the right by $\begin{pmatrix} W_1^0 & 0 \\ W_2^{0\prime} & W_3^0 \end{pmatrix}^{-T}$ and $\begin{pmatrix} W_1^0 & 0 \\ W_2^{0\prime} & W_3^0 \end{pmatrix}^{-1}$ respectively, and taking into account (15), Condition *ii)* is equivalent to:

$$P_e - \left\{K_{(i)}' - E_{(i)}'\right\} u_{0(i)}^{-2} \left\{K_{(i)} - E_{(i)}\right\} \geq 0 \quad i = 1\ldots r$$

Therefore, the satisfaction of *ii)* ensures that the ellipsoid I_0 is included in the polyhedral set $D(K\text{-}E, U_0)$ and for all $\eta \in I_0$ sector condition is satisfied.

Multiplying on the left and on the right relation *i)* by $\left[\eta' \begin{pmatrix} W_1^0 & W_2^0 \\ 0 & W_3^0 \end{pmatrix}^{-1} \middle| \psi' T^{-1} \right]$ and by its transpose respectively, it is obtained that the time-derivative of the quadratic Lyapunov function along the trajectories of the system (12) satisfies:

$$\dot{V}(\eta) \leq \dot{V}(\eta) - 2\psi(\eta)' T^{-1} \left(\psi(\eta) + E\eta\right) < 0$$

for any η in I_0. Since this reasoning is valid for any η in I_0, $\eta \neq 0$, one can conclude that the set I_0 is a set of asymptotic stability for the closed-loop system. Therefore, the conditions of Proposition 1 allow to obtain a solution to Problem 1. ◊

Certain conditions to be satisfied can be added to Proposition 1 in order to achieve particular control objectives.

Global stability is only possible if the open loop system is stable, in such case $D(K\text{-}E, U_0)$ must satisfy $D(K - E, U_0) = \mathbb{R}^{n+m}$ and E=K.

5. NUMERICAL RESULTS

Several numerical examples have been performed to illustrate the validity of the proposed design techniques.

The design is performed according to Proposition 1 in Section 4 with the goal of obtaining a set of initial conditions I_0 as large as possible.

It is worth to notice that the conditions in Proposition 1 are under LMI form in the decision variables. This fact is due to use of a modified sector condition. The use of classical sector condition as in (Gomes da Silva, *et al.*, 2002) does not lead to LMI conditions but to BMI conditions. Thus, in this case, the exhibition of a solution to Problem 1 maximizing the estimate of the region of stability should be done by means of iterative schemes. Such solutions are very sensitive to the initial considered guess and local sub-optimality can be guaranteed. In the current paper, the solution does not require initial guesses neither iterative schemes.

Different optimization criteria can be considered to maximize the size of the initial conditions set. In particular, the volume of the set I_0 is proportional to $\sqrt{\det(P_e^{-1})}$. Besides, according to the definition of P_e (17), it follows that:

$$\det(P_e^{-1}) = \det(W_1^0)\det(W_3^0)$$

Therefore, for design purposes, the initial condition set can be maximize by maximizing the size of W_1^0 and W_3^0

Consider the system (1) described by the following numerical data:

$$A_{11} = \begin{bmatrix} 0 & 0.4 \\ 0 & 0 \end{bmatrix}; A_{12} = \begin{bmatrix} 0 & 0 \\ 0.345 & 0 \end{bmatrix}; A_{21} = \begin{bmatrix} 0 & -0.524 \\ 0 & 0 \end{bmatrix};$$
$$A_{22} = \begin{bmatrix} 0.465 & -0.262 \\ 0 & 1 \end{bmatrix}; B_1 = \mathbb{I}_2; B_2 = \mathbb{I}_2 \tag{18}$$

To avoid obtaining an excessively large initial conditions set, a maximum value for each component of the vector state has been imposed:

$$\eta_{\max} = \begin{bmatrix} 100 & 100 & 10 & 10 \end{bmatrix}'$$

The controller design does not depend on ε. However, to simulate the system response, it is necessary to complete the system definition in (18) by specifying its value: $\varepsilon = 0.01$. Two cases are considered for the saturation value u_0. The LMI control toolbox for Matlab (Gahinet *et al.*, 1995) has been used for calculating the state feedback control gain *K*, with the following results:

- For $u_0 = 2$, the control gain is:

$$K = 1e^{+6} * \begin{bmatrix} 0.0004 & 0.0324 & -0.0058 & -0.0983 \\ 0.0093 & 0.8093 & -0.0990 & -2.5756 \end{bmatrix}$$

- For $u_0 = 4$, the controller is:

$$K = 1e^{+4} * \begin{bmatrix} 0.0211 & 1.0433 & -0.6613 & -1.9975 \\ 0.0857 & 4.1594 & -2.3843 & -8.4825 \end{bmatrix}$$

Fig 1 depicts the projections of I_0 for the two considered cases. Note that the size of this set is limited in certain directions due to the maximal value imposed for the state vector denoted η_{max}.

Simulations results for $u_0 = 2$ are displayed in Fig 2, where the convergence of the state vector from the initial conditions $[10\ 0.5\ 1\ 0.125]'$ to the equilibrium point can be observed. The transient response could be improved by changing the design criterion of maximizing I_0.

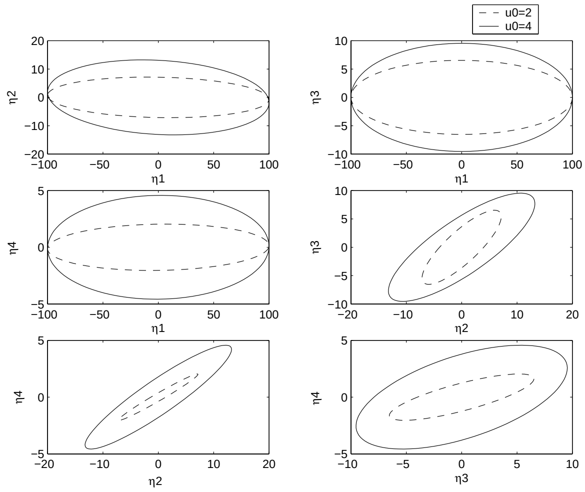

Fig. 1: Representation of I_0 by projection.

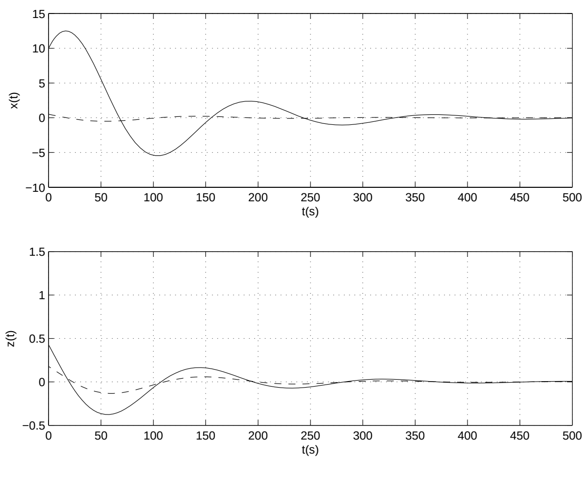

Fig. 2: Saturation Value $u_0 = 2$- Time evolution of vector state from $[10\ 0.5\ 1\ 0.125]'$.

6. CONCLUSION

In this paper, LMI conditions have been proposed to treat the state feedback gain design for linear singularly perturbed systems subject to actuator saturation. The obtained conditions are proposed via the use of a modified sector condition and a new non-decoupling control strategy which allows to cope with some drawback appearing when using other modeling for the closed loop system (like classical sector condition). In contrast with the usual approach for the singularly perturbed systems, the ill-conditioning problem has been avoided without decomposing the system model into slow and fast subsystems.

ACKNOWLEDGEMENTS

I. Lizarraga thanks to the UPV/EHU and to the MCYT for the financial support of this work through the projects 9/UPV00224.310-15254/2003 and DPI2002-04155-C02-01, respectively.

REFERENCES

Bernussou, J., P.L.D. Peres and C. Geromel (1989). A linear programming oriented procedure for quadratic stabilization of uncertain systems. *Systems and control letters*, **13**, 65-72.

Gahinet, P., A. Nemirovski., A. J. Laub and M. Chilali (1995) *LMI Control Toolbox* The Mathworks, Natick, MA.

Garcia, G. and S. Tarbouriech (2003). Control of singularly perturbed systems by bounded control. *Proc. of the ACC03*, Denver, USA, **5**, 4482-4487.

Gomes da Silva Jr., J.M., S. Tarbouriech and R. Reginatto (2002). Analysis of regions of stability for linear systems with saturating inputs through an anti-windup scheme. *Proc. of the IEEE Conference on Control Applications (CCA),* Glasgow, Scotland, UK, **2**, 1106-1111.

Gomes da Silva Jr., J.M. and S. Tarbouriech (2003). Anti-windup design with guaranteed regions of stability: an LMI-based approach. *Proc. of the IEEE CDC03,* Hawaii, USA, **5**, 4451-4456.

Kapila, V., A.G. Sparks, H. Pan (2001). Control of systems with actuator saturation non-linearities: an LMI approach. *Int. Journal of Control*, **74**, 586-599.

Kokotovic, P., H.K. Khalil and J. O'Reilly (1999). *Singular perturbation methods in control: Analysis and design*. Academic Press, New York.

Liu, P.-L. (2001). Stabilization of singularly perturbed multiple-time-delay systems with a saturating actuator. *Int. Journal of Systems Science*, **32**, 1041-1045.

Pare, T., H. Hindi, J. How (2002). Robust control design for systems with saturating nonlinearities. In: *Actuator saturation control* (V. Kapila and K. M. Grigoriadis, Ed.), pp. 163-187. Marcel Dekker, New York.