MODELING OF POWER AMPLIFIER NONLINEARITIES USING VOLTERRA SERIES

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Abstract: Radio Frequency (RF) power amplifiers (PAs) are one of the most important elements in radio telecommunication facilities, which are used to amplify the power of input signals. At the same time, they gradually become the bottleneck for modern telecommunication systems due to the confliction between the linearity and the efficiency, resulting in their relatively high level energy consumption. Volterra kernels are naturally good representations of intermodulation and harmonic distortions, the main distortions in typical power devices, because of their convolutional structures. Some work has been done to model the distortions by using Volterra series. However, Volterra series expansions are usually truncated severely, due to complexity of computation, so that they do not give accurate representations of the original system. It is intended ultimately to apply a time-invariant, discrete-time Volterra series method in the modelling of RF power amplifiers to assess distortion levels to eliminate or reduce them. Owing to our limited computational facilities, the experimental PA in this preliminary work is only excited in a moderate frequency range, 100Hz to 1kHz.

Keywords: Amplifiers, telecommunication, nonlinear distortion, system identification, intelligent machines, Volterra series

1. INTRODUCTION

Radio Frequency (RF) power amplifiers (PAs) are required in almost all electronic and digital telecommunication systems to amplify the power of input signals. One of the most general and important problems is the trade-off between the linearity and efficiency of RF PAs. In the modern telecommunication area, especially in a mobile or base station, linear amplification, based on a wide power range, is important. However, to achieve good linearity, a RF PA usually needs to be backed off, resulting in relatively low efficiency. Much work has been done to solve such a conflict (Kenington, 2000; Cripps, 1999; Cripps, 2002). Because of the similarity between their structures, Volterra kernels are theoretically good at representing the intermodulation and harmonic distortions. Hence, Volterra series (VS) are usually a good model for an inherently nonlinear PA as well as many other typical power devices (Vuolevi, 2001; Lee *et al.*, 1997; Ahmad and Gudimetla, 2002). But, in practice, the evaluation of "large " VS is usually computationally difficult, resulting in the truncation of the model and making the prediction performance less promising. To avoid the truncation of a VS model, which causes relatively poor representation, a kernel method is introduced to enable the computation of finite or infinite degree, finite memory length VS.

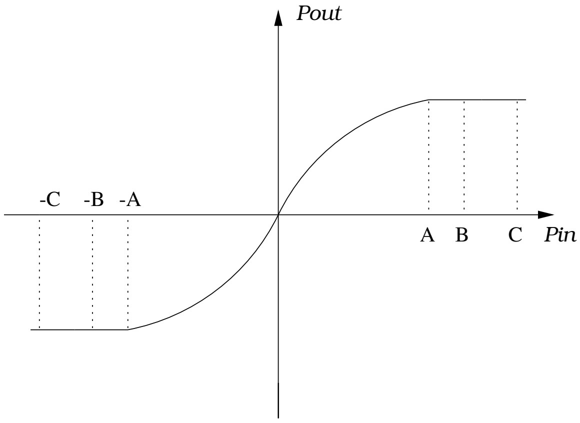

Fig. 1. Typical transfer property of a RF PA. $[-A, A]$ is a weakly nonlinear area, $[-B, B]$ is slightly into saturation and $[-C, C]$ is largely saturated.

In the next section, the conflict between linearity and power efficiency in RF PAs and the reasons for using kernel-based VS are discussed in more detail. In Section 3, the kernel method of identifying a PA's nonlinear transfer property is presented. The corresponding theoretical basis is also developed. Then, in Section 4, an experiment, based on a real but simple PA circuit, is analyzed. The motivation of the experiment is to test the effectiveness of the kernel based VS in identifying RF PAs' nonlinearities, but because of the limited computational facilities, the experimental PA is only excited from 100Hz to 1kHz to establish the principle. Finally, we summarize the paper and present some conclusions.

2. THE MOTIVATION

RF transistors are inherently nonlinear devices, resulting in the creation of distortions in the amplification process, as shown in Fig.(1).

The closer it is to the mid-point (origin in this case) of the curve, the more linear is the transfer property. That is the reason why, to achieve better linearity, a PA is often backed off, narrowing the input power range, hence, reducing efficiency.

By introducing the kernel method for identifying VS, we intend, first, to provide a good RF PA model for linear PA design and, second, to help improve distortion cancellation by means of some basic linearization techniques, e.g. the predistortion method (Cripps, 1999).

3. PA MODELING

Owing to their convolutional structures, Volterra kernels are naturally good representations of intermodulation and harmonic distortions, which are the main distortions in a RF PA (Vuolevi, 2001). Hence, Volterra models are often used to model intermodulation and harmonic distortions in RF PAs as well as in many other power semiconductor devices. The current problem is that a Volterra model is usually truncated severely to make the computation feasible. The accuracy of the model, thus, deteriorates severely.

It has been shown in (Harrison, 1999; Drezet, 2001; Dodd and Harrison, 2002a; Dodd and Harrison, 2002b; Wan *et al.*, 2003; Wan *et al.*, 2004) that a time-invariant, discrete-time, finite or infinite degree, finite memory length VS y

$$y\left(\underline{u}\right) = h_0 + \sum_{n=1}^{D}\left\{\sum_{m_1=0}^{M-1}\cdots\sum_{m_n=0}^{M-1} h_n\left(m_1,\cdots,m_n\right)\prod_{j=1}^{n} u_{m_j}\right\} \tag{1}$$

where $\underline{u} \triangleq \left[u_0,\cdots,u_{M-1}\right]^T$ is the vector of lagged input samples, can be represented in the form of

$$y(\underline{u}) = \sum_i \alpha_i k(\underline{u}_i, \underline{u})$$

in a reproducing kernel Hilbert space (RKHS) $\mathcal{H}$, equipped with a reproducing kernel k. In a practical system identification situation with N pairs of sample data, the corresponding least squares (LS) solution $\hat{y}$ is (Dodd and Harrison, 2002a; Wan *et al.*, 2003)

$$\hat{y}(\underline{u}) = \sum_{i=1}^{N} \alpha_i k(\underline{u}_i, \underline{u}) \tag{2}$$

Express Eq.(2) in vector form

$$\underline{\hat{y}} = K\underline{\alpha} \tag{3}$$

where

$$\underline{\hat{y}} = \left[\hat{y}(\underline{u}_1),\cdots,\hat{y}(\underline{u}_N)\right]^T$$
$$\underline{\alpha} = \left[\alpha_1,\cdots,\alpha_N\right]^T$$

and K is the kernel Gram matrix, $K_{ij} = k(\underline{u}_i, \underline{u}_j)$. Given that $\underline{u}_1,\cdots,\underline{u}_N$ are distinct, the kernel Gram matrix K is nonsingular, providing a unique solution (Zyla and De Figueiredo, 1983)

$$\underline{\alpha} = K^{-1}\underline{y} \tag{4}$$

In the case of identifying finite degree VS, a polynomial kernel $k_p = (1 + \langle\underline{u}, \underline{v}\rangle_{l_2})^D$ (Dodd and Harrison, 2002a) can be used, replacing k in Eq.(2), to reveal $\hat{y}$. If the target is infinite degree VS, an exponential kernel $k_e = \exp\left(\frac{\langle\underline{u}, \underline{v}\rangle_{l_2}}{p}\right)$ (Wan *et al.*, 2003) should be used. The use of these kernels can largely reduce the computational burden of this RKHS-based algorithm (Dodd and Harrison, 2002a; Wan *et al.*, 2003).

Thus, the problem of estimating a finite or infinite degree VS model is simplified to computing the parameter vector $\underline{\alpha} = [\alpha_1,\cdots,\alpha_N]^T$. Additionally, individual Volterra kernels can also be extracted from the linear combination of the corresponding polynomial terms

$$\hat{h}_n(m_1, \cdots, m_n) = \lambda_n(m_1, \cdots, m_n) \sum_{i=1}^{N} \alpha_i \prod_{j=1}^{n} u_{im_j} \tag{5}$$

in which $\lambda_n(m_1, \cdots, m_n)$ is a sequence of positive numbers(Dodd and Harrison, 2002b; Wan *et al.*, 2004).

Generalized frequency response functions (GFRFs) can be evaluated from h_ns through the Fourier transform,

$$H_n(\omega_1, \cdots, \omega_n) = \sum_{m_1=0}^{M-1} \cdots \sum_{m_n=0}^{M-1} h_n(m_1, \cdots, m_n) \times \exp\left(-j \sum_{a=1}^{n} \omega_a m_a\right) \tag{6}$$

where j is the imaginary unit.

Let us summarize the above algorithm. First, construct the polynomial or exponential kernel by using the training data and compute $\underline{\alpha}$ through Eq.(4). Second, based on the testing data, compute the LS approximated output $\hat{y}$ by using Eq.(2). Then, extract $\hat{h}_n$ through Eq.(5). At last, reveal the nth order GFRFs through Eq.(6) and the distortions of interest can be computed from the GFRFs (Bedrosian and Rice, 1971).

It should be noted that, in many practical environments, the fundamental condition that $\underline{u}_i, i = 1, 2, \cdots, N$, are distinct elements in $\mathbb{R}^M$ is not easily achieved, numerically, even in the noise-free situation, especially when N becomes large (Wan *et al.*, 2003). So, a regularization parameter ρ is often used to get a biased solution $\tilde{\alpha} = (K + \rho I)^{-1} \underline{y}$.

4. EXPERIMENT

A PA circuit, as shown in Fig.(2), was built and stimulated by a chirp input signal, sweeping from 100Hz to 1kHz in 10ms. The circuit's supply voltage V_{cc} was 25V and the sampling rate, 80kHz. There are 2000 pairs of v_{in} and v_{out} sampled for both training and testing. The data acquisition card samples the two ports, input and output, alternately, which means the sampled v_{in} and v_{out} are not truly synchronous. However, to identify the PA properly, synchronous input and output data are required. To tackle this problem, the original data are downsampled by a factor of 5 to be 400 pairs. Thus, the difference between the sample times of one pair of v_{in} and v_{out}, which is 12.5μs, is less significant and neglected in the following analysis so that v_{in} and v_{out} are treated as synchronous.

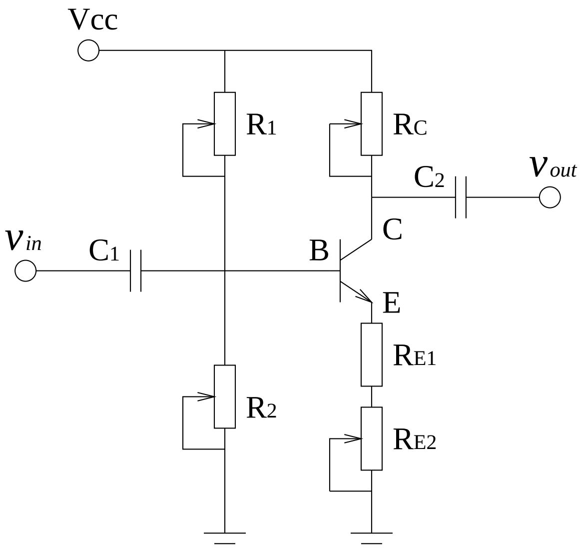

Fig. 2. Ampifier circuit with $R_1 = 370\Omega, R_2 = 430\Omega, R_C = 198\Omega, R_{E1} = 18\Omega, R_{E2} = 19\Omega$ and $C_1 = C_2 = 0.1$mf. The NPN epitaxial silicon transistor is BC546 from FAIRCHILD SEMICONDUCTOR®.

By adjusting the amplitude of v_{in}, the PA operates in three zones, weakly nonlinear ($[-A, A]$ in Fig.(1)), slightly into saturation ($[-B, B]$ in Fig.(1)) and largely into saturation ($[-C, C]$ in Fig.(1)).

In all the estimations, those model parameters, including memory length M, exponential constant p and polynomial degree D, are chosen by hold-out cross validation based on the minimum value of the total normalized mean square error (NMSE) [1]. The fast Fourier transform (FFT) is performed for computing the power spectrum of a discrete signal $x[n], n = 0, 2, \cdots, N-1$,

$$ps_i = \frac{1}{N}\mathcal{F}(y_i)\mathcal{F}^*(y_i)$$

where $\mathcal{F}(y_i)$ is the FFT of y_i and $\mathcal{F}^*(y_i)$ is its conjugate.

4.1 Model parameters and estimation results

When the PA works in $[-A, A]$, its property is estimated by an infinite degree VS at first with $M = 11, p = 0.5$ and $\rho = 1 \times 10^{-3}$. Then the PA is identified by a finite degree VS with $M = 11, D = 5$ and $\rho = 1 \times 10^{-3}$. The estimation results

[1] NMSE consists of two parts: normalized MSEs in the time domain (NTMSE) and the frequency domain (NPMSE), respectively, thus NMSE = NTMSE + NPMSE. $\text{NTMSE}(\underline{\hat{y}}) = \sum_{i=1}^{N}\left(\frac{y_i - \hat{y}_i}{y_i}\right)^2$ and $\text{NPMSE}(\underline{\hat{ps}}) = \sum_{i=1}^{N}\left(\frac{ps_i - \hat{ps}_i}{ps_i}\right)^2$, where $\underline{\hat{ps}} = [\hat{ps}_1, \cdots, \hat{ps}_N]^T$ is the power spectrum of $\underline{\hat{y}}$ and $\hat{y}_i = \hat{y}(\underline{u}_i)$.

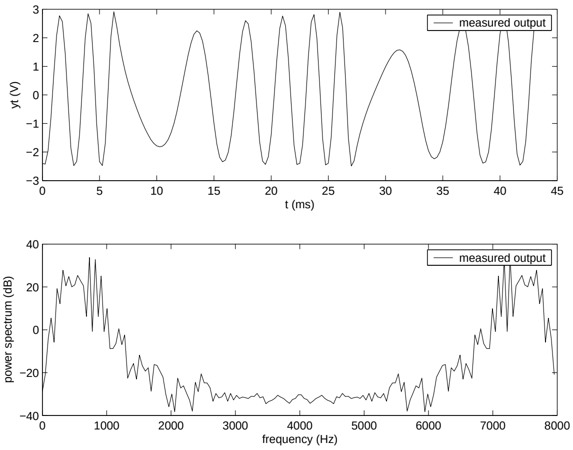

Fig. 3. The measured time series and frequency response when the PA works in $[-A, A]$.

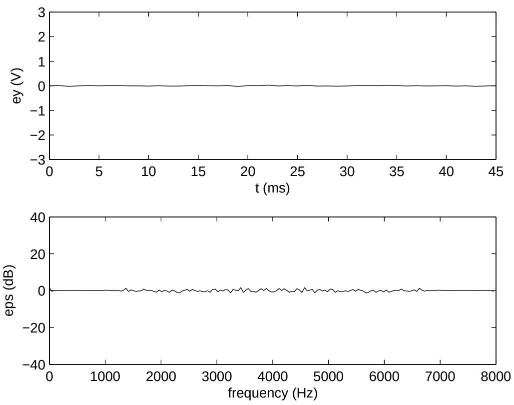

Fig. 4. The estimation errors in both the time and frequency domains given by the infinite VS model.

are shown in Fig.(3), (4) and (5), respectively. Note that, to make the corresponding figures comparable, their scales are set to be the same. In many cases, this means the errors look like zero all the time because they are trivial compared with the corresponding measured signals, whose exact values can be checked in Table (1) [2].

The predicted first order Volterra kernel vector $\underline{\hat{h}}_1$ of the infinite VS is

$$\underline{\hat{h}}_1 = \big[-3.6446\ 0.4168\ 0.9681\ 0.8722\ 0.4344\ \cdots$$
$$\cdots\ 0.2464\ 0.1421\ 0.1031\ 0.1613\ 0.1579\ 0.5421\big]$$

When the PA works in $[-B, B]$, both infinite and finite degree VS models are also identified. The parameters of the infinite VS are $M = 11, p = 0.28$ and $\rho = 1 \times 10^{-3}$. With the same M and ρ, the finite VS model is of degree 6. The corresponding results are displayed in Fig.(6), (7) and (8), respectively.

[2] The Inf and Fin in the table mean infinite and finite degree VS identifications, respectively.

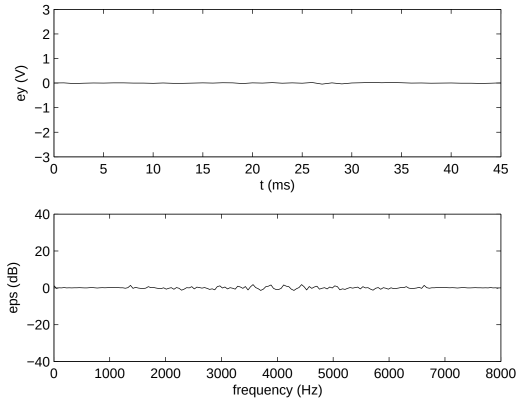

Fig. 5. The estimation errors in both the time and frequency domains given by the finite VS model.

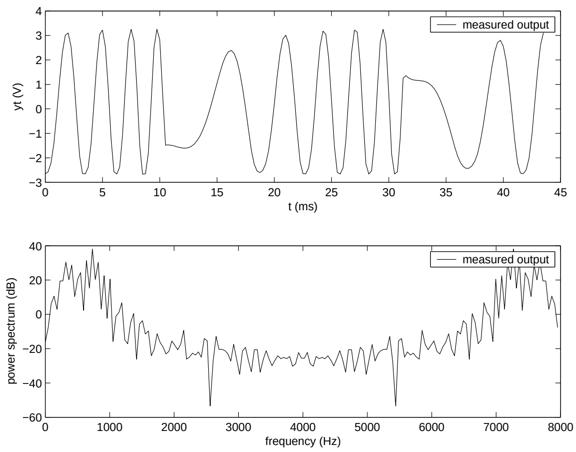

Fig. 6. The measured time series and frequency response when the PA works in $[-B, B]$.

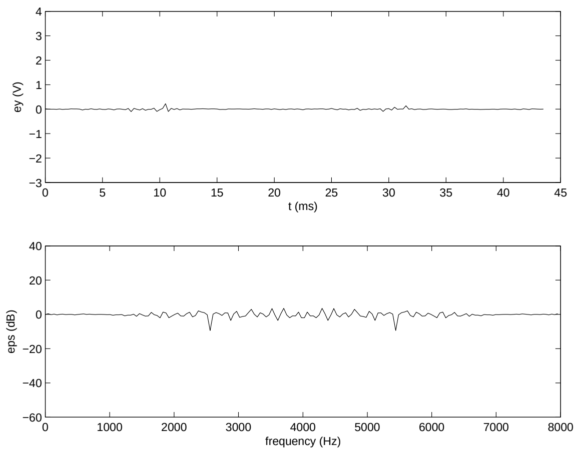

Fig. 7. The estimation errors in both the time and frequency domains given by the infinite VS model.

The predicted first order Volterra kernel vector $\underline{\hat{h}}_1$ of the infinite VS is

$$\underline{\hat{h}}_1 = \big[-3.2919\ 0.3381\ 0.6265\ 0.4091\ 0.3737\ \cdots$$
$$\cdots\ 0.4196\ 0.3461\ 0.1967\ 0.1359\ 0.2332\ 0.1446\big]$$

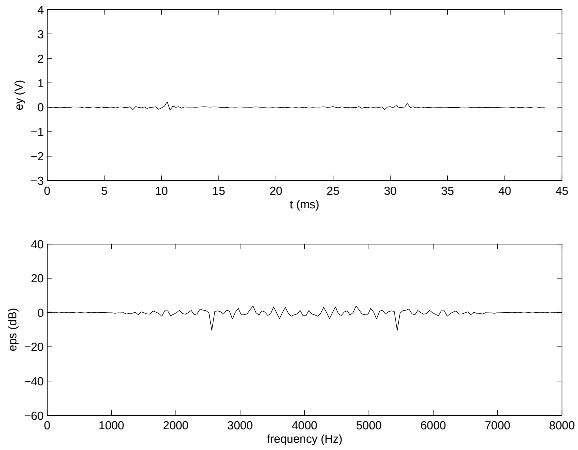

Fig. 8. The estimation errors in both the time and frequency domains given by the finite VS model.

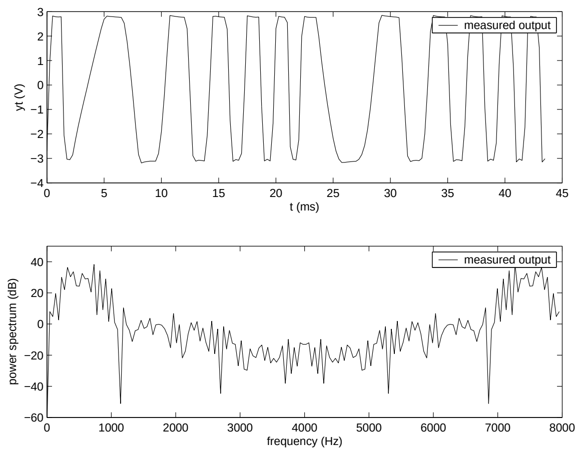

Fig. 9. The measured time series and frequency response when the PA works in $[-C, C]$.

Lastly, we run the PA in $[-C, C]$ and identify its property by an infinite and a finite degree VS model, respectively. The infinite VS model is of $M = 6, p = 0.18$ and $\rho = 1 \times 10^{-3}$ and the finite VS model has the same M and ρ with $D = 9$. The results are shown in Fig.(9), (10) and (11), respectively.

The predicted first order Volterra kernel vector $\underline{\hat{h}}_1$ of the infinite VS is

$$\underline{\hat{h}}_1 = \big[-3.1909 \ -0.4606 \ 0.9296 \ \cdots \\ \cdots \ 1.0559 \ 1.5730 \ 1.1146\big]$$

The predicted second order frequency response, given by the infinite VS model, is shown in Fig.(12).

The NTMSEs, NPMSEs and NMSEs for all the modeling processes are summerized in Table (1).

4.2 Analysis

By using the kernel methods presented in Section 3, higher order h_n and H_n can also be computed.

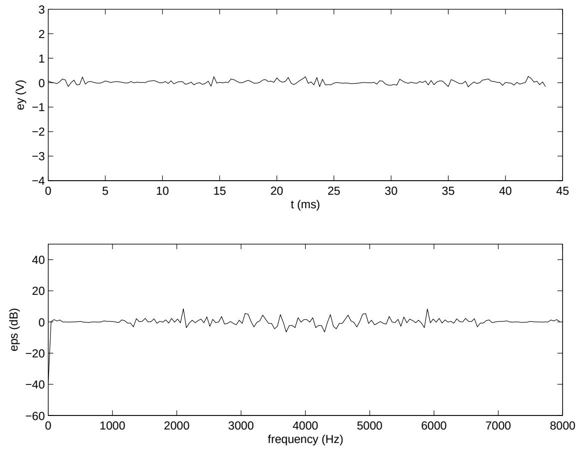

Fig. 10. The estimation errors in both the time and frequency domains given by the infinite VS model.

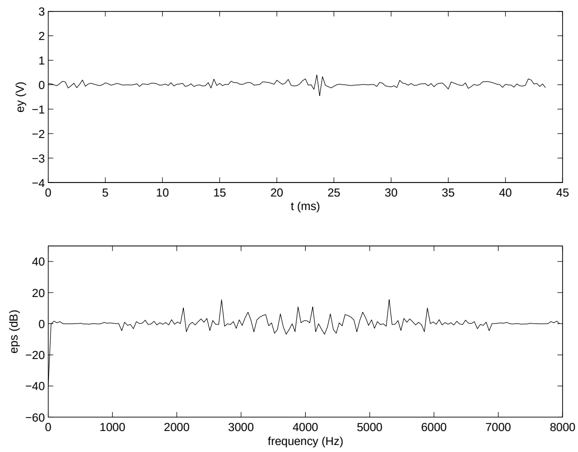

Fig. 11. The estimation errors in both the time and frequency domains given by the finite VS model.

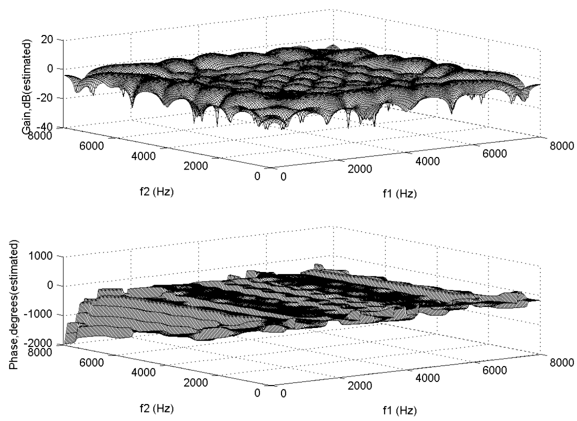

Fig. 12. Predicted second order frequency response given by the infinite VS model.

But because of the space limit, they are not listed here.

From the figures and the table, we can see that those "large" VS models do fit the PA well even in the severely nonlinear situation as shown in

Table 1. NTMSEs, NPMSEs and NMSEs for all the modeling processes.

	AA Inf	AA Fin	BB Inf	BB Fin	CC Inf	CC Fin
NTMSE(10^{-4})	0.30	0.32	2.18	2.25	10.00	13.00
NPMSE(10^{-5})	0.63	0.80	2.51	2.84	23.57	20.82
NMSE(10^{-4})	0.36	0.40	2.43	2.54	12.00	15.00

Fig.(10). On the other hand, the deeper the PA works into the saturation area, the more difficult it is to identify the nonlinearity, which is clearly shown by Table (1).

As mentioned before, the frequency range of the input chirp signal is from 100Hz to 1kHz. If we observe the frequency domain estimation results more carefully, it can found that the VS models work better on [100Hz, 1kHz]. Most estimation errors, that can be identified by eyes from the figures, happen on the outside, or distortion, area.

In this experiment, generally speaking, infinite degree VS models work better than finite degree ones in both the time and frequency domains, based on the same memory lengths and regularizations.

5. CONCLUSION

VS, especially high or even infinite degree VS, are powerful models in identifying PAs. By using the kernel method, the practical utilization of high and infinite degree VS is achievable and the computational burden of estimating a large number of Volterra coefficients is reduced. The use of high or infinite degree VS benefits identification of PA nonlinearities, which potentially could help reduce the distortions in PAs and increase their linearities while keeping reasonable efficiency.

Future work will focus on the more detailed identification and analysis of individual distortion components, mainly harmonic and intermodulation effects. Tests will be done based on RF inputs instead of the current relatively low frequency ([100Hz, 1kHz]) ones. Exact methods of using identified VS models to reduce distortions will also be investigated.

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