MIXED H_2/H_∞ SUB-OPTIMIZATION APPROACH FOR INTEGRATED AIRCRAFT/CONTROLLER DESIGN ⋆

Fang Liao [*,1], **Kai Yew Lum** [*] and
Jian Liang Wang [**]

[*] *Temasek Laboratories, National University of Singapore, Singapore 117508*
[**] *School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798.*

Abstract: This paper presents a new methodology to solve the optimization problem of integrated aircraft/dynamic output-feedback controller design where both polytopic model uncertainties and multi-missions are considered. This design optimization is based on mixed H_2/H_∞ performance requirement. According to the projection lemma, the integrated design optimization problem is separated into aircraft parameter optimization problem and controller optimization problem. An LMI-based sub-optimization approach is proposed for the aircraft parameter design. Then for the obtained sub-optimal aircraft parameters, an optimization approach is given to solve for the dynamic output-feedback controllers corresponding to different missions.

Keywords: integrated aircraft/controller design; optimization; mixed H_2/H_∞ control; polytopic uncertainty; parameter-dependent; linear matrix inequalities(LMIs).

1. INTRODUCTION

In the design of aircraft control systems, an aircraft is traditionally designed first, then a controller is designed for the given aircraft. However, combining these individually designed components into a single vehicle does not necessarily guarantee good vehicle performance. In addition, following the rapid and substantial progress in micro-electro-mechanical systems (MEMS), micro air vehicles (MAVs) are arousing the interest of aeronautical engineers. As MAVs with flapping wings have highly unsteady aerodynamics, the separated aircraft and flight controller design methods may not enable missions with stringent performance requirements and even may not stabilize them.

As a result of these real and predicted problems, and motivated by the fact that significant improvements in the overall system performance and cost are possible if the process of aircraft design and control system development were integrated, much research in recent years has been devoted to the development of integrated aircraft/controller design methods. Several integrated H_∞ aircraft/controller design approaches have been presented (Niewoehner and Kaminer, 1996; Grigoriadis and Wu, 1997; Yang and Lum, 2003). In (Niewoehner and Kaminer, 1996), an iterative LMI-based algorithm is suggested to solve the optimization problem. Unfortunately, convergence properties cannot be guaranteed for the proposed algorithm. In (Yang and Lum, 2003), Yang and Lum continue to pursue the integrated aircraft/controller design optimization problem. An iterative LMI-based algorithm with convergence properties is proposed. Nevertheless, in (Niewoehner and Kaminer, 1996; Yang

⋆ This work is supported under grant DSTA POD 1820, Singapore.
[1] Corresponding author. Tel: (65) 6874-5986, Fax: (65) 6872-6840, E-mail: tsllf@nus.edu.sg.

and Lum, 2003), only state-feedback control problem is considered. The dynamic output-feedback control problem was considered by Grigoriadis and Wu in (Grigoriadis and Wu, 1997) where a convergent iterative LMI algorithm was proposed. However, the assumption that the control input matrix is independent of design parameters of the plant seems too restrictive. Although all the above-mentioned approaches are proposed for the integrated aircraft/controller design, they only considered H_∞ performance requirement and used iterative LMI algorithms which greatly depended on selection of initial values. Moreover, model uncertainties and multi-mission operation were not involved by these approaches.

An LMI-based integrated plant/controller optimization approach has been presented by the authors in (Liao, et al., 2005) where only one plant parameter is to be determined. This paper investigates integrated aircraft/output-feedback controller design optimization problems with more aircraft parameters to be determined. And both polytopic model uncertainties and multi-missions are considered. By using the projection lemma, the integrated design optimization problem is separated into aircraft parameter optimization problem and controller optimization problem. An LMI-based sub-optimization approach is presented for the design of aircraft parameters, such as control surface sizes of aircraft, subject to existing a dynamic output-feedback controller for each mission that satisfies the closed-loop mixed H_2/H_∞ performance requirement. Then for the obtained sub-optimal aircraft parameters, an LMI-based optimization approach is proposed to solve for the dynamic output-feedback controllers corresponding to different missions.

2. PROBLEM FORMULATION

Consider a set of parameter-dependent dynamic systems $\mathcal{P}_l$ $(l = 1, 2, \cdots, L)$ with polytopic uncertainties, which are described by the following state-space representation.

$$\begin{bmatrix} \dot{\mathbf{x}}_l \\ \mathbf{z}_{2l} \\ \mathbf{z}_{\infty l} \\ \mathbf{y}_l \end{bmatrix} = \begin{bmatrix} \mathbf{A}_l(\xi,\Theta_l) & \mathbf{B}_{2l}(\xi,\Theta_l) & \mathbf{B}_{\infty l}(\xi,\Theta_l) & \mathbf{B}_l(\xi,\Theta_l) \\ \mathbf{C}_{2l} & \mathbf{0} & \mathbf{0} & \mathbf{D}_{2l} \\ \mathbf{C}_{\infty l} & \mathbf{0} & \mathbf{0} & \mathbf{D}_{\infty l} \\ \mathbf{C}_l(\xi) & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_l \\ \mathbf{w}_{2l} \\ \mathbf{w}_{\infty l} \\ \mathbf{u}_l \end{bmatrix} \tag{1}$$

Here the subscript l $(l = 1, 2, \cdots, L)$ represents the linear aircraft model corresponding to the lth mission to be executed, hence there are in total L missions to be executed. $\mathbf{x}_l(t) \in \mathbf{R}^{n_l}$ is the state vector, $\mathbf{u}_l(t) \in \mathbf{R}^{n_{ul}}$ is the control input signal, $\mathbf{y}_l(t) \in \mathbf{R}^{n_{yl}}$ is the measured output, $\mathbf{z}_{2l}(t) \in \mathbf{R}^{n_{z2l}}$ describes the H_2 performance output vector, $\mathbf{z}_{\infty l}(t) \in \mathbf{R}^{n_{z\infty l}}$ describes the H_∞ performance output vector, and $\mathbf{w}_{2l}(t) \in \mathbf{R}^{n_{w2l}}$ and $\mathbf{w}_{\infty l}(t) \in \mathbf{R}^{n_{w\infty l}}$ are the disturbance vectors. The matrices $\mathbf{C}_{2l}$, $\mathbf{D}_{2l}$, $\mathbf{C}_{\infty l}$ and $\mathbf{D}_{\infty l}$ are known constant matrices. And the matrices $\mathbf{A}_l(\xi,\Theta_l)$, $\mathbf{B}_{2l}(\xi,\Theta_l)$, $\mathbf{B}_{\infty l}(\xi,\Theta_l)$, $\mathbf{B}_l(\xi,\Theta_l)$ and $\mathbf{C}_l(\xi)$ are given by

$$\begin{aligned} \mathbf{A}_l(\xi,\Theta_l) &= \sum_{j=1}^{q_l}\left(\mathbf{A}_{0jl} + \sum_{i=1}^{r}\xi_i\mathbf{A}_{ijl}\right)\theta_{jl} \\ \mathbf{B}_{2l}(\xi,\Theta_l) &= \sum_{j=1}^{q_l}\left(\mathbf{B}_{20jl} + \sum_{i=1}^{r}\xi_i\mathbf{B}_{2ijl}\right)\theta_{jl} \\ \mathbf{B}_{\infty l}(\xi,\Theta_l) &= \sum_{j=1}^{q_l}\left(\mathbf{B}_{\infty 0jl} + \sum_{i=1}^{r}\xi_i\mathbf{B}_{\infty ijl}\right)\theta_{jl} \\ \mathbf{B}_l(\xi,\Theta_l) &= \sum_{j=1}^{q_l}\left(\mathbf{B}_{0jl} + \sum_{i=1}^{r}\xi_i\mathbf{B}_{ijl}\right)\theta_{jl} \\ \mathbf{C}_l(\xi) &= \mathbf{C}_{00l} + \sum_{i=1}^{r}\xi_i\mathbf{C}_{i0l} \end{aligned} \tag{2}$$

where the matrices $\mathbf{A}_{ijl}$, $\mathbf{B}_{ijl}$, $\mathbf{B}_{2ijl}$, $\mathbf{B}_{\infty ijl}$ and $\mathbf{C}_{i0l}$ $(i = 0, 1, \cdots, r, j = 1, 2, \cdots, q_l, l = 1, 2, \cdots, L)$ are known constant matrices of appropriate dimensions. The vector $\xi = [\xi_1 \ \xi_2 \ \cdots \ \xi_r]$ is the aircraft parameter vector to be optimized and belongs to the set

$$\Xi \triangleq \{[\xi_1 \ \xi_2 \ \cdots \ \xi_r] \in \mathbf{R}^r : \xi_i \geq 0, i = 1, 2, \cdots, r\} \tag{3}$$

Without loss of generality, we assume that $\xi = 0$ corresponds to the nominal aircraft parameters (e.g., the largest sizes of control surfaces) that have been chosen in a prior design stage. The parameter $\Theta_l = [\theta_{1l} \ \theta_{2l} \ \cdots \ \theta_{q_l l}]^T \in \mathbf{R}^{q_l}$ is the uncertain constant parameter vector satisfying

$$\Theta_l \in \mathbf{\Theta} \triangleq \Big\{[\theta_{1l} \ \theta_{2l} \cdots \theta_{q_l l}]^T \in \mathbf{R}^{q_l} : \theta_{jl} \geq 0, \\ j = 1, 2, \cdots, q_l, \ l = 1, 2, \cdots, L, \ \sum_{j=1}^{q_l}\theta_{jl} = 1\Big\} \tag{4}$$

Consider the following stabilizing dynamic output-feedback controllers

$$\mathcal{K}_l : \begin{cases} \dot{\mathbf{x}}_{\mathbf{k}l} = \mathbf{A}_{\mathbf{k}l}\mathbf{x}_{\mathbf{k}l} + \mathbf{B}_{\mathbf{k}l}\mathbf{y}_l \\ \mathbf{u}_l = \mathbf{C}_{\mathbf{k}l}\mathbf{x}_{\mathbf{k}l} + \mathbf{D}_{\mathbf{k}l}\mathbf{y}_l \end{cases}, \ l = 1, 2, \cdots, L \tag{5}$$

where $\mathbf{x}_{\mathbf{k}l} \in \mathbf{R}^{n_{kl}}$ is the state vector of the dynamic output-feedback controller corresponding to the lth mission. Denote this controller by

$$\mathbf{K}_l = \begin{bmatrix} \mathbf{D}_{\mathbf{k}l} & \mathbf{C}_{\mathbf{k}l} \\ \mathbf{B}_{\mathbf{k}l} & \mathbf{A}_{\mathbf{k}l} \end{bmatrix} \tag{6}$$

Furthermore, denote

$$\mathcal{A}_l(\xi,\Theta_l) = \begin{bmatrix} \mathbf{A}_l(\xi,\Theta_l) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{x}_{\mathbf{cl}l} = \begin{bmatrix} \mathbf{x}_l \\ \mathbf{x}_{\mathbf{k}l} \end{bmatrix}$$
$$\mathcal{B}_l(\xi,\Theta_l) = \begin{bmatrix} \mathbf{B}_l(\xi,\Theta_l) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \ \mathcal{C}_l(\xi) = \begin{bmatrix} \mathbf{C}_l(\xi) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

$$\mathcal{B}_{\infty l}(\xi,\Theta_l)=\begin{bmatrix}\mathbf{B}_{\infty l}(\xi,\Theta_l)\\ \mathbf{0}\end{bmatrix},\ \begin{matrix}\mathcal{C}_{\infty l}=[\mathbf{C}_{\infty l}\ \ \mathbf{0}]\\ \mathcal{D}_{\infty l}=[\mathbf{D}_{\infty l}\ \ \mathbf{0}]\end{matrix}$$

$$\mathcal{B}_{\mathbf{2}l}(\xi,\Theta_l)=\begin{bmatrix}\mathbf{B}_{\mathbf{2}l}(\xi,\Theta_l)\\ \mathbf{0}\end{bmatrix},\ \begin{matrix}\mathcal{C}_{\mathbf{2}l}=[\mathbf{C}_{\mathbf{2}l}\ \ \mathbf{0}]\\ \mathcal{D}_{\mathbf{2}l}=[\mathbf{D}_{\mathbf{2}l}\ \ \mathbf{0}]\end{matrix}$$

Then, the closed-loop systems $\mathbf{\Pi}_l$ $(l=1,2,\cdots,L)$ are described by

$$\begin{bmatrix}\dot{\mathbf{x}}_{\mathbf{cl}l}\\ \mathbf{z}_{\mathbf{2}l}\\ \mathbf{z}_{\infty l}\end{bmatrix}=\begin{bmatrix}\mathcal{A}_{\mathbf{cl}l}(\xi,\Theta_l,\mathbf{K}_l) & \mathcal{B}_{\mathbf{2}l}(\xi,\Theta_l) & \mathcal{B}_{\infty l}(\xi,\Theta_l)\\ \mathcal{C}_{\mathbf{2cl}l}(\xi,\mathbf{K}_l) & \mathbf{0} & \mathbf{0}\\ \mathcal{C}_{\infty\mathbf{cl}l}(\xi,\mathbf{K}_l) & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}_{\mathbf{cl}l}\\ \mathbf{w}_{\mathbf{2}l}\\ \mathbf{w}_{\infty l}\end{bmatrix} \quad (7)$$

with

$$\begin{aligned}\mathcal{A}_{\mathbf{cl}l}(\xi,\Theta_l,\mathbf{K}_l)&=\mathcal{A}_l(\xi,\Theta_l)+\mathcal{B}_l(\xi,\Theta_l)\mathbf{K}_l\mathcal{C}_l(\xi)\\ \mathcal{C}_{\mathbf{2cl}l}(\xi,\mathbf{K}_l)&=\mathcal{C}_{\mathbf{2}l}+\mathcal{D}_{\mathbf{2}l}\mathbf{K}_l\mathcal{C}_l(\xi)\\ \mathcal{C}_{\infty\mathbf{cl}l}(\xi,\mathbf{K}_l)&=\mathcal{C}_{\infty l}+\mathcal{D}_{\infty l}\mathbf{K}_l\mathcal{C}_l(\xi)\end{aligned}$$

The design objective is to optimize the aircraft parameters subject to the existence of a set of stabilizing dynamic output-feedback controllers as in (5) satisfying the mixed H_2/H_∞ closed-loop performance requirement. In other words, we seek to solve the following optimization problem:

$$\begin{gathered}\text{maximize } c\xi \text{ subject to}\\ ||\mathbf{\Pi}_{\mathbf{z_{2\mathit{l}}w_{2\mathit{l}}}}(s)||_2<\nu_l,\ ||\mathbf{\Pi}_{\mathbf{z}_{\infty l}\mathbf{w}_{\infty l}}(s)||_\infty<\gamma_l,\\ \mathbf{K}_l\in\mathcal{K}_l,\ \xi\in\Xi,\ l=1,2,\cdots,L\end{gathered} \quad (8)$$

where $c=[c_1c_2\cdots c_r]$ with $c_i>0$ $(i=1,2,\cdots,r)$ is a known constant vector, and $\mathbf{\Pi}_{\mathbf{z_{2\mathit{l}}w_{2\mathit{l}}}}(s)$ and $\mathbf{\Pi}_{\mathbf{z}_{\infty l}\mathbf{w}_{\infty l}}(s)$ denote the transfer functions of the lth closed-loop system of (7). Hence, both the aircraft parameter ξ and controller parameters $\mathbf{K}_l$ $(l=1,2,\cdots,L)$ are taken into account in the optimization problem (8).

3. INTEGRATED AIRCRAFT/CONTROLLER DESIGN OPTIMIZATION

Lemma 1. (Zhou and Doyle, 1998) Consider the closed-loop systems $\mathbf{\Pi}_l$ $(l=1,2,\cdots,L)$ as in (7). For given scalars $\gamma_l>0$ and $\nu_l>0$, $l=1,2,\cdots,L$, we have that $||\mathbf{\Pi}_{\mathbf{z_{2\mathit{l}}w_{2\mathit{l}}}}(s)||_2<\nu_l$ and $||\mathbf{\Pi}_{\mathbf{z}_{\infty l}\mathbf{w}_{\infty l}}(s)||_\infty<\gamma_l$ if and only if there exist symmetric positive-definite matrices $\mathbf{Y}_{\infty l}\in\mathbf{R}^{(n_l+n_{kl})\times(n_l+n_{kl})}$ and $\mathbf{Y}_{\mathbf{2}l}\in\mathbf{R}^{(n_l+n_{kl})\times(n_l+n_{kl})}$ and $\mathbf{Q}_l\in\mathbf{R}^{n_{w2l}\times n_{w2l}}$ $(l=1,2,\cdots,L)$, controller parameter matrices $\mathbf{K}_l$ $(l=1,2,\cdots,L)$ as in (6) and an aircraft parameter vector $\xi\in\Xi$ such that the following matrix inequalities are satisfied.

$$\begin{bmatrix}\mathcal{A}_{\mathbf{cl}l}(\xi,\Theta_l,\mathbf{K}_l)^T\mathbf{Y}_{\infty l}+\mathbf{Y}_{\infty l}\mathcal{A}_{\mathbf{cl}l}(\xi,\Theta_l,\mathbf{K}_l) & * & *\\ \mathcal{B}_{\infty l}^T(\xi,\Theta_l)\mathbf{Y}_{\infty l} & -\gamma_l\mathbf{I} & *\\ \mathcal{C}_{\infty\mathbf{cl}l}(\xi,\mathbf{K}_l) & \mathbf{0} & -\gamma_l\mathbf{I}\end{bmatrix}<0 \quad (9)$$

$$\begin{bmatrix}\mathcal{A}_{\mathbf{cl}l}(\xi,\Theta_l,\mathbf{K}_l)^T\mathbf{Y}_{\mathbf{2}l}+\mathbf{Y}_{\mathbf{2}l}\mathcal{A}_{\mathbf{cl}l}(\xi,\Theta_l,\mathbf{K}_l) & *\\ \mathcal{C}_{\mathbf{2cl}l}(\xi,\mathbf{K}_l) & -\mathbf{I}\end{bmatrix}<0 \quad (10)$$

$$\begin{bmatrix}\mathbf{Q}_l & *\\ \mathbf{Y}_{\mathbf{2}l}\mathcal{B}_{\mathbf{2}l}(\xi,\Theta_l) & \mathbf{Y}_{\mathbf{2}l}\end{bmatrix}>0 \quad (11)$$

$$trace(\mathbf{Q}_l)<\nu_l \quad (12)$$

$$l=1,2,\cdots,L$$

Note that here * denotes symmetric entries of a symmetric matrix. It is applicable to the rest of this paper.

Remark 1. Lemma 1 gives a necessary and sufficient condition to solve the integrated aircraft/controller design problem with mixed H_2/H_∞ performance requirement. As ξ, $\mathbf{Y}_{\mathbf{2}l}$, $\mathbf{Y}_{\infty l}$, $\mathbf{Q}_l$ and $\mathbf{K}_l$ $(l=1,2,\cdots,L)$ are variables, the conditions (9)-(11) are not LMIs.

Lemma 2. (*Projection Lemma*)(Apkarian, et al., 2001): Given a symmetric matrix $\mathbf{\Psi}$ and two matrices $\mathbf{P}$ and $\mathbf{Q}$, there exists an $\mathbf{X}$ such that the following LMI holds:

$$\mathbf{\Psi}+\mathbf{P}^T\mathbf{X}^T\mathbf{Q}+\mathbf{Q}^T\mathbf{X}\mathbf{P}<0 \quad (13)$$

if and only if the following projection inequalities are satisfied

$$\mathcal{N}_P^T\mathbf{\Psi}\mathcal{N}_P<0,\ \ \mathcal{N}_Q^T\mathbf{\Psi}\mathcal{N}_Q<0 \quad (14)$$

where $\mathcal{N}_P$ and $\mathcal{N}_Q$ denote arbitrary bases of the null spaces of $\mathbf{P}$ and $\mathbf{Q}$, respectively.

According to Lemma 2, the inequality (9) is equivalent to

$$\mathcal{N}_{\infty\mathbf{P}l}^T\begin{bmatrix}\mathcal{A}_l^T(\xi,\Theta_l)\mathbf{Y}_{\infty l}+\mathbf{Y}_{\infty l}\mathcal{A}_l(\xi,\Theta_l) & * & *\\ \mathcal{B}_{\infty l}^T(\xi,\Theta_l)\mathbf{Y}_{\infty l} & -\gamma_l\mathbf{I} & *\\ \mathcal{C}_{\infty l} & \mathbf{0} & -\gamma_l\mathbf{I}\end{bmatrix}\mathcal{N}_{\infty\mathbf{P}l}<0 \quad (15)$$

$$\mathcal{N}_{\infty\mathbf{Q}l}^T\begin{bmatrix}\mathbf{Y}_{\infty l}^{-1}\mathcal{A}_l^T(\xi,\Theta_l)+\mathcal{A}_l(\xi,\Theta_l)\mathbf{Y}_{\infty l}^{-1} & * & *\\ \mathcal{B}_{\infty l}^T(\xi,\Theta_l) & -\gamma_l\mathbf{I} & *\\ \mathcal{C}_{\infty l}\mathbf{Y}_{\infty l}^{-1} & \mathbf{0} & -\gamma_l\mathbf{I}\end{bmatrix}\mathcal{N}_{\infty\mathbf{Q}l}<0 \quad (16)$$

where $\mathcal{N}_{\infty\mathbf{P}l}$ and $\mathcal{N}_{\infty\mathbf{Q}l}$ are the null spaces of matrices $[\mathcal{C}_l(\xi)\ \ \mathbf{0}\ \ \mathbf{0}]$ and $\left[\mathcal{B}_l^T(\xi,\Theta_l)\ \ \mathbf{0}\ \ \mathcal{D}_{\infty l}^T\right]$, respectively, and the inequality (10) is equivalent to

$$\mathcal{N}_{\mathbf{2P}l}^T\begin{bmatrix}\mathcal{A}_l^T(\xi,\Theta_l)\mathbf{Y}_{\mathbf{2}l}+\mathbf{Y}_{\mathbf{2}l}\mathcal{A}_l(\xi,\Theta_l) & *\\ \mathcal{C}_{\mathbf{2}l} & -\mathbf{I}\end{bmatrix}\mathcal{N}_{\mathbf{2P}l}<0 \quad (17)$$

$$\mathcal{N}_{\mathbf{2Q}l}^T\begin{bmatrix}\mathbf{Y}_{\mathbf{2}l}^{-1}\mathcal{A}_l^T(\xi,\Theta_l)+\mathcal{A}_l(\xi,\Theta_l)\mathbf{Y}_{\mathbf{2}l}^{-1} & *\\ \mathcal{C}_{\mathbf{2}l} & -\mathbf{I}\end{bmatrix}\mathcal{N}_{\mathbf{2Q}l}<0 \quad (18)$$

where $\mathcal{N}_{\mathbf{2P}l}$ and $\mathcal{N}_{\mathbf{2Q}l}$ are the null spaces of matrices $[\mathcal{C}_l(\xi)\ \ \mathbf{0}]$ and $\left[\mathcal{B}_l^T(\xi,\Theta_l)\ \ \mathcal{D}_{\mathbf{2}l}^T\right]$, respectively.

It is noted that the controller parameter $\mathbf{K}_l$ is no longer included in the inequalities (15)-(18) which are equivalent to (9)-(10) in Lemma 1. Hence, the integrated aircraft/controller design optimization problem can be separated into the optimization problem of aircraft parameter vector ξ and that of controller parameters $\mathbf{K}_l$ $(l = 1, 2, \cdots, L)$. In the following, the aircraft parameters and controller parameters are individually optimized.

3.1 Sub-Optimization of Aircraft Parameter Design

Before presenting the main result, partition $\mathbf{Y}_{\infty l}$ and $\mathbf{Y}_{\infty l}^{-1}$ as in (15) and (16) into

$$\mathbf{Y}_{\infty l}=\begin{bmatrix}\mathbf{S}_{\infty l} & \mathbf{N}_{\infty l}\\ \mathbf{N}_{\infty l}^T & \#_{\infty l}\end{bmatrix},\ \mathbf{Y}_{\infty l}^{-1}=\begin{bmatrix}\mathbf{R}_{\infty l} & \mathbf{M}_{\infty l}\\ \mathbf{M}_{\infty l}^T & \$_{\infty l}\end{bmatrix} \quad (19)$$

and denote

$$\mathbf{Z}_{\infty \mathbf{1} l}=\begin{bmatrix}\mathbf{I} & \mathbf{R}_{\infty l}\\ \mathbf{0} & \mathbf{M}_{\infty l}^T\end{bmatrix},\quad \mathbf{Z}_{\infty \mathbf{2} l}=\begin{bmatrix}\mathbf{S}_{\infty l} & \mathbf{I}\\ \mathbf{N}_{\infty l}^T & \mathbf{0}\end{bmatrix} \quad (20)$$

Assuming that the matrix $\mathbf{M}_{\infty l}$ is invertible, performing a congruence transformation with $\mathbf{Z}_{\infty \mathbf{1} l}$ on $\mathbf{Y}_{\infty l} > 0$, we obtain

$$\begin{bmatrix}\mathbf{S}_{\infty l} & \mathbf{I}\\ \mathbf{I} & \mathbf{R}_{\infty l}\end{bmatrix} > 0 \quad (21)$$

Similarly, partition $\mathbf{Y}_{\mathbf{2}l}$ and $\mathbf{Y}_{\mathbf{2}l}^{-1}$ as in (17) and (18) into

$$\mathbf{Y}_{\mathbf{2}l}=\begin{bmatrix}\mathbf{S}_{\mathbf{2}l} & \mathbf{N}_{\mathbf{2}l}\\ \mathbf{N}_{\mathbf{2}l}^T & \#_{\mathbf{2}l}\end{bmatrix},\quad \mathbf{Y}_{\mathbf{2}l}^{-1}=\begin{bmatrix}\mathbf{R}_{\mathbf{2}l} & \mathbf{M}_{\mathbf{2}l}\\ \mathbf{M}_{\mathbf{2}l}^T & \$_{\mathbf{2}l}\end{bmatrix} \quad (22)$$

and denote

$$\mathbf{Z}_{\mathbf{21}l}=\begin{bmatrix}\mathbf{I} & \mathbf{R}_{\mathbf{2}l}\\ \mathbf{0} & \mathbf{M}_{\mathbf{2}l}^T\end{bmatrix},\quad \mathbf{Z}_{\mathbf{22}l}=\begin{bmatrix}\mathbf{S}_{\mathbf{2}l} & \mathbf{I}\\ \mathbf{N}_{\mathbf{2}l}^T & \mathbf{0}\end{bmatrix} \quad (23)$$

Assuming that the matrix $\mathbf{M}_{\mathbf{2}l}$ is invertible, performing a congruence transformation with $\mathbf{Z}_{\mathbf{21}l}$ on $\mathbf{Y}_{\mathbf{2}l} > 0$, we obtain

$$\begin{bmatrix}\mathbf{S}_{\mathbf{2}l} & \mathbf{I}\\ \mathbf{I} & \mathbf{R}_{\mathbf{2}l}\end{bmatrix} > 0 \quad (24)$$

As a result, the LMI $\mathbf{Y}_{\infty l} > 0$ is equivalent to (21) and $\mathbf{Y}_{\mathbf{2}l} > 0$ is equivalent to (24).

Substituting the partitions (19), (22) and (2) into (15)-(18) and introducing slack matrix variables $\mathbf{E}_{\infty l}$, $\mathbf{H}_{\infty l}$, $\mathbf{E}_{\mathbf{2}l}$ and $\mathbf{H}_{\mathbf{2}l}$, we have the following Theorem 1. Denote

$$\Delta_{\infty \mathbf{P}0jl}(\mathbf{S}_{\infty l}, \mathbf{E}_{\infty l}, \gamma_l) = \begin{bmatrix}\mathbf{A}_{0jl}^T\mathbf{S}_{\infty l} + \mathbf{S}_{\infty l}\mathbf{A}_{0jl} & * & *\\ \mathbf{B}_{\infty 0jl}^T\mathbf{S}_{\infty l} & -\gamma_l\mathbf{I} & *\\ \mathbf{C}_{\infty l} & \mathbf{0} & -\gamma_l\mathbf{I}\end{bmatrix} + \mathbf{E}_{\infty l}\begin{bmatrix}\mathbf{C}_{00l}^T\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}^T + \begin{bmatrix}\mathbf{C}_{00l}^T\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{E}_{\infty l}^T$$

$$\Delta_{\infty \mathbf{P}ijl}(\mathbf{S}_{\infty l}, \mathbf{E}_{\infty l}) = \begin{bmatrix}\mathbf{A}_{ijl}^T\mathbf{S}_{\infty l} + \mathbf{S}_{\infty l}\mathbf{A}_{ijl} & * & *\\ \mathbf{B}_{\infty ijl}^T\mathbf{S}_{\infty l} & \mathbf{0} & *\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix} + \mathbf{E}_{\infty l}\begin{bmatrix}\mathbf{C}_{i0l}^T\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}^T + \begin{bmatrix}\mathbf{C}_{i0l}^T\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{E}_{\infty l}^T$$

$$\Delta_{\infty \mathbf{Q}0jl}(\mathbf{R}_{\infty l}, \mathbf{H}_{\infty l}, \gamma_l) = \begin{bmatrix}\mathbf{A}_{0jl}\mathbf{R}_{\infty l} + \mathbf{R}_{\infty l}\mathbf{A}_{0jl}^T & * & *\\ \mathbf{B}_{\infty 0jl}^T & -\gamma_l\mathbf{I} & *\\ \mathbf{C}_{\infty l}\mathbf{R}_{\infty l} & \mathbf{0} & -\gamma_l\mathbf{I}\end{bmatrix} + \mathbf{H}_{\infty l}\begin{bmatrix}\mathbf{B}_{0jl}\\ \mathbf{0}\\ \mathbf{D}_{\infty l}\end{bmatrix}^T + \begin{bmatrix}\mathbf{B}_{0jl}\\ \mathbf{0}\\ \mathbf{D}_{\infty l}\end{bmatrix}\mathbf{H}_{\infty l}^T$$

$$\Delta_{\infty \mathbf{Q}ijl}(\mathbf{R}_{\infty l}, \mathbf{H}_{\infty l}) = \begin{bmatrix}\mathbf{A}_{ijl}\mathbf{R}_{\infty l} + \mathbf{R}_{\infty l}\mathbf{A}_{ijl}^T & * & *\\ \mathbf{B}_{\infty ijl}^T & \mathbf{0} & *\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix} + \mathbf{H}_{\infty l}\begin{bmatrix}\mathbf{B}_{ijl}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}^T + \begin{bmatrix}\mathbf{B}_{ijl}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{H}_{\infty l}^T$$

$$\Delta_{\mathbf{2P}0jl}(\mathbf{S}_{\mathbf{2}l}, \mathbf{E}_{\mathbf{2}l}) = \begin{bmatrix}\mathbf{A}_{0jl}^T\mathbf{S}_{\mathbf{2}l} + \mathbf{S}_{\mathbf{2}l}\mathbf{A}_{0jl} & *\\ \mathbf{C}_{\mathbf{2}l} & -\mathbf{I}\end{bmatrix} + \mathbf{E}_{\mathbf{2}l}\begin{bmatrix}\mathbf{C}_{00l}^T\\ \mathbf{0}\end{bmatrix}^T + \begin{bmatrix}\mathbf{C}_{00l}^T\\ \mathbf{0}\end{bmatrix}\mathbf{E}_{\mathbf{2}l}^T$$

$$\Delta_{\mathbf{2P}ijl}(\mathbf{S}_{\mathbf{2}l}, \mathbf{E}_{\mathbf{2}l}) = \begin{bmatrix}\mathbf{A}_{ijl}^T\mathbf{S}_{\mathbf{2}l} + \mathbf{S}_{\mathbf{2}l}\mathbf{A}_{ijl} & *\\ \mathbf{0} & \mathbf{0}\end{bmatrix} + \mathbf{E}_{\mathbf{2}l}\begin{bmatrix}\mathbf{C}_{i0l}^T\\ \mathbf{0}\end{bmatrix}^T + \begin{bmatrix}\mathbf{C}_{i0l}^T\\ \mathbf{0}\end{bmatrix}\mathbf{E}_{\mathbf{2}l}^T$$

$$\Delta_{\mathbf{2Q}0jl}(\mathbf{R}_{\mathbf{2}l}, \mathbf{H}_{\mathbf{2}l}) = \begin{bmatrix}\mathbf{A}_{0jl}\mathbf{R}_{\mathbf{2}l} + \mathbf{R}_{\mathbf{2}l}\mathbf{A}_{0jl}^T & *\\ \mathbf{C}_{\mathbf{2}l}\mathbf{R}_{\mathbf{2}l} & -\mathbf{I}\end{bmatrix} + \mathbf{H}_{\mathbf{2}l}\begin{bmatrix}\mathbf{B}_{0jl}\\ \mathbf{D}_{\mathbf{2}l}\end{bmatrix}^T + \begin{bmatrix}\mathbf{B}_{0jl}\\ \mathbf{D}_{\mathbf{2}l}\end{bmatrix}\mathbf{H}_{\mathbf{2}l}^T$$

$$\Delta_{\mathbf{2Q}ijl}(\mathbf{R}_{\mathbf{2}l}, \mathbf{H}_{\mathbf{2}l}) = \begin{bmatrix}\mathbf{A}_{ijl}\mathbf{R}_{\mathbf{2}l} + \mathbf{R}_{\mathbf{2}l}\mathbf{A}_{ijl}^T & *\\ \mathbf{0} & \mathbf{0}\end{bmatrix} + \mathbf{H}_{\mathbf{2}l}\begin{bmatrix}\mathbf{B}_{ijl}\\ \mathbf{0}\end{bmatrix}^T + \begin{bmatrix}\mathbf{B}_{ijl}\\ \mathbf{0}\end{bmatrix}\mathbf{H}_{\mathbf{2}l}^T$$

$$\Delta_{\mathbf{2}0jl}(\mathbf{S}_{\mathbf{2}l}, \mathbf{R}_{\mathbf{2}l}, \mathbf{Q}_l) = \begin{bmatrix}\mathbf{Q}_l & * & *\\ \mathbf{S}_{\mathbf{2}l}\mathbf{B}_{\mathbf{2}0jl} & \mathbf{S}_{\mathbf{2}l} & *\\ \mathbf{B}_{\mathbf{2}0jl} & \mathbf{I} & \mathbf{R}_{\mathbf{2}l}\end{bmatrix}$$

$$\Delta_{\mathbf{2}ijl}(\mathbf{S}_{\mathbf{2}l}) = \begin{bmatrix}\mathbf{0} & * & *\\ \mathbf{S}_{\mathbf{2}l}\mathbf{B}_{\mathbf{2}ijl} & \mathbf{0} & *\\ \mathbf{B}_{\mathbf{2}ijl} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

Theorem 1. Consider the closed-loop systems $\mathbf{\Pi}_l$ $(l = 1, 2, \cdots, L)$ as in (7). For given scalars $\gamma_l > 0$ and $\nu_l > 0$ $(l = 1, 2, \cdots, L)$, if there exist an aircraft parameter vector $\xi \in \Xi$, symmetric positive-definite matrices $\mathbf{S}_{\infty l} \in \mathbf{R}^{n_l \times n_l}$, $\mathbf{R}_{\infty l} \in \mathbf{R}^{n_l \times n_l}$, $\mathbf{S}_{\mathbf{2}l} \in \mathbf{R}^{n_l \times n_l}$, $\mathbf{R}_{\mathbf{2}l} \in \mathbf{R}^{n_l \times n_l}$ and $\mathbf{Q}_{\mathbf{2}l} \in \mathbf{R}^{n_{w2l} \times n_{w2l}}$, and matrices $\mathbf{E}_{\infty l}$, $\mathbf{H}_{\infty l}$, $\mathbf{E}_{\mathbf{2}l}$ and $\mathbf{H}_{\mathbf{2}l}$ $(l = 1, 2, \cdots, L)$ such that for all $l = 1, 2, \cdots, L$ and $j = 1, 2, \cdots, q_l$,

$$\sum_{i=1}^{r} \xi_i \Delta_{\infty \mathbf{P}ijl}(\mathbf{S}_{\infty l}, \mathbf{E}_{\infty l}) < -\Delta_{\infty \mathbf{P}0jl}(\mathbf{S}_{\infty l}, \mathbf{E}_{\infty l}, \gamma_l) \quad (25)$$

$$\sum_{i=1}^{r} \xi_i \Delta_{\infty \mathbf{Q}ijl}(\mathbf{R}_{\infty l}, \mathbf{H}_{\infty l}) < -\Delta_{\infty \mathbf{Q}0jl}(\mathbf{R}_{\infty l}, \mathbf{H}_{\infty l}, \gamma_l) \quad (26)$$

$$\sum_{i=1}^{r}\xi_i\Delta_{\mathbf{2P}ijl}(\mathbf{S_{2}}_l,\mathbf{E_{2}}_l)<-\Delta_{\mathbf{2P}0jl}(\mathbf{S_{2}}_l,\mathbf{E_{2}}_l) \quad (27)$$

$$\sum_{i=1}^{r}\xi_i\Delta_{\mathbf{2Q}ijl}(\mathbf{R_{2}}_l,\mathbf{H_{2}}_l)<-\Delta_{\mathbf{2Q}0jl}(\mathbf{R_{2}}_l,\mathbf{H_{2}}_l) \quad (28)$$

$$\sum_{i=1}^{r}\xi_i\Delta_{\mathbf{2}ijl}(\mathbf{S_{2}}_l)>-\Delta_{\mathbf{2}0jl}(\mathbf{S_{2}}_l,\mathbf{R_{2}}_l,\mathbf{Q}_l) \quad (29)$$

$$\begin{bmatrix}\mathbf{S}_{\infty l} & \mathbf{I}\\ \mathbf{I} & \mathbf{R}_{\infty l}\end{bmatrix}>0,\ \begin{bmatrix}\mathbf{S_{2}}_l & \mathbf{I}\\ \mathbf{I} & \mathbf{R_{2}}_l\end{bmatrix}>0 \quad (30)$$

$$trace(Q_l)<\nu_l \quad (31)$$

then there exist dynamic output-feedback controller parameters $\mathbf{K}_l$ $(l = 1, 2, \cdots, L)$ as in (6) such that the closed-loop systems $\mathbf{\Pi}_l$ $(l = 1, 2, \cdots, L)$ in (7) are robustly stabilized and satisfy $||\mathbf{\Pi}_{\mathbf{z}_{\infty l}\mathbf{w}_{\infty l}}(s)||_\infty < \gamma_l$ and $||\mathbf{\Pi}_{\mathbf{z_2}_l\mathbf{w_2}_l}(s)||_2 < \nu_l$ $(l = 1, 2, \cdots, L)$.

Based on the solution of the generalized eigenvalue problem (GEVP) (Gahinet, et al., 1995), the solution for sub-optimal aircraft parameter ξ_{opt} is given as follows.

Step 1 *Optimize a single aircraft parameter while maintain the other aircraft parameters as the nominal ones, this is, if only optimize the aircraft parameter* ξ_i, *then choose* $\xi = [0 \ \cdots \ 0 \ \xi_i \ 0 \ \cdots \ 0]$. *For given closed-loop* H_∞ *performance upper bounds* γ_{l0} $(l = 1, 2, \cdots, L)$ *and* H_2 *performance upper bounds* ν_l $(l = 1, 2, \cdots, L)$, *minimize* λ_i *over* $\mathbf{S}_{\infty l}$, $\mathbf{R}_{\infty l}$, $\mathbf{E}_{\infty l}$, $\mathbf{H}_{\infty l}$, γ_l, $\mathbf{S_{2}}_l$, $\mathbf{R_{2}}_l$, $\mathbf{E_{2}}_l$, $\mathbf{H_{2}}_l$ *and* $\mathbf{Q}_l$ $(l = 1, 2, \cdots, L)$ *subject to LMIs (30)-(31) and*

$$\Delta_{\infty\mathbf{P}0jl}(\mathbf{S}_{\infty l},\mathbf{E}_{\infty l},\gamma_l) < 0 \quad (32)$$

$$\Delta_{\infty\mathbf{Q}0jl}(\mathbf{R}_{\infty l},\mathbf{H}_{\infty l},\gamma_l) < 0 \quad (33)$$

$$\Delta_{\mathbf{2P}0jl}(\mathbf{S_{2}}_l,\mathbf{E_{2}}_l) < 0 \quad (34)$$

$$\Delta_{\mathbf{2Q}0jl}(\mathbf{R_{2}}_l,\mathbf{H_{2}}_l) < 0 \quad (35)$$

$$\Delta_{\mathbf{2}0jl}(\mathbf{S_{2}}_l,\mathbf{R_{2}}_l,\mathbf{Q}_l) > 0 \quad (36)$$

$$\gamma_l<\gamma_{l0} \quad (37)$$

$$\Delta_{\infty\mathbf{P}ijl}(\mathbf{S}_{\infty l},\mathbf{E}_{\infty l})<-\lambda_i\Delta_{\infty\mathbf{P}0jl}(\mathbf{S}_{\infty l},\mathbf{E}_{\infty l},\gamma_l)$$
$$\Delta_{\infty\mathbf{Q}ijl}(\mathbf{R}_{\infty l},\mathbf{H}_{\infty l})<-\lambda_i\Delta_{\infty\mathbf{Q}0jl}(\mathbf{R}_{\infty l},\mathbf{H}_{\infty l},\gamma_l)$$
$$\Delta_{\mathbf{2P}ijl}(\mathbf{S_{2}}_l,\mathbf{E_{2}}_l)<-\lambda_i\Delta_{\mathbf{2P}0jl}(\mathbf{S_{2}}_l,\mathbf{E_{2}}_l)$$
$$\Delta_{\mathbf{2Q}ijl}(\mathbf{R_{2}}_l,\mathbf{H_{2}}_l)<-\lambda_i\Delta_{\mathbf{2Q}0jl}(\mathbf{R_{2}}_l,\mathbf{H_{2}}_l)$$
$$\Delta_{\mathbf{2}ijl}(\mathbf{S_{2}}_l)>-\lambda_i\Delta_{\mathbf{2}0jl}(\mathbf{S_{2}}_l,\mathbf{R_{2}}_l,\mathbf{Q}_l)$$
$$l = 1, 2, \cdots, L, \quad j = 1, 2, \cdots q_l$$

Then we have λ_{iopt} *and the optimized aircraft parameter* $\xi_{iopt} = 1/\lambda_{iopt}$. *Repeat the above optimization procedures, we have all* ξ_{iopt}, $i = 1, 2, \cdots, r$.

Step 2 *Let* $\xi = [c_1\xi_{1opt} \ c_2\xi_{2opt} \ \cdots \ c_r\xi_{ropt}]\xi_c$ *where* ξ_c *is a variable,* $\xi_{iopt}(i = 1, 2, \cdots, r)$ *are obtained from Step 1 and* $c = [c_1 \ c_2 \ \cdots \ c_r]$ *with* $c_i > 0$ $(i = 1, 2 \cdots, r)$ *is the given penalty vector as in (8). For given closed-loop* H_∞ *performance upper bounds* γ_{l0} $(l = 1, 2, \cdots, L)$ *and* H_2 *performance upper bounds* ν_l $(l = 1, 2, \cdots, L)$, *minimize* λ_c *over* $\mathbf{S}_{\infty l}$, $\mathbf{R}_{\infty l}$, $\mathbf{E}_{\infty l}$, $\mathbf{H}_{\infty l}$, γ_l, $\mathbf{S_{2}}_l$, $\mathbf{R_{2}}_l$, $\mathbf{E_{2}}_l$, $\mathbf{H_{2}}_l$ *and* $\mathbf{Q}_l$ *subject to LMIs (30)-(37) and*

$$\sum_{i=1}^{r}c_i\xi_{iopt}\Delta_{\infty\mathbf{P}ijl}(\mathbf{S}_{\infty l},\mathbf{E}_{\infty l})<-\lambda_c\Delta_{\infty\mathbf{P}0jl}(\mathbf{S}_{\infty l},\mathbf{E}_{\infty l},\gamma_l)$$
$$\sum_{i=1}^{r}c_i\xi_{iopt}\Delta_{\infty\mathbf{Q}ijl}(\mathbf{R}_{\infty l},\mathbf{H}_{\infty l})<-\lambda_c\Delta_{\infty\mathbf{Q}0jl}(\mathbf{R}_{\infty l},\mathbf{H}_{\infty l},\gamma_l)$$
$$\sum_{i=1}^{r}c_i\xi_{iopt}\Delta_{\mathbf{2P}ijl}(\mathbf{S_{2}}_l,\mathbf{E_{2}}_l)<-\lambda_c\Delta_{\mathbf{2P}0jl}(\mathbf{S_{2}}_l,\mathbf{E_{2}}_l)$$
$$\sum_{i=1}^{r}c_i\xi_{iopt}\Delta_{\mathbf{2Q}ijl}(\mathbf{R_{2}}_l,\mathbf{H_{2}}_l)<-\lambda_c\Delta_{\mathbf{2Q}0jl}(\mathbf{R_{2}}_l,\mathbf{H_{2}}_l)$$
$$\sum_{i=1}^{r}c_i\xi_{iopt}\Delta_{\mathbf{2}ijl}(\mathbf{S_{2}}_l)>-\lambda_c\Delta_{\mathbf{2}0jl}(\mathbf{S_{2}}_l,\mathbf{R_{2}}_l,\mathbf{Q}_l)$$
$$l = 1, 2, \cdots, L, \quad j = 1, 2, \cdots q_l$$

Then we have $\xi_{copt} = 1/\lambda_{copt}$ *and the sub-optimized aircraft parameter vector* $\xi_{opt} = [c_1\xi_{1opt} \ c_2\xi_{2opt} \ \cdots \ c_r\xi_{ropt}]\xi_{copt}$.

Remark 2. It is noted that ξ_c in Step 2 is a common factor of the aircraft parameter vector ξ that is to be optimized. The scalars c_i $(i = 1, 2, \cdots, r)$ are used to penalize different aircraft parameters according to the design requirement, while the optimal solutions ξ_{iopt} $(i = 1, 2, \cdots, r)$ obtained from Step 1 are used to normalize the corresponding aircraft parameters $\xi_i(i = 1, 2, \cdots, r)$ so that the designed aircraft parameter vector ξ_{opt} is closer to the optimal one. It is noted that the above proposed approach can not guarantee to achieve the optimal aircraft parameters. Once the sub-optimal parameter vector ξ_{opt} is obtained, the optimal dynamic output-feedback controllers $\mathbf{K}_l$ $(l = 1, 2, \cdots, L)$ as in (6) can be solved by using the following approach.

3.2 Optimization of Controller Design

Performing congruence transformation with *diag* $\{\mathbf{Z}_{\infty\mathbf{1}l}, \mathbf{I}, \mathbf{I}\}$ on (9) and with *diag*$\{\mathbf{Z_{21}}_l, \mathbf{I}\}$ on (10), respectively, and partitioning $\mathbf{Z}_{\infty\mathbf{1}l}$, $\mathbf{Z}_{\infty\mathbf{2}l}$, $\mathbf{Z_{21}}_l$ and $\mathbf{Z_{22}}_l$ as in (20) and (23), we have the following equivalent matrix inequalities

$$\sum_{j=1}^{q_l}\mathbf{\Omega}_{\infty jl}(\mathbf{A_{k}}_l,\mathbf{B_{k}}_l,\mathbf{C_{k}}_l,\mathbf{D_{k}}_l)\theta_{jl} < 0 \quad (38)$$

$$\sum_{j=1}^{q_l}\mathbf{\Omega_{2}}_{jl}(\mathbf{A_{k}}_l,\mathbf{B_{k}}_l,\mathbf{C_{k}}_l,\mathbf{D_{k}}_l)\theta_{jl} < 0 \quad (39)$$

where $\mathbf{\Omega}_{\infty jl}(\mathbf{A_k}_l,\mathbf{B_k}_l,\mathbf{C_k}_l,\mathbf{D_k}_l) =$

$$\begin{bmatrix} \mathbf{\Omega}^{11}_{\infty jl}(\mathbf{B_k}_l,\mathbf{D_k}_l) & * & * & * \\ \mathbf{\Omega}^{21}_{\infty jl}(\mathbf{A_k}_l,\mathbf{B_k}_l,\mathbf{C_k}_l,\mathbf{D_k}_l) & \mathbf{\Omega}^{22}_{\infty jl}(\mathbf{C_k}_l,\mathbf{D_k}_l) & * & * \\ \mathbf{B}^T_{\infty jl}(\xi_{opt})\mathbf{S}_{\infty l} & \mathbf{B}^T_{\infty jl}(\xi_{opt}) & -\gamma_l \mathbf{I} & * \\ \mathbf{C}_{\infty l}+\mathbf{D}_{\infty l}\mathbf{D_k}_l\mathbf{C}_l(\xi_{opt}) & \mathbf{\Omega}^{42}_{\infty l}(\mathbf{C_k}_l,\mathbf{D_k}_l) & \mathbf{0} & -\gamma_l\mathbf{I} \end{bmatrix}$$

and $\mathbf{\Omega}_{\mathbf{2}jl}(\mathbf{A_k}_l,\mathbf{B_k}_l,\mathbf{C_k}_l,\mathbf{D_k}_l) =$

$$\begin{bmatrix} \mathbf{\Omega}^{11}_{\mathbf{2}jl}(\mathbf{B_k}_l,\mathbf{D_k}_l) & * & * \\ \mathbf{\Omega}^{21}_{\mathbf{2}jl}(\mathbf{A_k}_l,\mathbf{B_k}_l,\mathbf{C_k}_l,\mathbf{D_k}_l) & \mathbf{\Omega}^{22}_{\mathbf{2}jl}(\mathbf{C_k}_l,\mathbf{D_k}_l) & * \\ \mathbf{C}_{\mathbf{2}l}+\mathbf{D}_{\mathbf{2}l}\mathbf{D_k}_l\mathbf{C}_l(\xi_{opt}) & \mathbf{\Omega}^{42}_{\mathbf{2}l}(\mathbf{C_k}_l,\mathbf{D_k}_l) & -\mathbf{I} \end{bmatrix}$$

with

$$\begin{aligned}
&\mathbf{\Omega}^{11}_{\mathbf{x}jl}(\mathbf{B_k}_l,\mathbf{D_k}_l) = \mathbf{S_x}_l\mathbf{A}_{jl}(\xi_{opt}) + \mathbf{A}^T_{jl}(\xi_{opt})\mathbf{S_x}_l \\
&+\mathbf{C}^T_l(\xi_{opt})[\mathbf{N_x}_l\mathbf{B_k}_l+\mathbf{S_x}_l\mathbf{B}_{jl}(\xi_{opt})\mathbf{D_k}_l]^T \\
&+[\mathbf{N_x}_l\mathbf{B_k}_l+\mathbf{S_x}_l\mathbf{B}_{jl}(\xi_{opt})\mathbf{D_k}_l]\mathbf{C}_l(\xi_{opt}) \\
&\mathbf{\Omega}^{21}_{\mathbf{x}jl}(\mathbf{A_k}_l,\mathbf{B_k}_l,\mathbf{C_k}_l,\mathbf{D_k}_l) = \mathbf{R_x}_l\mathbf{C}^T_l(\xi_{opt})\mathbf{B}^T_{\mathbf{k}l}\mathbf{N}^T_{\mathbf{x}l} \\
&+\mathbf{M_x}_l\mathbf{C}^T_{\mathbf{k}l}\mathbf{B}^T_{jl}(\xi_{opt})\mathbf{S_x}_l + \mathbf{M_x}_l\mathbf{A}^T_{\mathbf{k}l}\mathbf{N}^T_{\mathbf{x}l} \\
&+\mathbf{B}_{jl}(\xi_{opt})\mathbf{D_k}_l\mathbf{C}_l(\xi_{opt}) + \mathbf{R_x}_l\mathbf{A}^T_{jl}(\xi_{opt})\mathbf{S_x}_l \\
&+\mathbf{A}_{jl}(\xi_{opt}) + \mathbf{R_x}_l\mathbf{C}^T_l(\xi_{opt})\mathbf{D}^T_{\mathbf{k}l}\mathbf{B}^T_{jl}(\xi_{opt})\mathbf{S_x}_l \\
&\mathbf{\Omega}^{22}_{\mathbf{x}jl}(\mathbf{C_k}_l,\mathbf{D_k}_l) = \mathbf{A}_{jl}(\xi_{opt})\mathbf{R_x}_l + \mathbf{R_x}_l\mathbf{A}^T_{jl}(\xi_{opt}) \\
&+\mathbf{B}_{jl}(\xi_{opt})[\mathbf{C_k}_l\mathbf{M}^T_{\mathbf{x}l}+\mathbf{D_k}_l\mathbf{C}_l(\xi_{opt})\mathbf{R_x}_l] \\
&+[\mathbf{C_k}_l\mathbf{M}^T_{\mathbf{x}l}+\mathbf{D_k}_l\mathbf{C}_l(\xi_{opt})\mathbf{R_x}_l]^T\mathbf{B}^T_{jl}(\xi_{opt}) \\
&\mathbf{\Omega}^{42}_{\mathbf{x}l}(\mathbf{C_k}_l,\mathbf{D_k}_l) = \mathbf{C_x}_l\mathbf{R_x}_l \\
&+\mathbf{D_x}_l[\mathbf{C_k}_l\mathbf{M}^T_{\mathbf{x}l}+\mathbf{D_k}_l\mathbf{C}_l(\xi_{opt})\mathbf{R_x}_l] \\
&\mathbf{N_x}_l = (\mathbf{I} - \mathbf{S_x}_l\mathbf{R_x}_l)(\mathbf{M}^{-1}_{\mathbf{x}l})^T \qquad (40)
\end{aligned}$$

where the subscript $\mathbf{x}$ is either 2 or ∞ and

$$\mathbf{A}_{jl}(\xi)=\mathbf{A}_{0jl}+\sum_{i=1}^{r}\xi_i\mathbf{A}_{ijl}, \quad \mathbf{B}_{jl}(\xi)=\mathbf{B}_{0jl}+\sum_{i=1}^{r}\xi_i\mathbf{B}_{ijl}$$

According to the above congruence transformation, the mixed H_2/H_∞ conditions (9)-(12) in Lemma 1 are equivalent to the conditions (38), (39), (29)-(31). Unfortunately, for the known aircraft parameter vector ξ_{opt}, the conditions (38), (39), (29)-(31) are still nonconvex. However, they are LMIs with respect to the control parameters $\mathbf{A_k}_l$, $\mathbf{B_k}_l$, $\mathbf{C_k}_l$, $\mathbf{D_k}_l$ if the matrices $\mathbf{S}_{\infty l}$, $\mathbf{R}_{\infty l}$, $\mathbf{M}_{\infty l}$, $\mathbf{N}_{\infty l}$, $\mathbf{S}_{\mathbf{2}l}$, $\mathbf{R}_{\mathbf{2}l}$, $\mathbf{M}_{\mathbf{2}l}$ and $\mathbf{N}_{\mathbf{2}l}$ are fixed. Here we use the approach proposed by Theorem 1 to solve for $\mathbf{S}_{\infty l}$, $\mathbf{R}_{\infty l}$, $\mathbf{S}_{\mathbf{2}l}$ and $\mathbf{R}_{\mathbf{2}l}$. As Theorem 1 provides a sufficient condition for Lemma 1, the solutions of Theorem 1 must satisfy the conditions of Lemma 1, namely, the matrix inequalities (38), (39), (29)-(31).

Hence, an approach for the optimal dynamic output-feedback controllers $\mathbf{K}_{lopt}$ $l = 1, 2, \cdots, L$ is proposed as follows: For chosen invertible matrices $\mathbf{M}_{\infty l}$ and $\mathbf{M}_{\mathbf{2}l}$, and known symmetric positive-definite matrices $\mathbf{S}_{\infty l}$, $\mathbf{R}_{\infty l}$, $\mathbf{S}_{\mathbf{2}l}$ and $\mathbf{R}_{\mathbf{2}l}$ obtained from Step 2 of the aircraft parameter optimization in Section 3.1, the matrices $\mathbf{N}_{\infty l}$ and $\mathbf{N}_{\mathbf{2}l}$ can be obtained from (40). For the given scalar $\gamma_l > 0$, find the matrix variables $\mathbf{A_k}_l$, $\mathbf{B_k}_l$, $\mathbf{C_k}_l$, $\mathbf{D_k}_l$ such that

$$\mathbf{\Omega}_{\infty jl}(\mathbf{A_k}_l,\mathbf{B_k}_l,\mathbf{C_k}_l,\mathbf{D_k}_l) < 0 \qquad (41)$$

$$\mathbf{\Omega}_{\mathbf{2}jl}(\mathbf{A_k}_l,\mathbf{B_k}_l,\mathbf{C_k}_l,\mathbf{D_k}_l) < 0 \qquad (42)$$

$$j = 1, 2, \cdots, q_l$$

Then we obtain the dynamic output-feedback controller parameters $\mathbf{A}_{\mathbf{k}lopt}$, $\mathbf{B}_{\mathbf{k}lopt}$, $\mathbf{C}_{\mathbf{k}lopt}$, $\mathbf{D}_{\mathbf{k}lopt}$.

4. CONCLUSIONS

This paper proposes a sub-optimization approach for integrated aircraft/controller parameter design. Based on the projection lemma Lemma 2, the design of aircraft parameters can be separated from the design of dynamic output-feedback controller parameters. As the sub-optimization approach is based on linear matrix inequalities, the computational amount required by the proposed approach is greatly less than that required by the iterative-LMI-based approaches, especially when disturbances, polytopic uncertainties, multi-mission operation and mixed H_2/H_∞ performance requirement are considered simultaneously.

REFERENCES

Apkarian, P., H. D. Tuan and J. Bernussou (2001), "Continuous-Time Analysis, Eigenstructure Assignment, and H_2 Synthesis With Enhanced Linear Matrix Inequalities (LMI) Characterizations", *IEEE Trans. Automat. Contr.*, Vol. 46, pp. 1941-1946.

Gahinet, P., A. Nemirovski, A. J. Laub and M. Chilali (1995), "LMI Control Toolbox," Natick, MA: *The Mathworks,*.

Grigoriadis, K.M. and F. Wu (1997), "Integrated H_∞ Plant/Controller Design via Linear Matrix Inequalities," in *Proc. 36th IEEE Conference on Decision & Control,* San Diego, California USA, pp. 789-790.

Liao, F., K. Y. Lum and J. L. Wang (2005), "An LMI-Based Optimization Approach for Integrated Plant/Output-Feedback Controller Design," accepted for presenting in *American Control Conference 2005,* Potland, Oregon USA, June, 2005.

Niewoehner, R.J. and I. I. Kaminer (1996), "Integrated Aircraft-Controller Design Using Linear Matrix Inequalities," *Journal of Guidance, Control and Dynamics,* Vol. 19, No. 2, pp. 445-452.

Yang, G. H. and K. Y. Lum (2003), "An optimization Approach to Integrated Aircraft-Controller Design," in *Proc. American Control Conference,* Denver, Colorado USA, pp. 1649-1654.

Zhou, K. M. and J. C. Doyle (1998), "Essentials of Robust Control", *Prentice-Hall Int. Inc.*.