A DATA-COMPRESSED TECHNIQUE OF A REFERENCE GOVERNOR IN A PIECEWISE AFFINE FUNCTION

Kiminao KOGISO [*] **Kenji HIRATA** [**]

[*] *Department of Information Systems, Nara Institute of Science and Technology; 8916-5 Takayamacho, Ikoma, Nara 6300192, JAPAN,* kogiso@is.naist.jp
[**] *Department of Computer-controlled Mechanical Systems, Osaka University; 2-1 Yamadaoka, Suita, Osaka 5650871, JAPAN,* hirata@mech.eng.osaka-u.ac.jp

Abstract: This paper demonstrates how to construct a reference governor for which the size of its implemented data can be adjusted with an integer parameter ν, ensuring fulfillment of a pointwise-in-time constraint. Using a technique called "blocking," a management method is established that changes the external reference every νT_s period, where T_s denotes a sampling period of a constrained system. Constraint fulfillment for an infinite time is achieved by satisfying a terminal condition using a maximal output admissible set. The reference governor is finally obtained with an explicit solution to the convex quadratic programming problem. Simulation and experimental evaluations demonstrate its effectiveness at constraint fulfillment and data compression.

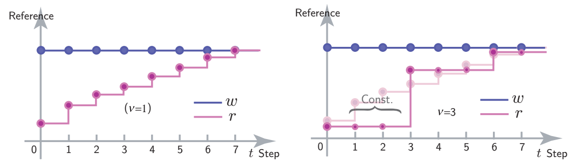
Keywords: Constraints, Closed-loop, Data compression, Optimization problems, Quadratic programming, Discrete-time systems

1. INTRODUCTION

Constraints are inherent characteristics in almost all practical control systems. They appear most commonly as actuator bounds on control variables, but physical limits on state variables are also ubiquitous. It is known that violations of such constraints drastically degrade system performance, and in the worst cases, lead to instability(Gilbert, 1992).

Recently, much research has been done on control approaches to systems with input and/or state constraints, with reference governor control schemes above all having received considerable attention(Bemporad and Mosca, 1998; Gilbert and Kolmanovsky, 2002; Hirata and Fujita, 1999; Hirata and Kogiso, 2001; Kapasouris *et al.*, 1988; Kogiso and Hirata, 2002; Sugie and Yamamoto, 2001; Kogiso and Hirata, 2003; Kogiso and Hirata, 2004). The most important and distinctive role of reference governors is to modify a reference signal supplied to a closed-loop system so as to enforce the fulfillment of constraints. Another property is that the problem of obtaining a good local control design for each specification can be decoupled from the problem of meeting constraints that typically becomes an issue when there is a large change in reference signals.

In practical control or implementation of the reference governor, it is vital to take into account its data size due to the finite specifications of a digital computer, for example, CPU performance, memory size and so on. If the size of the constructed reference governor is too large, the computer may not operate accurately or calculate tasks in real time. Therefore, it is worth considering a reference governor that has a parameter for adjusting its implemented data size. However, there has been no literature of how to construct a reference governor that considers not only the constraint fulfillment but also such a practical viewpoint. This paper, therefore, considers a reference governor to be constructed under a policy of data compression.



(a) Reference management of (Kogiso and Hirata, 2003) every sampling period.

(b) Reference management of this method every νT_s sampling period.

Fig. 1. The main idea of how to manage a given external reference w into an actual input r to a constrained system.

Our previous work (Kogiso and Hirata, 2003) has been extended to a method that additionally has a parameter ν to be able to adjust the size of its implemented data. The basic idea of the constraint fulfillment is the same as in the previous work. To achieve the goal of data compression, this approach employs a technique called "blocking" (Keller and Anderson, 1992) or a "ν-lifting operator" (Ishii and Francis, 2002), and realizes the reference governor, which manages the external reference(w) into an actual input(r) to the constrained system. Fig. 1 illustrates the difference between (Kogiso and Hirata, 2003) and this approach. Fig. 1(a) shows the previous method of managing the reference every sampling period, while Fig. 1(b) displays the way to manage every νT_s sampling period. This difference between methods causes data compression of the reference governor. In addition, to verify this reference governor's effectiveness at constraint fulfillment and data compression, some simulation and experimental results will be presented.

The structure of this paper is as follows. In Section 2, the constrained system is formulated by using the blocking technique, and in Section 3 show a terminal condition for constraint fulfillment. In Section 4, a reference governor is constructed, and in Section 5 perform a simulation and experimental evaluation. Finally, Section 6 concludes this paper.

2. CONSTRAINED SYSTEM

Consider a linear discrete-time closed-loop system Σ illustrated in Fig. 2, which consists of a plant Σ_p and a controller Σ_c. The system will be described in the formulation in Section 2.1 as an original, and another formulation for a data-compressed reference governor is shown in Section 2.2.

2.1 Formulation of an original system

The linear discrete-time system Σ with the sampling period T_s can be written by;

$$\Sigma: \quad \begin{aligned} x(t+1) &= Ax(t)+Bw(t), && (1a)\\ z_1(t) &= C_1x(t), && (1b)\\ z_0(t) &= C_0x(t)+D_0w(t), && (1c) \end{aligned}$$

where $t \in \mathbb{Z}^+$(non-negative integers) is a step of (1), $x = [x_p' \;\; x_c']' \in \Re^n \, (n = n_p + n_c)$ is a state vector of Σ, and $x_p \in \Re^{n_p}$ and $x_c \in \Re^{n_c}$ are respectively state vectors of Σ_p and Σ_c, and are measurable. An initial state is given by $x(0) = x_0 \in \Re^n$, $w \in \Re^{p_1}$

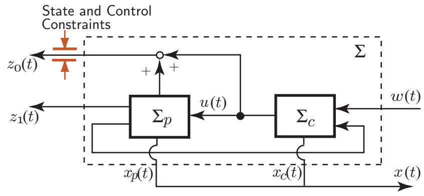


Fig. 2. The closed-loop system with state and/or control constraints.

is a reference, and $z_1 \in \Re^{p_2}$ is a controlled output. Additionally, $z_0 \in \Re^{p_0}$ is a vector to be constrained within the prescribed subset $\mathscr{Z} \subset \Re^{p_0}$ as

$$z_0(t) \in \mathscr{Z} \qquad \forall\, t \in \mathbb{Z}^+. \tag{2}$$

A control objective is achievement of a good tracking performance of z_1 to w under constraint fulfillment (2).

Remark 1. $\mathscr{Z}$ is a convex polyhedral set and is written by matrix $M_z \in \Re^{s_z \times p_0}$ and vector $m_z \in \Re^{s_z}$, i.e., $\mathscr{Z} = \{ z_0 \in \Re^{p_0} \,|\, M_z z_0 \leq m_z \}$. Note that inequalities in the above equations imply that it is component-wise.

Remark 2. In a correct and exact formulation, which includes a term of the sampling period T_s of Σ, T_s has to be explicitly described, that is, $x(T_s(t+1)) = Ax(T_st)+Bw(T_st)$ in (1a), $z_0(T_st) \in \mathscr{Z} \forall\; t \in \mathbb{Z}^+$ in (2) and so on. However, to escape these complicated descriptions, T_s is normalized, i.e., $T_s = 1$.

Remark 3. Our interest is focused on an additional reference management technique applied to the primary designed closed-loop system Σ. It is assumed that the controller Σ_c has already been designed by using abundant results of linear control theories, and in the absence of specified constraints, the controller Σ_c provides the desired tracking performance.

2.2 Reformulation of the original system by blocking

In constructing such a reference governor that the reference is managed every νT_s sampling period, a system with a sampling period νT_s is needed. The system can be derived from the original formulation (1) with a "blocking"(Keller and Anderson, 1992). Moreover, since a signal managed by the reference governor is the actual input to Σ in Fig. 3, the character w in (1) is replaced with the input vector $r \in \Re^{p_1}$.

Define a blocked vector, and make an assumption about the blocked signal. Here, $(\mathscr{B}_\nu r)[k] \in \Re^{\nu p_1}$, $k \in \mathbb{Z}^+$, denotes a blocked signal of r with ν successive signals:

$$(\mathscr{B}_\nu r)[k] = \begin{bmatrix} r(\nu k) \\ r(\nu k+1) \\ \vdots \\ r(\nu(k+1)-1) \end{bmatrix},$$

and it is assumed that the reference governor holds the first vector value $r(\nu k)$ for the νT_s sampling period. Then, we have the following assumption.

Assumption 4. All component vectors r of $(\mathscr{B}_\nu r)[k]$ at step $k \in \mathbb{Z}^+$ are the same, and the vector is denoted by $\bar{r}[k]$, i.e., $\bar{r}[k] = r(\nu k + i) = r(\nu k + (i+1)), \forall i \in \{0, 1, \cdots, \nu - 2\}$.

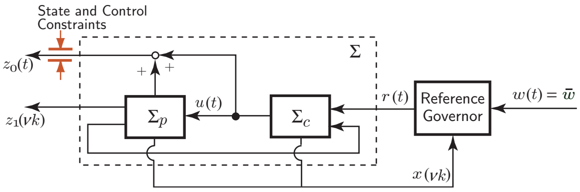


Fig. 3. The constrained system equipped with the proposed reference governor.

Remark 5. Without *Assumption 4*, we can construct data-compressed reference governors. However, since it is far more effective for data compression, this paper makes *Assumption 4*. The reason will be stated in Section 5.1.

Using the blocking and the notation $\bar{r}[k]$ under *Assumption 4*, the reformulated system below which has the sampling period νT_s, can be easily derived from (1) because of its linearity,

$$x(\nu(k+1)) = \tilde{A}x(\nu k) + \tilde{B}\bar{r}[k], \tag{3a}$$

$$z_1(\nu k) = \tilde{C}_1 x(\nu k), \tag{3b}$$

$$(\mathscr{B}_\nu z_0)[k] = \tilde{C}_0 x(\nu k) + \tilde{D}_0 \bar{r}[k], \tag{3c}$$

where $k \in \mathbb{Z}^+$ is a step of (3), T_s is normalized, $\tilde{A} = A^\nu$, $\tilde{C}_1 = C_1$, and $\tilde{C}_0$ is an observability matrix of the pair (A, C), and $\tilde{B} \in \Re^{n\times p_1}$ and $\tilde{D}_0 \in \Re^{\nu p_0 \times p_1}$ are written as follow: $\tilde{B} = A^{\nu-1}B + A^{\nu-2}B + \cdots + AB + B$, and

$$\tilde{D}_0 = \begin{bmatrix} D_0 \\ C_0B + D_0 \\ C_0AB + C_0B + D_0 \\ \vdots \\ C_0A^{\nu-2}B + C_0A^{\nu-3}B + \cdots + C_0B + D_0 \end{bmatrix}.$$

It is a key point that from the original constraint (2), the constraint corresponding to the system (3) is equivalently written below,

$$(\mathscr{B}_\nu z_0)[k] \in \tilde{\mathscr{Z}} \qquad \forall k \in \mathbb{Z}^+, \tag{4}$$

where $\tilde{\mathscr{Z}}$ is a convex polyhedral set and formulated by matrix $\tilde{M}_z = \mathrm{diag}(M_z, M_z, \cdots, Mz) \in \Re^{\nu s_z \times \nu p_0}$ and vector $\tilde{m}_z = [m_z' \ m_z' \cdots m_z']' \in \Re^{\nu s_z}$, i.e., $\tilde{\mathscr{Z}} = \{(\mathscr{B}_\nu z_0) \in \Re^{\nu p_0} \,|\, \tilde{M}_z z_0 \le \tilde{m}_z\}$. Note that although in (4) the blocked vector $(\mathscr{B}_\nu z_0)[k]$ is used, the inclusion (4) is equivalent to the original constraint (2).

2.3 Assumptions about references

This paper considers how to construct to a reference governor under the constant reference. Consequently, in this section we state some assumptions about the reference.

Assumption 6. An external reference is constant, i.e., $w(t) = \bar{w} \in \Re^{p_1} \, \forall t \in \mathbb{Z}^+$.

Remark 7. The limitation on the class of allowable external reference signals may not necessarily restrict the applicability for some specific but wide class of practical applications, for example, step-like reference inputs for the position control problem of mechanical systems.

The tracking to an arbitrary $\bar{w}$ may be impossible under the state and/or control constraints. It is, thereby, nontrivial to consider that the specified state and control constraints will be satisfied at the equilibrium state (corresponding to the external constant reference input $\bar{w}$). The equilibrium point $\bar{x}$ corresponding to the constant reference input $\bar{w}$ is given by $\bar{x} = (I-A)^{-1}B\bar{w}$ and the output at the equilibrium state by $\bar{z}_0 = (D_0 + C_0(I-A)^{-1}B)\bar{w}$. Regarding constraint fulfillment at the equilibrium state, this paper contains the following.

Assumption 8. A constant reference $\bar{w}$ satisfies

$$\bar{w} \in \mathrm{int}W$$
$$W = \{\bar{w} \in \Re^{p_1} \,|\, M_Z(D_0 + C_0(I-A)^{-1}B)\bar{w} \le m_Z\},$$

where int$W$ is an interior of a set W.

Therefore, this paper considers the control object for the system (3) that under the constant reference $\bar{w} \in \mathrm{int}W$, a good tracking performance of z_1 in (3b) is attained, fulfilling the constraint (4).

Remark 9. In comparison with the original formulation of (1), the information of the output z_1 described by (3b) partially drops, because the formulation of (4) considers only the state transition every νT_s sampling period.

3. TERMINAL CONDITION FOR CONSTRAINT FULFILLMENT

In this section a condition is clarified to fulfill the constraint (4), which is equivalent to (2). To do that, a maximal output admissible set is introduced below.

Definition 10. (**Maximal Output Admissible Set**) Let $z_0(t; x_0, \bar{w})$ denote the output (1c) of Σ for an initial state x_0 and a constant reference $\bar{w} \in \mathrm{int}W$. Define the $\bar{w}$ dependent maximal output admissible set by (Gilbert and Tan, 1991; Hirata and Fujita, 1999)

$$O_\infty(\bar{w}) = \{x_0 \in \Re^n \,|\, z_0(t; x_0, \bar{w}) \in \mathscr{Z} \ \ \forall t \in \mathbb{Z}^+\}.$$

Remark 11. Linear programming-based computational procedures of $O_\infty(\bar{w})$ have been proposed (Gilbert and Tan, 1991; Hirata and Fujita, 1999). The set $O_\infty(\bar{w})$ is a convex polyhedral set and can be constructed in the form of $O_\infty(\bar{w}) = \{x \in \Re^n \,|\, Mx \le m\}$, where $M \in \Re^{s\times n}$ and $m \in \Re^s$ are the matrices to describe linear constraints that specify $O_\infty(\bar{w})$.

Remark 12. Consider the constrained systems (1) under *Assumptions 6* and *8*. Then, from the definition of $O_\infty(\bar{w})$ it is obvious that the necessary and sufficient condition for constraint fulfillment (2) is that $x(0) \in O_\infty(\bar{w})$.

In this approach, the maximal output admissible set $O_\infty(\bar{w})$ is applied to the reformulated system (3), although it is defined for the constrained system (1) under the constant $\bar{w} \in \mathrm{int}W$, because it is reported in (Gilbert and Tan, 1991) that an inclusion $O_\infty^{T_{s1}}(\bar{w}) \subseteq O_\infty^{T_{s2}}(\bar{w})$ holds for the sampling period $T_{s1} \le T_{s2}$, where $O_\infty^{T_{s1}}(\bar{w})$ denotes the maximal output admissible set for the system discretized by a zero-order hold with T_{s1}. This means that the state $x(\nu T_k) \in O_\infty(\bar{w})$ given by (3a) for a step $k = T_k$ implies $x(\nu T_k) \in O_\infty^{\nu T_s}(\bar{w})$.

To fulfill the constraint, therefore, it is the most important to satisfy a terminal condition $x(\nu T_k) \in O_\infty(\bar{w})$

for a given step $k = T_k \in \mathbb{Z}^+$, being similar to the technique of constraint fulfillment in (Hirata and Kogiso, 2001). Once the terminal condition is satisfied at step $k = T_k$, the constraint $z_0(t) \in \mathscr{Z}\ \forall t \geq \nu T_k$ is fulfilled with $r(t) = \bar{w} \in \mathrm{int}W\ \forall t \geq \nu T_k$, that is, the output z_0 of (3c) fulfills the constraint (4) for all $k \geq \nu T_k$. To achieve constraint fulfillment for an infinite step, the reference governor must alter the reference $\bar{w}$ till the terminal condition $x(\nu T_k) \in O_\infty(\bar{w})$ for a given step $k = T_k \in \mathbb{Z}^+$ is satisfied.

4. REFERENCE GOVERNOR

This section formulates a method of managing the reference as an optimization problem for the system (3), and show how to derive a piecewise affine function that becomes a reference governor.

When an initial state x_0 of Σ in (3) and a reference $\bar{w} \in \mathrm{int}W$ are given under *Assumptions 4*, *6*, and *8*, a method of managing $\bar{w}$ so as to avoid violating the constraint (4) reduces to the following optimization problem:

$$\min_{\hat{\bar{r}}^\infty} ||\hat{z}_1^\infty - \hat{\bar{w}}^\infty|| + ||\hat{\bar{r}}^\infty - \hat{\bar{w}}^\infty|| \tag{5a}$$

$$\text{s.t. the constraint (4)}, \tag{5b}$$

where $||x||$ is defined as an appropriate norm of a vector x, while $\hat{z}_1^\infty$ denotes a sequence of the vector z_1 with an infinity horizon. The optimization (5) is difficult to solve in general, because it has the infinite dimension of the decision variable $\hat{\bar{r}}^\infty$ and the infinite number of the inequality constraints (5b). However, with the terminal condition stated in Section 3, the optimization (5) can be equivalently reduced to one with the finite dimension of the decision variable and the finite inequality constraints. The optimization problem is as follows:

$$\min_{\hat{\bar{r}}^{T_k^*} \in \Re^{T_k^* p_1}} ||\hat{z}_1^{T_k^*} - \hat{\bar{w}}^{T_k^*}||_{2,P} + ||\hat{\bar{r}}^{T_k^*} - \hat{\bar{w}}^{T_k^*}||_{2,Q} \tag{6a}$$

$$\text{s.t. } x(\nu T_k^*) \in O_\infty(\bar{w}), \tag{6b}$$

$$(\mathscr{B}_\nu z_0)[\tau] \in \tilde{\mathscr{Z}},\ \ \tau = 0, \cdots, T_k^* - 1, \tag{6c}$$

where the decision variable $\hat{\bar{r}}^{T_k^*}$ is a vector lengthwise-arranged $\bar{r}[k]$ from step 0 to $T_k^* - 1$, i.e., $\hat{\bar{r}}^{T_k^*} = [\bar{r}[0]'\ \bar{r}[1]'\ \cdots\ \bar{r}[T_k^* - 1]']'$, and $\hat{z}_1^{T_k^*} \in \Re^{p_1 T_k^*}$ similarly denotes a vector lengthwise-arranged z_1, which is written in $\hat{z}_1^{T_k^*} = Q_1 x_0 + Q_2 \hat{\bar{r}}^{T_k^*}$ by a linearity of (3) using appropriate matrices $Q_1 \in \Re^{p_1 T_k^* \times n}$ and $Q_2 \in \Re^{p_1 T_k^* \times p_1 T_k^*}$. In addition, $\hat{\bar{w}}^{T_k^*}$ consists of all the same components of $\bar{w}$. From the statement in Section 3, only the maximal output admissible set $O_\infty(\bar{w})$ in (6b) corresponds to the system Σ in (1). Finally, the step T_k^* denotes the shortest one of all such steps in which both the terminal condition $x(\nu T_k^*) \in O_\infty(\bar{w})$ and $(\mathscr{B}_\nu z_0)[\tau] \in \tilde{\mathscr{Z}}\ \tau \in \{0, 1, \cdots, T_k^* - 1\}$ are satisfied for a given initial state x_0 and $\bar{w} \in \mathrm{int}W$. The shortest step can be calculated by the method given in the previous work (Kogiso and Hirata, 2003).

Remark 13. The first term of the objective (6a) represents an error between the controlled output and the reference with a weight matrix P. The second term of (6a) is added for the signals obtained from the problem (6) in order to minimize vibration, for there is a danger of breaking the closed-loop system in practical use if such signals are inputted.

Although the reference governor works based on the quadratic optimization problem (6), what is implemented on the computer as the reference governor is the explicit solution to (6). The explicit solution can be derived from (6) through the multi-parametric quadratic programming with the Karush-Kuhn-Tacker condition(see details in (Bemporad *et al.*, 2002)). The obtained explicit solution is expressed as the following piecewise affine functions of a state:

$$\hat{\bar{r}}^{T_k^*} = F_i x + g_i\ \forall x \in CR_i,\ \ i = 0, 1, \cdots, T_k^* - 1, \tag{7}$$

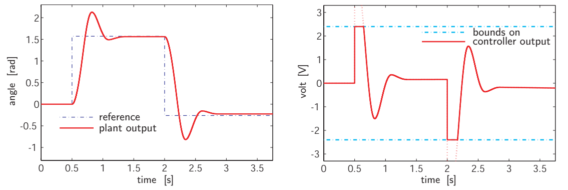
where $F_i \in \Re^{p_1 T_k^* \times n}$ and $g_i \in \Re^{p_1 T_k^*}$ denote an appropriate matrix and vector, and a critical region, $CR_i \subset \Re^n$, is a polytopic set on the state space. Here, consider a critical region CR_1 and a state $x^1 \in CR_1$. We choose another state parameter $x^2 \notin CR_1$, then similarly derive the corresponding explicit solution(F_2 and g_2) in the piecewise affine function of the state to be active over another critical region $CR_2 \ni x^2$, where $CR_1 \cap CR_2 = \phi$. An effective method of clarifying such critical regions is proposed in (Bemporad *et al.*, 2002; Tøndel *et al.*, 2003). By repeatedly calculating each (7) off-line, we can finally construct the reference management rule (7) in the form of a piecewise affine function over the specified parameter space.

With the state measured, the rule (7) gives the optimal solution to (6). Therefore, the reference governor inputs on-line the first signal $r(t) = \hat{\bar{r}}^{T^*}(1), \nu k \leq t \leq \nu(k+1)$ into the constrained system Σ every sampling period νT_s, and once the terminal condition (6b) is satisfied, then it directly inputs $\bar{w}$ into Σ without any change. Consequently, under *Assumptions 4*, *6*, and *8*, it becomes possible to show the following result of constraint fulfillment.

Theorem 14. For an initial state $x_0 \in \Re^n$ and a reference $\bar{w} \in \mathrm{int}W$, a constrained system equipped with the reference governor (7) illustrated in Fig. 3 can fulfill the prescribed constraints (2).

Proof. The proof is simple, and is omitted here. See *Theorem 1* in (Kogiso and Hirata, 2003).

The factor of data compression in comparison with $\nu = 1$, which corresponds to the previous work(Kogiso and Hirata, 2003), is the number of affine functions of the state and the critical region in (7). For example, in the case of $\nu = 10$ the reference governor finds such a single signal $\bar{r}[k]$ that $(\mathscr{B}_{10} z_0)[k] \in \tilde{\mathscr{Z}}$ is satisfied at step k. However, in the case of $\nu = 1$ the reference governor must find ten such managed signals that $z_0(t) \in \mathscr{Z}$ for each $t \in \{t, t+1, \cdots, t+9\}$. This case requires ten times the number of management rules in comparison with $\nu = 1$. On the other hand, this approach holds the advantage that the tracking performance of z_1 is evaluated in the sense of not every T_s sampling period (1b), but every νT_s period (3b).



(a) Reference $w(t)$ and controlled output $z_1(t)$.

(b) Control $z_0(t)$ and the constraint (8).

Fig. 4. Experimental result of step responses of the constrained system Σ without the proposed reference governor.

Therefore, we can say that this approach sacrifices partial tracking performance for data compression, where the constraint is fulfilled at every T_s sampling period.

Remark 15. Under this approach, it is difficult to accurately predict to what extent data will be compressed in comparison to the previous method (Kogiso and Hirata, 2003). However, it is certain that the more effective the data compression, the larger the ν, because the number of equations in the management rule (7) is decreased.

The next section validates the constraint fulfillment and data compression at both the simulation and experimental levels.

5. SIMULATION AND EXPERIMENTAL VALIDATION

Here we consider a practical plant Σ_p that consists of a DC motor, a gear, a hard shaft and a load. For additional information on the plant, see (Kogiso and Hirata, 2003). Defining $x_p = [\theta_L \ \dot{\theta}_L]'$, where θ_L is the load rotation angle, a model of Σ_p can be described by the following state-space form,

$$\dot{x}_p(t) = \begin{bmatrix} 0 & 1 \\ 0 & -9.8 \end{bmatrix} x_p(t) + \begin{bmatrix} 0 \\ 49 \end{bmatrix} u(t),$$
$$z_1(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p(t),$$

where the control $u = V$ and the output $z_1 = \theta_L$. In addition, Σ_p has a saturation constraint about the input voltage $z_0 = u$, which is given by

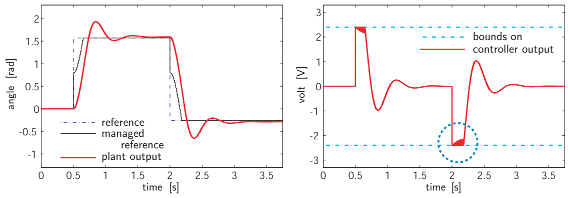
$$-2.4 \leq u(t) \leq 2.4 \qquad \forall t \in \mathbb{Z}^+. \tag{8}$$

A controller Σ_c was designed by loop-shaping, considering the specifications of stability and tracking performance. Here, Σ_c is represented as the following transfer function from an error $r - z_1$ to a control u, $\Sigma_c(s) = 3\left(1 + \frac{1}{3s}\right)$. It was then discretized by the sampling period of $T_s = 1.0$ ms with zero-order hold, and implemented on an Intel Pentium 3 733 MHz, 256 MB, on which RT-Linux v3.1 is installed as the operating system.

The external reference signal $w(t)$ is as follows,

$$w(t) = \begin{cases} 0, & 0 \leq t < 0.5 \text{ [s]}, \\ \bar{w}_1, & 0.5 \leq t < 2.0 \text{ [s]}, \\ \bar{w}_2, & 2.0 \leq t \text{ [s]}, \end{cases} \tag{9}$$

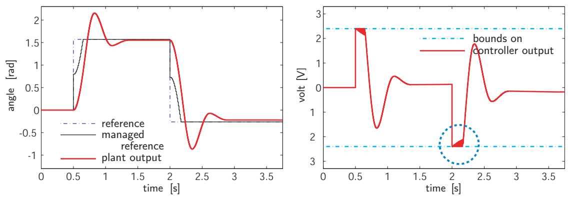
where $\bar{w}_1 = 1.5708$ rad and $\bar{w}_2 = -0.2618$ rad. The initial states $x_p(0)$ and $x_c(0)$ are all zeros. Without the proposed reference governor, then, we have obtained the result of the responses $z_1 = \theta_L$ and $z_0 = u$ in Fig. 4. From Fig. 4(a), the time-response of z_1 shows a very fast response for the reference (9), but also an inadmissible voltage input z_0 for the first reference change at 0.5 s and the second at 2.0 s, as shown in Fig. 4(b). Note that because of saturation, the dotted line in Fig. 4(b) is not supplied by the DC-motor, but is just a value for the controller Σ_c to calculate.



(a) Reference $w(t)$, managed reference $r(t)$ and output $z_1(t)$.

(b) Control $z_0(t)$ and the constraint (8).

Fig. 5. Simulation results of time responses of the constrained system Σ in the case of the proposed reference governor under $\nu = 10$.



(a) Reference $w(t)$, managed reference $r(t)$ and output $z_1(t)$.

(b) Control $z_0(t)$ and the constraint (8).

Fig. 6. Experimental results of time responses of the constrained system Σ in the case of the proposed reference governor under $\nu = 10$.

5.1 Simulation and experimental results

Under the reference (9), set $\nu = 10$. Fig. 5 illustrates a simulation result for time responses in the case of the proposed reference governor. From Fig. 5(a), the reference (9) is managed into a thin solid line $r(t)$, while the plant output $z_1(t)$ is plotted by a thick solid line. Fig. 5(b) shows the time response of the control $z_0(t)$ and the constraint condition. From this graph it can be seen that the response features good tracking performance with respect to the reference (9), and the constraint (2) is fulfilled. Though from Fig. 5(b) the control $z_0(t)$ does not violate the constraint condition (8), it does show oscillations while the proposed reference governor operates about the circle-marked region in Fig. 5(b).

Fig. 6 illustrates an experimental result of time responses in the experimental closed-loop system. Similar to the simulation result, from Figs. 6(a) and (b) it can be seen that there is good tracking performance with respect to the reference, the constraint is fulfilled, and the oscillations appear about the marked region in Fig. 6(b).

Remark 16. If only *Assumption 4* is removed out of this approach, oscillations such as those appearing in Figs. 5(b) and 6(b) can be erased. However, it is true that in this case the constructed reference governor becomes less effective at data compression than the

Table 1. Data compression on implemented reference governor program under the sampling time of $T_s = 1.0$ ms.

	Original Reference			
	from 0 to 90 deg		from 90 to -15 deg	
$v = 1$	518	(unexecutable)	633	(unexecutable)
$v = 10$	156	(-69.8%)	228	(-63.9%)

unit: Kbyte

case with *Assumption 4*. The reason is because the reference governor has ten times the number of management rules that give the signal that each component of $(\mathscr{B}_{10} z_0)[k] \in \tilde{\mathscr{Z}}$ is satisfied, and the more number of consituent inequalities of a critical region.

From both the simulation and experimental results, the data-compression reference governor's effectiveness at constraint fulfillment could be verified.

5.2 Data compression

This section shows to what extent the implemented data in the experiment is compressed, and compares setting $v = 10$ with $v = 1$, which also corresponds to (Kogiso and Hirata, 2003). Because RT-Linux is utilized as the operating system to perform the control experiments, the reference governor is made to be a function of a module program, which consists of if-then rules, in a kernel space. The size of the compiled program files is presented in Table 1.

Table 1 shows that data compression is successfully performed by setting $v = 10$. In this case, 69.8 % from 0 to $\bar{w}_1$ at 0.5 s, and 63.9 % from $\bar{w}_1$ to $\bar{w}_2$ at 2.0 s are respectively data-compressed, in comparison with the case of setting $v = 1$, which is too large for the complied program to execute in real time. From these concrete numbers, therefore, it can be seen that the reference governor in this approach has a policy of data compression, and it can adjust the implemented data, ensuring constraint fulfillment.

6. CONCLUSION

This paper proposed a reference governor that can consider data compression using a parameter v. The main idea is to manage the given reference every vT_s period so as to fulfill the constraint. To attain such a goal, we considered a reformulated system with a sampling period vT_s from the original system with T_s, and introduced the terminal condition described by the maximal output admissible set for the reformulated system. The reference management rule was derived from the explicit solution to the convex quadratic programming problem, which includes a state parameter. Furthermore, from the simulation and experimental verifications, we demonstrated the reference governor's effectiveness at constraint fulfillment and data compression.

REFERENCES

Bemporad, A. and E. Mosca (1998). Fulfilling hard constraints in uncertain linear systems by reference manageing. *Automatica* **34**(4), 451–461.

Bemporad, A., M. Manfred et al. (2002). The explicit linear quadratic regulator for constrained systems. *Automatica* **38**(1), 3–20.

Gilbert, E. G. (1992). Linear control systems with pointwise-in-time constraints: What do we do about them?. In: *Proceedings of the 1992 American Control Conference*. Chicago. p. 2565.

Gilbert, E. G. and I. Kolmanovsky (2002). Nonlinear tracking control in the presence of state and control constraints: A generalized reference governor. *Automatica* **38**(12), 2063–2073.

Gilbert, E. G. and K. T. Tan (1991). Linear systems with state and control constraints: Thte theory and application of maximal output admissible sets. *IEEE Transactions on Automatic Control* **36**(9), 1008–1030.

Hirata, K. and K. Kogiso (2001). A performance improving off-line reference management for systems with state and control constraints. In: *Proceedings of the 40th IEEE Conference on Decision and Control*. Orlando. pp. 4645–4650.

Hirata, K. and M. Fujita (1999). Set of admissible reference signals and control of systems with state and control constraints. In: *Proceedings of the 38th IEEE Conference on Decision and Control*. Phoenix. pp. 1427–1432.

Ishii, H. and B. A. Francis (2002). Stabilizing a linear system by switching control with dwell time. *IEEE Transactions on Automatic Control* **47**(12), 1962–1973.

Kapasouris, P., M. Athans and G. Stein (1988). Design of feedback control systems for stable plants with saturating actuators. In: *Proceedings of the 27th IEEE Conference on Decision and Control*. Austin. pp. 469–479.

Keller, J. P. and B. D. O. Anderson (1992). A new approach to the discretization of continuous-time controllers. *IEEE Transactions on Automatic Control* **37**(2), 214–223.

Kogiso, K. and K. Hirata (2002). An off-line reference management technique for linear constrained systems with time-varying uncertainties. In: *Proceedings of the 4th Asian Control Conference*. Singapore. pp. 77–82.

Kogiso, K. and K. Hirata (2003). A reference governor in a piecewise state affine function. In: *Proceedings of the 42th IEEE Conference on Decision and Control*. Maui. pp. 1747–1752.

Kogiso, K. and K. Hirata (2004). A reference governor for constrained systems in a block representation. In: *Proceedings of the 10th IFAC/IFORS/IMACS/IFIP Symposium on Large Scale Systems: Theory and Applications*. Vol. 2. Osaka. pp. 652–657.

Sugie, T. and H. Yamamoto (2001). Reference management for closed loop systems with state and control constraints. In: *Proceedings of the 2001 American Control Conference*. pp. 1426–1431.

Tøndel, P., T. A. Johansen and A. Bemporad (2003). An algorithm for multi-parametric quadratic programming and explicit MPC solutions. *Automatica* **39**(3), 489–497.