FAULT DETECTION WITH NON-LINEAR NUISANCE PARAMETERS AND SAFE TRAIN NAVIGATION

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Abstract: Fault Detection and Identification (FDI) problems often have to take into account the interference of nuisance parameters in the elaboration of decision processes. There are many works addressing cases in which nuisance parameters interfere in a linear and additive way, most of them in a deterministic framework. The main contribution of the paper is to propose a fully statistical methodology for dealing with non-linear nuisance parameters. The results obtained allow an analysis of the risks (in terms of non-detection and false alarm probabilities) attached to a statistical test designed for such non-linear models. The developed method is applied to the integrity monitoring of GNSS train navigation and some simulations demonstrate the worthiness of this approach.
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1. OUTLINE

The goal of the paper is to consider the FDI problem in the case of nuisance parameters (or faults) given by a non-linear static regressive model. The main contribution of the paper is the adaptation of the general statistical theory of detection with nuisance parameters to a certain class of non-linear models. A very special effort is done to estimate the statistical quality of the obtained tests and to warrant their ε-optimality. Finally, the developed methodology is used in the context of the integrity monitoring of GNSS train navigation.

2. PROBLEM STATEMENT

Let us consider the following static non-linear model

$$Y = H(X) + \varepsilon[+M\theta],\ X \in K \subset \mathbf{R}^m \qquad (1)$$

where $Y \in \mathbf{R}^n$ is the measured output, X is a unknown nuisance parameter (typically the state of a system), it is assumed that $n > m$, K is a compact set of vectors, $\theta \in \mathbf{R}^p$ is an informative parameter vector, i.e. the target fault to be detected, $\varepsilon \sim \mathcal{N}(0, \sigma^2 I_n)$ is a zero-mean Gaussian noise, M is a known full column rank $n \times p$ matrix and $H(X) = (h_1(X), \ldots, h_n(X))^T$ is a known function assumed to be at least continuous w.r.t. X. The target fault θ detection problem is stated as follows : one wishes to design a statistically optimal decision rule (based on the output Y) between hypotheses

$$\mathcal{H}_0 : \theta = 0, X \in K \subset \mathbf{R}^m \text{ (null hypothesis) and}$$
$$\mathcal{H}_1 : \theta \neq 0, X \in K \subset \mathbf{R}^m \text{ (alternative one).} \qquad (2)$$

The assumption that the vector X belongs to a compact set K can be justified in practice by some physical limitations on the values of variable X (the altitude of an aircraft is always positive, the power of an engine is limited), even if the real value of X remains unknown, besides it has been proved that such an information (if available) may be useful to improve the statistical quality of the decision process (Lacresse and Grall, 2001).

The statistical performance of a binary decision test $\delta\ : Y \mapsto \{\mathcal{H}_0,;\ \mathcal{H}_1\}$ is defined with the probability of false alarm: $\alpha = \Pr_0(\delta \neq \mathcal{H}_0)$ and the power function : $\beta_\delta(\theta) = \Pr_\theta(\delta = \mathcal{H}_1)$,where $\Pr_\theta$ stands for the vector Y being generated by model (1) when the true target fault vector is equal to θ. In case of a vector parameter θ, the crucial issue is to find an optimal solution over a set of alternatives which is rich enough. Unfortunately, uniformly most powerful (UMP) tests scarcely exist, except for some family of distributions when the parameter θ is scalar (Borovkov, 1987). The general theory of the composite hypotheses testing problem was originally proposed by Wald. His idea is to impose an additional constraint on the class of considered tests, namely, a *constant power function* $\beta_\delta(\theta)$ over a family of surfaces defined on the parameter space Θ (see details in (Wald, 1943)).

The optimal statistical decision problem has been solved in the case of linear nuisance parameter $H(X) = HX$, where H is a matrix of adequate size, which allows to rewrite (1) as

$$Y = HX + \varepsilon[+M\theta]. \tag{3}$$

Model (1) remains quite general, indeed many discrete-time models involving an informative parameter (target fault) can be re-written as regression models similarly to (1) (or (3)) (see details in (Basseville, 1997)), provided the initial detection problem is off-line.

3. LINEAR MODEL

An optimal solution based on the invariant uniformly best constant power (UBCP) test will be briefly recalled in this section (see details in (Basseville and Nikiforov, 2002; Fouladirad and Nikiforov, 2003; Fouladirad and Nikiforov, n.d.; Nikiforov, 2002)). The statistical testing problem between two hypotheses given by equation (2) in the case of model (3) is invariant under the group of transformations $G = \{Y \mapsto g(Y) = Y + HX\}$ of the output data Y, and one should then seek a statistics based on Y that is a maximal invariant statistics w.r.t G to design an optimal test. Let us define the column space $R(H)$ of the matrix H. The standard solution to this invariant problem is the projection of Y on the orthogonal complement $R(H)^\perp$ of the column space $R(H)$ (“parity space" in the analytical redundancy literature). The parity vector $Z = WY$ is the transformation of the measured output Y into a set of $n-q$ linearly independent variables (q being the rank of H). If the matrix H is full column rank, i.e. $q = m$, the matrix W satisfies the following conditions

$$WH = 0,\ \ WW^T = I_{n-m},\ \ W^TW = P_H,$$

with $P_H = I_n - H(H^TH)^{-1}H^T$. If $q < m$ then the matrix $(H^TH)^{-1}$ is replaced by a generalized inverse matrix (Koch, 1999, Ch. 1.5.3). The parity vector $Z = WY$ is indeed a maximal invariant for the group of transformations G (see (Fouladirad and Nikiforov, n.d.) for details) and transformation by W removes the interference of the nuisance parameter X since $Z = WY = W\varepsilon\ [+WM\theta]$. By using Wald’s methodology (Wald, 1943), one can define the statistics corresponding to this problem :

$$\Lambda(Y) = \frac{1}{\sigma^2} Y^T P_H M (M^T P_H M)^{-1} M^T P_H Y \tag{4}$$

It has been shown (Fouladirad and Nikiforov, 2003; Fouladirad and Nikiforov, n.d.) that the test δ^*

$$\delta^*(Y) = \begin{cases} \mathcal{H}_1 & \text{if} \quad \Lambda(Y) \geq h(\alpha) \\ \mathcal{H}_0 & \text{if} \quad \Lambda(Y) < h(\alpha) \end{cases}, \tag{5}$$

where the threshold $h(\alpha)$ is determined so that

$$\mathbf{Pr}_0(\delta^*(Y) = \mathcal{H}_1) = \int_{h(\alpha)}^{\infty} f_0(z)dz = \alpha$$

with $f_0(z) = \frac{z^{\frac{n-m}{2}-1} e^{-\frac{z}{2}}}{2^{\frac{n-m}{2}} \Gamma\left(\frac{n-m}{2}\right)}$, $\Gamma(x)$ is the gamma function and α is a prescribed probability of false alarm, is UBCP over the family of surfaces S_c : $\frac{1}{\sigma^2}\|WM\theta\|_2^2 = c^2$, $c > 0$. It is thus convenient to present the power of a such a test as a function of c^2 : $c^2 \mapsto \beta_{\delta^*}(c^2)$. It is easy to see that c^2 is the parameter of non centrality of the χ^2 law under $\mathcal{H}_1$. Hence, the power function is given by

$$\beta_{\delta^*}(c^2) = \mathbf{Pr}_{c^2}(\delta^*(Y)) = \int_{h(\alpha)}^{\infty} f_{c^2}(z)dz \tag{6}$$

where $f_\lambda(z) = f_0(z)e^{-\frac{\lambda}{2}} G\left(\frac{n-m}{2}, \frac{\lambda z}{4}\right)$ is the density of the non central $\chi^2_{n-m,\lambda}$ with $n-m$ degrees of freedom, λ is the non-centrality parameter, and $G(\kappa, x)$ is the hyper-geometric function. It was also shown (Nikiforov, 2002) that the invariant UBCP test given by equations (4) - (5) coincides with the *generalized likelihood ratio* test.

4. NONLINEAR MODEL

4.1 Preliminary remarks

At first glance, it seems that the methodology described in (Wald, 1943) could allow the treatment of of the fault detection problem in the case of non-linear model (1) re-formulated as follows

$$\mathcal{H}_0 : \nu(\theta) = 0, \mathcal{H}_1 : \nu(\theta) \neq 0$$

by using the observation vector Y whose distribution $\mathcal{F}_\theta$ is a function of parameter $\theta \in \Theta \subset \mathbf{R}^p$ and the function $\nu : \Theta \subset \mathbf{R}^p \to \mathbf{R}^r (r \leq p)$ being potentially non-linear. The feasibility of Wald’s scheme is conditioned by the possibility to complete the following set of r equations, $\nu(\theta) = 0$, corresponding to hypothesis $\mathcal{H}_0$, with $p - r$ similar equations to obtain a differentiable one-to-one map of the set Θ onto itself. The conditions on the family of distributions $\mathcal{F}_\theta$ are *a priori* weak but the properties of this family should allow to compute the maximum likelihood estimate

$\hat{\theta}(Y_1^N)$ from a sample Y_1^N of statistics Y to comply with the requirements of Wald's approach.

Unfortunately, it can be stated with many examples that the application of Wald's scheme to non-linear models encounters serious intrinsic difficulties. These problems can be quite easily explained with the following simple particular case of (1)

$$Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \sin x \\ \cos x \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} [+\theta] \quad (7)$$

with $x \in \mathbf{R}^1$, $\theta = (\theta_1, \theta_2)^T$ a possible fault and $\varepsilon = (\varepsilon_1 \varepsilon_2)^T \sim \mathcal{N}(0, \sigma^2 I_2)$. The initial decision problem is $\mathcal{H}_0 : \theta = 0$ vs. $\mathcal{H}_1 : \theta \neq 0$ but, in order to use Wald's methodology, model (7) and the initial formulation of $\mathcal{H}_0$ and $\mathcal{H}_1$ ought to be replaced by other more appropriate though not equivalent forms. Having no direct way to eliminate the nuisance x, it is assumed that $Y \sim \mathcal{N}\left((m_1, m_2)^T; \sigma^2 I_2\right)$ and the "new" decision problem is defined by using

$$\mathcal{H}_0 : \nu(m_1, m_2) = m_1^2 + m_2^2 - 1 = 0$$
$$\text{and } \mathcal{H}_1 : \nu(m_1, m_2) \neq 0$$

The first difficulty is that the function ν and the

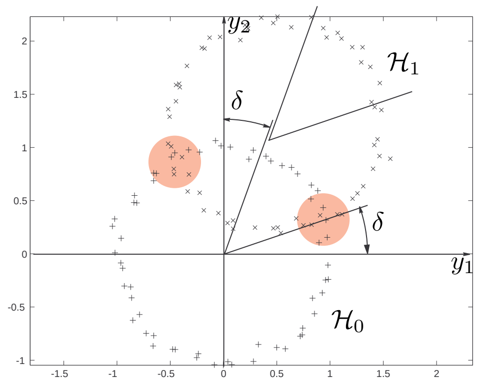

Fig. 1. The two hypotheses : the values of nuisance x ranges over the interval $[0; 2\pi]$

auxiliary parameters (m_1, m_2) do not fulfill Wald's requirements, i.e. in this case the completion of equation $m_1^2 + m_2^2 - 1 = 0$ with another similar equation to obtain a one-to-one map of $\mathbf{R}^2$ is impossible, and the statistical separability of the two hypotheses is not ensured (see figure 1). It is however interesting to retain the idea that a hypothesis testing problem involving a non-linear model with linear hypotheses can be turned into a problem with a linear model and non-linear hypotheses, even if the two formulations are not exactly describing the same problem.

The second difficulty is that even if a suitable function ν could be found to satisfy Wald's conditions, another problem would still persist : since the nuisance parameter x interferes within output Y in a non-linear way the optimality of the test based on Wald's statistic would only be asymptotic, whereas in the linear case optimality can be achieved without asymptotic conditions. This means that getting reliable statistical performances requires infinitely many measurements of output Y, whereas the detection processes under consideration should use very few measurements and possibly only one to be relevant to practical needs.

4.2 Usual linearization schemes

The use of linearization scheme is the most obvious way to overcome the difficulties mentioned in the previous section when dealing with the following non-linear model

$$Y = H(X) + \varepsilon[+\theta], X \in K$$

where H is a non-linear function. It is usually assumed that H is smooth enough to write, for any $X_0 \in K$, $H(X) = H(X_0) + \mathcal{J}_H(X_0) \cdot (X - X_0) + \|X - X_0\| \varepsilon(X - X_0)$ with $\lim_{X \to X_0} \varepsilon(X - X_0) = 0$ and $\mathcal{J}_H(X_0)$ the Jacobian matrix of H at point X_0. The non explicit term of this expansion is then neglected to treat statistics Y as if

$$Y \sim \mathcal{N}\left(H(X_0) + \mathcal{J}_H \cdot (X_0)(X - X_0) + \theta, \sigma^2 I_n\right)$$

If W_{X_0} is a projection on the left null-space of $\mathcal{J}_H(X_0)$, the affine interference of the nuisance X can be rejected by using the statistics $Z = W_{X_0} \cdot (Y - H(X_0))$ (assuming there is no rank problem with this projection). Following the methodology applied to linear models, the statistics to be used in this situation is $\Lambda(Y) = \frac{1}{\sigma^2} Z^T Z$. This statistics is distributed according to a χ^2 type law whose approximate non-centrality parameter (with a fault θ) is $\lambda_a = \|\frac{1}{\sigma^2} W_{X_0} \theta\|^2$ whereas its exact expression is $\lambda_t = \|\frac{1}{\sigma^2} W_{X_0} \cdot (H(X) - H(X_0) + \theta)\|^2$. To assess the performance of the test based on such an approximation scheme, it would be useful to compute

$$\sup_{X \in K} \|W_{X_0} \cdot (H(X) - H(X_0))\|^2$$

or even $\inf_{X_0 \in K} \sup_{X \in K} \|W_{X_0} \cdot (H(X) - H(X_0))\|^2$ to see if there is a "best" linearization point X_0 within the compact set K. The main difficulty arising in the case of this double optimization is that there is no general analytical/numerical solution to it. For this reason, it is not obvious to warrant certain statistical performances of the corresponding fault detection algorithm.

4.3 A more convenient approximation scheme

The "best" approximation scheme for a non-linear model like (1), taking into account the fact that X belongs to a compact set K, would allow to compute, for each component function h_i of H, the optimal coefficients $a_i^* \in \mathbf{R}^m$ and $b_i^* \in \mathbf{R}$ so that

$$(a_i^*, b_i^*) = \arg \inf_{(a_i, b_i) \in \mathbf{R}^m \times \mathbf{R}} J(a_i, b_i) \quad (8)$$

where $J(a_i, b_i) = \sup_{X \in K} |h_i(X) - a_i^T X - b_i|$. This leads to an approximation $X \mapsto a_i^T X + b_i$ of each function h_i and consequently to an approximation $X \mapsto AX + B$ of the function H *adapted* to the compact set K where the nuisance vector X lies.

Unfortunately, these optimizations problems are again intrinsically difficult if no particular assumption is made on the structure of H. To avoid this difficulty, the expression of the criterion $J(a_i, b_i)$ can be replaced by a more tractable expression to allow easier computations. For instance, if the components h_i of function H are accessible only via a discrete sample of their ranges on the compact set K at k different points $X_1, X_2, \ldots, X_k$ of K, one may replace $J(a_i, b_i)$ by a "quadratic" criterion such as

$$\tilde{J}(a_i, b_i) = \sum_{1 \le j \le k} \left(h_i(X_j) - (a_i^T X_j + b_i)\right)^2 \quad (9)$$

The above quadratic criterion yields straightforward computations of the coefficients and will be used in the rest of this contribution.

The coefficients a_i and b_i computed by a minimization of an alternative criterion $\tilde{J}(a_i, b_i)$ are not optimal, but they are useful for the statistical decision problem under consideration. The quality of such an approximation should be assessed through the computation of $\sup_{X \in K} |h_i(X) - a_i^T X - b_i|$, which is still a difficult optimization problem. To overcome this new obstacle, one can use the following easily computable expression $\rho_i = \sqrt{\tilde{J}(a_i, b_i)}$ or even

$$\tilde{\rho}_i = \max_{1 \le j \le k} |h_i(X_j) - (a_i^T X_j + b_i)| \quad (10)$$

as indicators of the approximation quality for each function h_i. Indeed $\tilde{\rho}_i$ is the only accessible estimate of $\sup_{X \in K} |h_i(X) - a_i^T X - b_i|$ if the functions h_i are only known via a discreet sample of values : $h_i(X_1), h_i(X_2), \ldots, h_i(X_k)$. As it will be shown in section 5, this kind of situation can arise in some applications and it should not be considered as an excessive simplification of the initial optimization problem.

The information contained in the values of $\tilde{\rho}_i$ for each component h_i of function H can be used in the following way. Setting $\mathcal{E}(X) = H(X) - AX - B$ and $Z = W \cdot (Y - B)$, with W a linear projection on the left null-space of matrix A, the non-centrality parameter of the χ^2 statistics $\Lambda(Y) = \frac{1}{\sigma^2} \mathcal{Z}^T \mathcal{Z}$ under hypothesis $\mathcal{H}_1$ is $\lambda_t(\theta) = \frac{1}{\sigma^2} \|W\theta + W\mathcal{E}(X)\|^2$ (with $\theta \neq 0$). This non-centrality parameter can be bounded, independently of the value of $X \in K$, using the norm of vector $R = (\tilde{\rho_1}, \ldots, \tilde{\rho_n})^T$

$$\lambda_1(\theta) = \frac{1}{\sigma^2} \max(\|W\theta\| - |||W||| \|R\|; 0)]^2 \le \lambda_t(\theta)$$
$$\le \lambda_2(\theta) = \frac{1}{\sigma^2} (\|W\theta\| + |||W||| \|R\|)^2,$$

with $|||W|||$ the Euclidean norm of the linear projection W (one can chose the projection matrix W so that $|||W||| = 1$), from the triangular inequality.

For a given test level α, $0 < \alpha < 1$, the power function β_t of the test based on statistics $\Lambda(Y) \sim \chi^2_{n-m, \lambda_t(\theta)}$ (with $m = \mathrm{rank}(W)$) have to be enveloped by the power functions β_1 and β_2 of the tests of the same level, based respectively on the distributions $\chi^2_{n-m, \lambda_1(\theta)}$ and $\chi^2_{n-m, \lambda_2(\theta)}$. β_1 and β_2 reflect the possible deviations of the power function of a test based on $\Lambda(Y)$. Setting $t = \|W\theta\|$, the linearization error of the model may cause a power loss reaching $e_1 = \sup_{t \in \mathbf{R}^+} [\beta(t) - \beta_1(t)]$ or a power gain reaching $e_2 = \sup_{t \in \mathbf{R}^+} [\beta_2(t) - \beta(t)]$ with β the power function of an "ideal" test based on $\chi^2_{n-m, \frac{t}{\sigma^2}}$ (β_t is not directly accessible). Of course, both expressions e_1, e_2 are useful, but for some applications (like the one described in section 5) slightly modified indicators can be used based on the idea that a potential loss of power is more worrying than a power gain, especially for critically important values of θ.

5. APPLICATION : SAFE GNSS TRAIN NAVIGATION

For many safety-critical applications, a major problem of the existing navigation systems consists in its lack of integrity. The integrity monitoring concept requires that a navigation system detects faults and removes them from the navigation solution before they sufficiently contaminate the output. The recent researches show that the detection/exclusion of the navigation message contamination is crucially important for the radio-navigation. It is proposed “to encourage all the transportation modes to give attention to autonomous integrity monitoring of GPS signals" (John A. Volpe Center, 2001). Only detection function is discussed in this paper; the *exclusion function* of the integrity monitoring algorithm is not studied here.

5.1 Navigation model

As it follows from (Nikiforov and Choquette, 2003), the train navigation measurement model is based on the assumption that the accurate train track is available under the form $(x, y, z)^T = (\phi(l), \varphi(l), \eta(l))^T$, where $l \in I$ is a curvilinear parameter and I is an interval. The pseudo-ranges r_i from the locomotive to the n satellites of the GNSS constellation are given by

$$r_i = d_i(\phi(l), \varphi(l), \eta(l)) + ct_r + \varepsilon_i, \; i = 1, 2, \ldots, n, \quad (11)$$

where $d_i(l) = d_i(\phi(l), \varphi(l), \eta(l))$ is the true distance between satellite number i and the locomotive, t_r is the clock-bias of the receiver (unknown but non random), $c \simeq 2.9979 \cdot 10^8$m/s is the speed of light and ε_i is a measurement noise, assumed to be Gaussian, zero mean with variance σ_i^2.

This model is non-linear w.r.t. l but linear w.r.t. t_r and the "usual" linearization scheme of this model around a "working point" writes

$$Y = (r_1 - d_1(l_0), \ldots, r_n - d_n(l_0))^T \simeq H \cdot (X - X_0) + \Xi, \quad (12)$$

with H a Jacobian matrix and $X = (l, ct_r)^T$ the state (the nuisance) vector of the model, $X_0 = (l_0, 0)^T$ the working point of the model and $\Xi = (\varepsilon_1, \ldots, \varepsilon_n)^T$ the measurement noise. This linearized model is used iteratively to obtain an estimate $\hat{X} = (\hat{l}, \hat{ct_r})^T$ of X by minimizing the following quadratic function

$$X \mapsto [Y - H \cdot (X - X_0)]^T \Sigma^{-1} [Y - H \cdot (X - X_0)]$$

where Σ is a diagonal covariance matrix of pseudo-range variances σ_i^2 ($1 \leq i \leq n$). The “snapshot" (i.e. based on one observation vector) Receiver Autonomous Integrity Monitoring (RAIM) algorithm is defined by the following stopping rule :

$$N_s = \inf\{k : \Lambda(Y_k) \geq h_s\} \tag{13}$$

where the threshold h_s is defined by using the characteristics of the statistics $\Lambda(Y_k) = (WY_k)^T \Sigma^{-1} (WY_k)$ and the knowledge of the maximum acceptable false alarm rate α (whose value is given *a priori*) and W is a linear projection on the left null-space of matrix H to detect a potential fault θ_i in model (11).

5.2 Integrity monitoring

The integrity monitoring of such a navigation system is based on the concept of an unacceptable positioning error, so-called “positioning failure", i.e. a situation when a positioning error of the locomotive is incompatible with some predefined safety requirements. For this purpose, an “Along the Track Protection Interval" (ATPI) can be defined as a curvilinear segment $[\underline{C}(l); \overline{C}(l)]$ centered on the true position of the locomotive $S(l)$, which is required to contain the indicated locomotive position with a given probability $1 - p_r$. The “length" L of this interval can be stated directly through a range of values of the scalar parameter l (see details in (Nikiforov and Choquette, 2003)). It is assumed that only one satellite channel can be contaminated by an additional bias θ in the measurement model (11).

Because the linearized model given by equation (12) is used, it is assumed that at each time k, $\Lambda(Y_k)$ has a central χ^2 distribution with $n - 2$ degrees of freedom if $\theta = 0$ (no positioning failure), and a non central χ^2 distribution with a non-centrality parameter $\lambda(\theta)$ if a positioning failure occurs, $\theta \neq 0$. The performances of the RAIM algorithm are adjusted by the definition of its maximum false alarm rate α and *availability*. The value of α allows the computation of a threshold h_s that defines the *stopping rule* of the algorithm (see equation (13)). If a fault $\theta_i = (0, \ldots, 0, v_i, 0, \ldots, 0)^T \neq 0$ ($1 \leq i \leq n$) occurs at time k, $\Lambda(Y_k)$ has a non centrality parameter $\lambda(\theta_i) = \lambda(v_i)$ whose magnitude depends on v_i. The corresponding non-detection probability $\mathbf{Pr}_{nd}$ can be expressed by $p(i, v_i) = \mathbf{Pr}_{nd} = \int_0^{h_s} f_{\lambda(\theta_i)}(u) du$ with density $f_\lambda(u) = f_0(u) e^{-\frac{\lambda}{2}} G\left(\frac{n-2}{2}, \frac{\lambda u}{4}\right)$. For each satellite channel, the length L of the ATPI interval determines a critical absolute value v_i^* below which a fault $\theta_i = (0, \ldots, 0, v_i, 0, \ldots, 0)^T$ is acceptably "small". These computations involve the probability of integrity risk p_r (used to define the ATPI interval), the probability p_f of a GNSS satellite channel integrity failure and the distribution of the statistics $|\hat{l}_k - l_k|$, where $\hat{l}_k$ is the estimated curvilinear parameter l_k. By computing the probability $p^* = \max_{1 \leq i \leq n} p(i, v_i^*)$, the algorithm is considered as “available" if $p^* \leq \gamma$ with γ the upper bound fixed for the non-detection rate.

As it follows from the above discussion, the integrity risk analysis is heavily based on the linearity hypothesis. The goal of the following section is to study the relation between the train track non linearity “magnitude" and its impact on the probabilities of non-detection and false alarm.

5.3 Non linearity magnitude and risk analysis

Instead of linearizing model (11) around a working point, a compact interval $K = [l_{min}; l_{max}]$ can be defined for l_k according to physical considerations on the motion of the train. Since the clock bias t_r interferes linearly in (11), this allows the computation of a affine approximation, as suggested in 4.3,

$$Y_k = (r_1, \ldots, r_n)^T - B \simeq AX_k + \Xi_k[+\theta].$$

The vector $R = (\tilde{\rho}_1, \ldots, \tilde{\rho}_n)^T$ (see equation (10)) is an indicator of the “quality" of this approximation. The “new" statistics $\Lambda(Y_k) = (W_A Y_k)^T \Sigma_k^{-1} (W_A Y_k)$ uses a projection $W_A = W(A)$ on the left null-space of matrix A and the consequences of the linear approximation on the statistical quality of a test based on $\Lambda(Y_k)$ can be assessed as follows. A critical value of the non-centrality parameter $\Lambda(Y_k)$ is defined as

$$\lambda_{max} = \max_{1 \leq i \leq n} \|W(0, \ldots, v_i^*, \ldots, 0)^T\|^2$$

and allows a better estimation of the availability of the RAIM-snapshot algorithm. Indeed, for some fault θ, the following situation may occur

$$\|W\theta + W(D(l_k) - AX_k - B)\|^2 < \lambda_{max}$$
$$\text{with } \|W\theta\|^2 > \lambda_{max}$$

(where $D(l_k) = (d_1(l_k), \ldots, d_n(l_k))^T$) and certain fault could be hidden by the linearization error committed. To avoid the underestimation of the non-detection rate, the following indicator can be used $e_1 = \beta(\lambda_{max}) - \beta_1(\lambda_{max})$.

For the RAIM algorithm based on the statistics $\Lambda(Y_k)$, this indicator represents the potential power loss, due to non-linearities, for a given false alarm level within the critical range of non-centrality parameters of this statistics. A more accurate estimate of the non-detection risks attached to this algorithm is then $p^* + e_1$ and the availability of the algorithm has to be decided on the basis of the following comparison $p^* + e_1 \gtreqless \gamma$ with γ the pre-requested non-detection rate.

Another useful indicator interfering with the performances of the algorithm is the value of $e_2 = \beta_2(0) - \beta(0)$ which corresponds to a potential increase of the false alarm rate of the algorithm. Indeed, if the effective false alarm rate of the algorithm is much greater than the value of α (requested by the user), the algorithm becomes hardly useable generating too many false alarms.

A numerical simulation has been performed with the following track equation

$$\begin{cases} x(l) = & x_0 + k_d l \\ y(l) = & y_0 + \upsilon \sin(2\pi l) \\ z(l) = & z_0 \end{cases}, l \in [0;1]$$

where the parameter υ is used to vary the "non-linearity" of the track. The values of α and γ have been chosen as, respectively, 10^{-5} and 10^{-3} that corresponds to typical integrity monitoring requirements. Figure 2 shows clearly that e_1 and e_2 quickly become non negligible in comparison to α and γ as υ (x axis) increases. The influence of the linear approximation error is thus to be taken into account for such integrity problems.

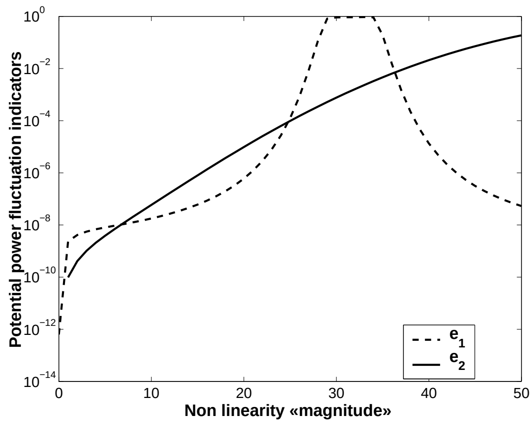

Fig. 2. Evolution of indicators e_1 et e_2 as functions of the non-linearity υ of the track.

6. CONCLUSION

The detection of a target fault in a stochastic model with non-linear nuisance parameters (or nuisance faults) is discussed in the paper. It is assumed that the nuisance parameters belong to a given compact set. The linearization of such a non-linear model allows the partial elimination of the nuisance parameter from the decision process but the usual linearization schemes does not warrant that the impact of such approximation errors on the statistical performances of an invariant test adapted to a linear model are negligible in every situation. The original aspect of the linearization scheme proposed in the paper is that the impact of the non-linearity of the model on the false alarm and non-detection rates of the test based on the corresponding linear approximation can be assessed through adequate indicators.

The developed theoretical results are applied to the risk analysis of the RAIM algorithm in the case of the GNSS-based along the track train navigation. It has been shown that replacing a non-linear measurement model by an approximate linear model is not always without serious consequences. Indeed, the example developed in this paper illustrates how a RAIM algorithm, working properly when the measurement equation is "mildly" non-linear, becomes useless and even dangerous for the user in the case when the non-linearity of this equation is too "strong".

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