ON CHATTERING-FREE DISCRETE-TIME SLIDING MODE CONTROL DESIGN

Seung-Hi Lee *

* *Samsung Advanced Institute of Technology, Suwon, KOREA*
shl@sait.samsung.co.kr

Abstract: A sliding mode control method is presented for linear multi-variable discrete-time systems, without introducing a discontinuous term to eliminate control chattering. Instead of a strong nonlinear function like switching introducing chattering problem in discrete-time, a linear control is designed around the sliding surface. Although not producing a strong acting control around the sliding surface, the discrete-time controller which becomes linear in the boundary layer is shown to deliver a bounded motion about the sliding surface even in the presence of plant uncertainties. An application example demonstrates excellent sliding mode behavior with a bounded motion about the sliding surface while eliminating control chattering. *Copyright©2005 IFAC*

Keywords: Control systems, Variable structure systems, Discrete time systems, Digital control.

1. INTRODUCTION

Sliding mode control (SMC) well known for its design simplicity and robustness is widely used in many control applications. Sliding mode is conventionally generated by means of discontinuous control signals about a surface which is attained from any initial condition in a finite time interval. Then, the plant motion being constrained to the sliding mode becomes robust against model uncertainty (Utkin, 1992; Edwards and Spurgeon, 1998; Young *et al.*, 1999). In continuous-time SMC, a discontinuous term is usually employed to generate finite time convergence to the surface, to enforce a sliding mode in the plane and to guarantee the closed-loop stability. Switching in the SMC well known to deliver robustness, however, also causes problematic control chattering that becomes more serious in discrete-time implementation.

In continuous-time, in order to generate finite time convergence to the surface, a continuous-time switching term is usually employed as a function of sliding surface. The discontinuous control is designed to enforce a sliding mode in the plane. This fast acting and simple control essentially applies infinite gain around the sliding surface, ensuring very smooth sliding and immunity to system uncertainties and disturbances (assuming no delay in the system). This is the key benefit of the SMC.

In discrete-time however, the discontinuous control action can be impractically fast, this limits some benefits of the SMC, introducing chattering problem. Accordingly in discrete-time sliding mode control (DSMC), it is impossible to maintain ideal sliding. Instead, a state feedback controller for discrete-time systems is introduced near the sliding mode (Spurgeon, 1992), where the feedback gain can be chosen to adjust convergence rate to the sliding mode (Lee *et al.*, 2000). In discrete-time control implementation, the control signal is held constant during the sample period and thereby it is not possible in general to attain ideal sliding as the required control must switch at infinite frequency. Since sampling at high frequency to closely approximate continuous-time is not always possible, the invariance properties of continuous-time SMC are lost. For this reason the DSMC has been studied in many papers (e.g. (Young *et al.*, 1999; Milosavljevic, 1985; Sarputurk *et al.*, 1987; Furuta, 1990; Spurgeon, 1992; Pieper and Surgenor, 1992; Su *et al.*, 1996)). In this case, the control law needs to be designed to keep the states as close as possible to the sliding surface. Introducing a boundary layer (e.g. (Spurgeon, 1992)) along the switching manifold and/or using estimated values in switching (e.g. (Young *et al.*, 1999)) eliminates/reduces control chattering. Removing such switching mechanism causes robustness degradation, but improves to some degree the control chattering problem.

This paper presents a sliding mode control method to eliminate control chattering for linear discrete-time systems. A modified sliding mode control method is presented for linear multi-variable discrete-time systems. Instead of a strong nonlinear function like switching introducing chattering problem in discrete-time, a linear control is designed around the sliding surface. This paper also analyze the resulting linear sliding mode controller to examine if it delivers a stable closed-loop and a bounded motion about the sliding surface in the presence of plant uncertainties. Then, although the resulting linear sliding mode controller may not have a strong acting control around the sliding surface, the controller producing a bounded motion about the sliding surface without a nonlinear function that causes control chattering must deliver desired plant behavior that can only be produced via enforced sliding mode. An application example is also provided to demonstrate sliding mode behavior with a bounded motion about the sliding surface while eliminating control chattering.

2. PROBLEM DESCRIPTION

To begin with, consider a discrete-time linear time-invariant plant model

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x(\cdot) \in \mathbb{R}^n$ is the state, $y(\cdot) \in \mathbb{R}^{n_y}$ is the output measurement, $u(\cdot) \in \mathbb{R}^{n_u}$ is the control input, $\Phi \in \mathbb{R}^{n\times n}$, $\Gamma \in \mathbb{R}^{n\times n_u}$, and $C \in \mathbb{R}^{n_y\times n}$.

Assumption 1. The triplet (Φ, Γ, C) is controllable and observable.

Assuming all the system states are available is not very realistic for practical problems and has motivated the need for the state estimator (Franklin *et al.*, 1990)

$$\begin{aligned} \bar{x}(k+1) &= \Phi\hat{x}(k) + \Gamma u(k) \\ \hat{x}(k) &= \bar{x}(k) + L_c\left(y(k) - C\bar{x}(k)\right) \end{aligned} \tag{2}$$

where $\bar{x}(\cdot)$ and $\hat{x}(\cdot)$ are predicted and corrected state estimates, respectively, $L_c \in \mathbb{R}^{n_y}$ is the state estimator gain to be determined for desired state estimation with a diagonal eigenvalue matrix Λ_{est}.

An observer based sliding dynamical sequence in the state space is defined as

$$s(k) = S\hat{x}(k) + r(k). \tag{3}$$

Here, $r(k)$ is the reference and

$$s(k) = \begin{bmatrix} s_1(k) \\ \vdots \\ s_{n_u}(k) \end{bmatrix}, \quad S = \begin{bmatrix} S_1 \\ \vdots \\ S_{n_u} \end{bmatrix} \in \mathbb{R}^{n_u\times n},$$

the sliding mode matrix.

Assumption 2. The sliding mode matrix $S \in \mathbb{R}^{n_u\times n}$ is determined such that $S\Gamma$ is nonsingular.

The problem we are concerned with is to design a sliding mode $s(\cdot)$ and a corresponding discrete-time sliding mode control law $u(\cdot)$ without introducing a discontinuous term, and to analyze the closed-loop system. Switching in the sliding mode control is well known to deliver robustness. However, it also causes problematic control chattering that becomes more serious in discrete-time control. Accordingly, no discontinuous term is going to be employed and the switching function is expressed in terms of a linear function using estimated values in the boundary layer. However, the resulting controller must guarantee the closed-loop's robust stability and bounded motion about the sliding surface. In this regards, this paper also analyze the closed-loop system's eigenvalues, robust stability, bounded motion about the sliding surface in the presence of plant uncertainties.

3. SLIDING MODE CONTROL DESIGN

The ideal sliding mode condition $s(k+1) = s(k) = 0$ gives the equivalent control

$$u_{eq}(k) = -K_{eq}\hat{x}(k) = -\left(S\Gamma\right)^{-1} S\Phi\hat{x}(k). \tag{4}$$

The ideal sliding mode dynamics is then described by

$$\hat{x}(k+1) = \Phi_{eq}\hat{x}(k) = \left(\Phi - \Gamma K_{eq}\right)\hat{x}(k) \tag{5}$$

with its eigenvalue matrix

$$\Lambda_{eq} = \mathrm{diag}\left(\Lambda_s, 0\right) \tag{6}$$

where $\Lambda_s = \mathrm{diag}\left(\Lambda_{s,1}, \cdots, \Lambda_{s,n_u}\right)$ with $\Lambda_{s,i} = \mathrm{diag}\left(\lambda_{i,1}, \cdots, \lambda_{i,n-1}\right), \quad i = 1, \cdots, n_u$, and corresponding eigenvector matrix

$$V_{eq} = \left[\, V_s \;\, V_n \,\right]. \tag{7}$$

The nonzero eigenvalue matrix satisfies $|\Lambda_s| < I$ as it is designed for and the corresponding eigenvector matrix

$$V_s = \left[\, V_{s,1} \ \cdots \ V_{s,n_u} \,\right]$$

with $V_{s,i} = \left[\, v_{i,1} \ \cdots \ v_{i,n-1} \,\right], \ i = 1, \cdots, n_u$, belongs to the null space $\mathcal{N}(S)$.

If the convergent sliding mode (Drakunov and Utkin, 1992; Milosavljevic, 1985; Su *et al.*, 1996) is not the ideal sliding mode, then the sliding points lie inside the layer after a certain time dependent upon the layer width.

Corollary 1. A sufficient condition for the existence of the discrete-time sliding mode is that there exists a positive integer k_o such that for $k \geq k_o$ and $1 \leq i \leq n_u$

$$|s_i(k+1)| \leq \alpha_i(k)\,|s_i(k)|\,, \ 0 \leq \alpha_i(k) < 1, \qquad (8)$$

in the region $\mathcal{N}_\epsilon = \{|s_i(k)| = |S_i\hat{x}(k)| < \epsilon\}$.

The sufficient condition (8) is equivalent to

$$-\alpha_i(k)\,|s_i(k)| \leq s_i(k+1) \leq \alpha_i(k)\,|s_i(k)|\,,$$

for $0 \leq \alpha_i(k) < 1$, $1 \leq i \leq n_u$. Here, we let

$$s(k+1) = w(k)|s(k)|$$

for a diagonal matrix $w(k)$. Then, a sufficient condition for the existence of the sliding mode is $|w(k)| < I$.

Using $s(k+1) = S\hat{x}(k+1) = -S\Gamma u_{eq}(k) + S\Gamma u(k)$ we have

$$u(k) = u_{eq}(k) + (S\Gamma)^{-1}\, w(k)|s(k)|. \qquad (9)$$

Introducing

$$M(k) = \text{diag}\left(\text{sgn}s_1(k), \cdots, \text{sgn}s_{n_u}(k)\right)$$

we have

$$u(k) = u_{eq}(k) + (S\Gamma)^{-1}\, w(k)M(k)s(k). \qquad (10)$$

Since a discontinuous term causing problematic control chattering is not going to be employed here, the switching function is expressed in terms of a constant diagonal matrix $W \in \mathbb{R}^{n_u \times n_u}$ referred to as *convergence rate factor matrix*, such that

$$w(k) = WM(k) \qquad (11)$$

and the control becomes

$$u(k) = u_{eq}(k) + (S\Gamma)^{-1}\, Ws(k). \qquad (12)$$

Furthermore, introducing a control bound, we have

$$u(k) = \text{sat}\left[u_{eq}(k) + (S\Gamma)^{-1}\, Ws(k)\right] \qquad (13)$$

where

$$\text{sat}[u] = \begin{cases} \text{sgn}[u]u_{\max} & \text{if } |u| > u_{\max}, \\ u & \text{otherwise} \end{cases}$$

is the saturation function and $u_{\max}$ is a vector of control bounds. Accordingly, the *control regions* are defined as follows:

$\mathcal{S}_+ = \{x : u = u_{\max}\}$;
$\mathcal{S}_- = \{x : u = -u_{\max}\}$;
$\mathcal{U} = \{x : |u| < u_{\max}\}$: boundary layer.

Observation 1. The resulting DSMC system becomes *linear* in the boundary layer $\mathcal{U}$ with the state feedback equivalent sliding mode control $u(k) = -K_{dsmc}x(k)$ where

$$K_{dsmc} = K_{eq} - (S\Gamma)^{-1}\, WS. \qquad (14)$$

Also, the shape of the boundary layer $\mathcal{U}$ can be computed from (13) in terms of W.

As a matter of fact, a linear control is applied around the sliding surface. The sliding mode control may be preferred to linear control because of its strong acting control around the sliding surface. The strong control achieves disturbance rejection. In general, the linear control does not have such desirable properties. In discrete-time however, the discontinuous control action can become impractically fast, this limits some benefits of the sliding mode control, introducing control chattering. In this paper, instead of a strong nonlinear function like switching, a linear control is designed around the sliding surface to eliminate chattering problem. A question that may naturally arise at this point is if the resulting controller guarantees a bounded motion about the sliding surface, a very useful characteristics of sliding mode control. The next section is dedicated to discussing a bounded motion about the sliding surface.

4. ANALYSIS OF THE SLIDING MODE CONTROLLER

Let us examine if the resulting linear sliding mode controller delivers a stable closed-loop and a bounded motion about the sliding surface. For this purpose, it is necessary to examine the eigenvalues of the closed-loop in the boundary layer $\mathcal{U}$ where the control becomes linear. Let the control closed-loop eigenvalue matrix

$$\Lambda = \text{diag}\left(\Lambda_1, \cdots, \Lambda_{n_u}\right) \qquad (15)$$

with $\Lambda_i = \text{diag}\left(\lambda_{i,1}, \cdots, \lambda_{i,n}\right), \quad i = 1, \cdots, n_u$. Also, let

$$\tilde{\Lambda} = \text{diag}\left(\tilde{\Lambda}_s, \tilde{\Lambda}_n\right), \qquad (16)$$

the rearranged control closed-loop eigenvalue matrix corresponding to the eigenvector matrix

$$\tilde{V} = \left[\, \tilde{V}_s \ \tilde{V}_n \,\right]. \qquad (17)$$

Proposition 1. Introducing the diagonal matrix $W \in \mathbb{R}^{n_u \times n_u}$ satisfies

$$\tilde{\Lambda}_s = \Lambda_s, \ \tilde{\Lambda}_n = W \qquad (18)$$

representing $\lambda_{i,n} = W_i$ for $i = 1, \cdots, n_u$.

Proof: Skipped due to space limitation. ■

Comparing the two matrices $\tilde{\Lambda}_n$ and W, we find $\tilde{\Lambda}_n = \text{diag}\left(\lambda_{1,1}, \cdots, \lambda_{n_u,n}\right)$ Also, observe that each row vector S_i in the matrix S is a left eigenvector of $\Phi_{eq} + \Gamma\left(S\Gamma\right)^{-1} WS$ corresponding to $\lambda_{i,n}$, $i = 1, \cdots, n_u$ such that

$$S\left(\Phi_{eq} + \Gamma\left(S\Gamma\right)^{-1} WS\right) = \tilde{\Lambda}_n S. \tag{19}$$

Introducing the diagonal convergence rate matrix $W \in \mathbb{R}^{n_u \times n_u}$, determining the convergence rate to the sliding mode, never affect the eigenvalues associated with the eigenvectors which belongs to the null space $\mathcal{N}\left(S\right)$. Thereby the sliding mode dynamics is preserved. The convergence rate factor W determines how fast the off-sliding surface plant trajectory points converge to the sliding surface. The resulting control closed-loop system matrix $\Phi_{eq} + \Gamma\left(S\Gamma\right)^{-1} WS$ yields an eigenvalue matrix $\tilde{\Lambda} = \text{diag}\left(\Lambda_s, W\right)$ with the corresponding eigenvector matrix $\tilde{V}$, whilst the equivalent system Φ_{eq} yields $\Lambda_{eq} = \text{diag}\left(\Lambda_s, 0\right)$ with the corresponding eigenvector matrix V_{eq}.

The result is summarized as follows.

Theorem 1. Given a discrete-time system (1) the state estimator gain L_c is determined for (2) such that $|\Lambda_{est}| < I$. Then, for any diagonal matrix $W \in \mathbb{R}^{n_u \times n_u}$ with $|W| < I$ producing convergent discrete sliding modes, the resulting sliding mode control law

$$u(k) = \text{sat}\left[u_{eq}(k) + \left(S\Gamma\right)^{-1} W s(k)\right]$$

delivers an internally globally asymptotically stable discrete-time closed-loop system with $|\Lambda| < I$.

Proof: The points outside the boundary layer (in the saturation region $\mathcal{S}_+$ and $\mathcal{S}_-$) move into the boundary layer $\mathcal{U}$ in finite time is well known and simple to prove (e.g. (Lee *et al.*, 2003)). Inside the boundary layer, the discrete sliding mode control system in fact becomes linear and thereby its stability can be simply determined in terms of closed-loop eigenvalues, Λ_{est} and $\tilde{\Lambda} = \text{diag}\left(\Lambda_s, W\right)$. Closed-loop stability in the boundary layer is assured from $|\Lambda_{est}| < I$, $|\Lambda_s| < I$ and $|W| < I$. ■

Let us consider the closed-loop system subject to a bounded matched plant uncertainty $g(k)$ described by

$$\mathbf{x}(k+1) = \Phi_{cl}\mathbf{x}(k) + \tilde{g}(k) \tag{20}$$

where $\mathbf{x} = \begin{bmatrix} x(k) \\ x(k) - \bar{x}(k) \end{bmatrix}$, $\tilde{g} = \begin{bmatrix} g(x) \\ g(x) \end{bmatrix}$ and

$$\Phi_{cl} = \begin{bmatrix} \Phi - \Gamma K_{dsmc} & \Gamma K_{dsmc}\left(I - L_c C\right) \\ 0 & \Phi - \Phi L_c C \end{bmatrix}.$$

The plant uncertainty is bounded such that $\|g(k)\| \le \rho \|\mathbf{x}(k)\|$ for a positive constant ρ. This uncertainty bound is realistic since state estimation error can become sufficiently small even in the presence of unknown plant uncertainty (Lee and Chung, 2003). Let $\Lambda_{cl} = \text{diag}\left(\Lambda, \Lambda_{est}\right)$, the closed-loop eigenvalue matrices, then the robust closed-loop stability can be stated as follows.

Corollary 2. If $\|\Lambda_{cl}\| + \sqrt{2}\rho < 1$, for $0 \le \|\mathbf{x}(k_o)\| < r$, the uncertainty closed-loop system (20) is globally uniformly asymptotically stable about the ball $\mathcal{B}(d) = \left\{\mathbf{x} \in \mathbb{R}^{2n} : \|\mathbf{x}\| \le d\right\}$ such that $\|\mathbf{x}(k)\| \le d(r, \Lambda_{cl}, \rho, k) = \left(\|\Lambda_{cl}\| + \sqrt{2}\rho\right)^{k-k_o} r$ for all $k \ge k_o$.

Proof: Skipped due to space limitation. ■

This result can be applied to state a bounded motion about the sliding surface.

Corollary 3. The distance from the sliding mode is bounded by

$$\|s\| \le \|S\| \|d(r, \Lambda, \rho, k)\|.$$

Proof: Skipped due to space limitation. ■

The resulting discrete-time sliding mode controller is shown to deliver a bounded motion about the sliding surface even in the presence of plant uncertainties. Although not producing a strong acting control around the sliding surface, the controller which becomes linear in the boundary layer $\mathcal{U}$ is shown to enforce the plant trajectory points robustly converge to the sliding surface against uncertainties in Corollaries 2 and 3, that can not happen in the linear control design. The resulting discrete-time sliding mode controller, producing a bounded motion about the sliding surface even in the presence of plant uncertainties, thereby must deliver desired plant behavior that can only be produced via enforced sliding mode. This is the key benefit of the proposed sliding mode control.

5. AN APPLICATION EXAMPLE

Let us consider a plant with one output measurement and two control inputs described by a triplet (A, B, C) and its discrete-time equivalent (Φ, Γ, C) at a sampling frequency of 19.2 kHz:

$$\Phi = \begin{bmatrix} 1 & 2.4479 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -4.7436\times10^{-3} & 0.6044 \\ 0 & 0 & -0.41936 & -0.3421 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 3.0335 & 0 \\ 2.4784 & 0 \\ -0.4255 & 0.04172 \\ -0.1776 & 0.017413 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}.$$

The plant has a lightly damped resonance that can be easily excited by control chattering. The proposed sliding mode control scheme is applied to enforce a sliding mode to attain a desired output response while avoiding resonance excitation.

In order to use state estimates, a current state estimator gain

$$L_c = \begin{bmatrix} 0.3365 & 2.1532{\times}10^3 & 0.6471 & -4.9861{\times}10^3 \end{bmatrix}^T$$

is chosen such that

$$\Lambda_{est} = \mathrm{diag}(0.69179{+}0.22722i, 0.69179{-}0.22722i, 0.06308{+}0.34667i, 0.06308{-}0.34667i).$$

Then, the sliding mode is determined by choosing $S_1 = \begin{bmatrix} 0.0935 & 1 & 0 & 0 \end{bmatrix}$ and $S_2 = \begin{bmatrix} 0.209 & 0 & 0.2707 & 1 \end{bmatrix}$ such that

$$S = \begin{bmatrix} 0.09354 & 1 & 0 & 0 \\ 0.20902 & 0 & 0.27072 & 1 \end{bmatrix}.$$

In this case, we have

$$K_{eq} = \begin{bmatrix} 0.033865 & 0.44493 & 0 & 0 \\ 6.8785 & 12.534 & -14.653 & -6.2155 \end{bmatrix}$$

and

$$\Lambda_s = \mathrm{diag}\,(0.79454, 0.37276)\,.$$

As mentioned earlier, the convergence rate matrix W determines the shape of each boundary layer. Choosing $W = \mathrm{diag}\,(0.1, 0.1)$ appears to produce desired convergence to the sliding modes (Fig. 1). Then, we compute the state feedback equivalent sliding mode control gain

$$K_{dsmc} = \begin{bmatrix} 0.030478 & 0.40873 & 0 & 0 \\ 6.1906 & 12.965 & -15.596 & -9.699 \end{bmatrix}$$

and find

$$\tilde{\Lambda}_n = W = \mathrm{diag}\,(0.1, 0.1)$$

and

$$\Lambda = \mathrm{diag}\,(0.79454, 0.1, 0.37276, 0.1)\,.$$

The DSMC system is tested through hybrid-time simulations (with the continuous-time plant and the discrete-time controller for more realistic results). Fig. 2 shows convergence of each sliding sequence. Plant trajectory points are being forced to slide down along each fixed hyperplane ($s_1(k)$ and $s_2(k)$, respectively) in Fig. 3 resulting in an excellent time response as shown in Fig. 4. Observe the resulting discrete-time sliding mode controller delivering a bounded motion about the sliding surface. Enforced sliding mode produces a desired output response while avoiding resonance excitation that can happen by control chattering. Also observe that the discrete-time control without chattering delivers a very smooth response.

6. CONCLUSIONS

This paper has presented a sliding mode control method for linear discrete-time systems, without

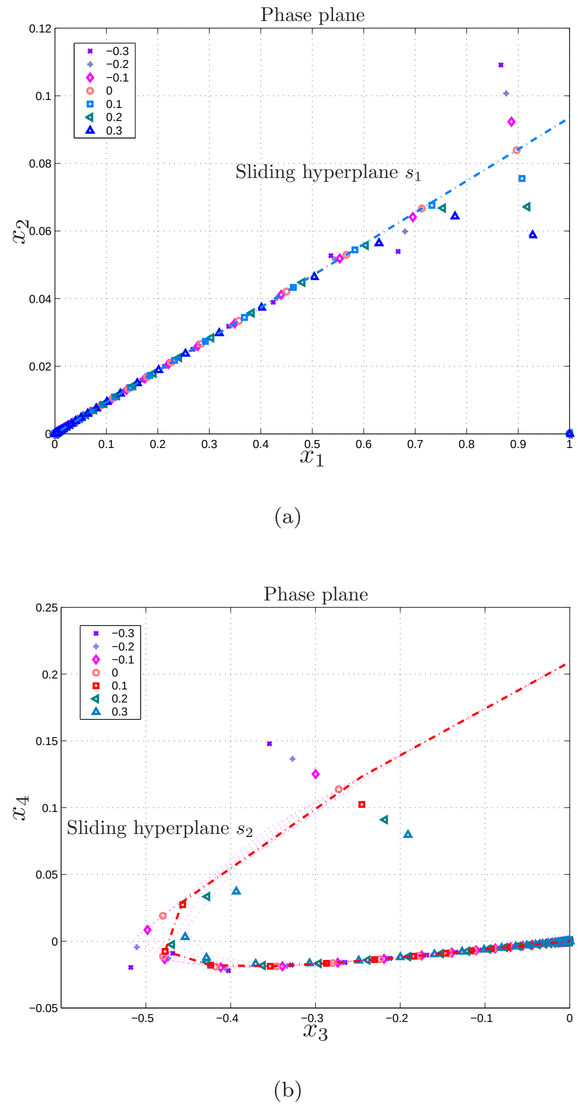

Fig. 1. Plant trajectory points converging to each sliding hyperplane with W: Observe the bounded motion about the sliding surface.

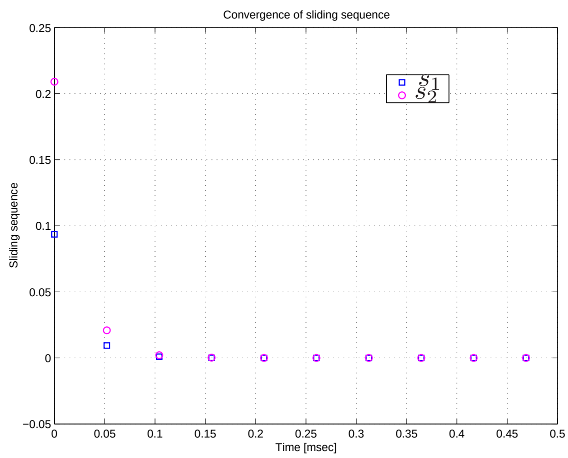

Fig. 2. Convergence of sliding sequences

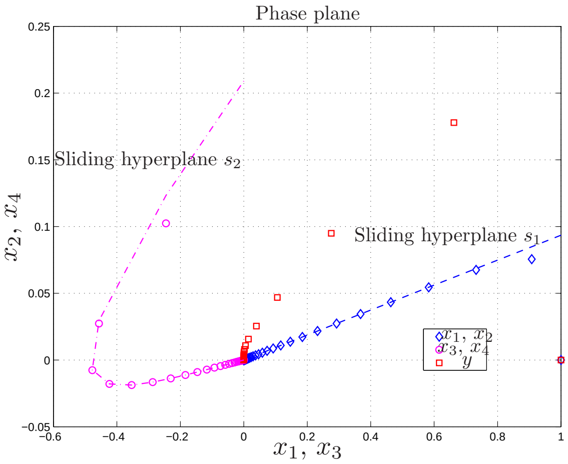

Fig. 3. Sliding hyperplanes and plant trajectories: Observe the bounded motion about the sliding surface.

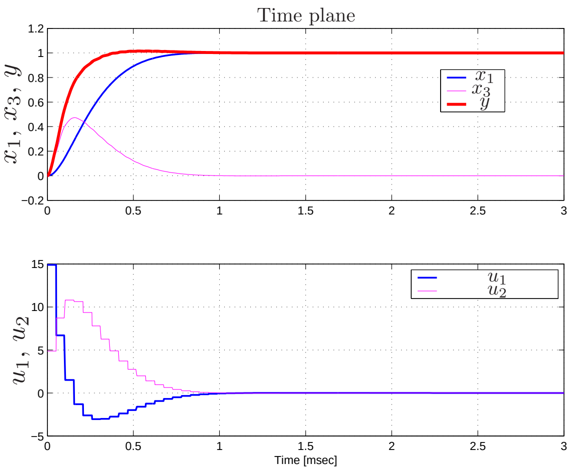

Fig. 4. Step response: Discrete-time control without chattering delivers a very smooth response.

introducing a discontinuous term to eliminate control chattering. Instead of a strong nonlinear function like switching introducing chattering problem in discrete-time, a linear control was designed around the sliding surface. Although not producing a strong acting control around the sliding surface, the controller which becomes linear in the boundary layer was shown to enforce the plant trajectory points robustly converge to the sliding surface against uncertainties, that can not happen in the linear control design. An application example demonstrated excellent sliding mode behavior with a bounded motion about the sliding surface while eliminating control chattering. The resulting discrete-time sliding mode controller, which guarantees a bounded motion about the sliding surface without a nonlinear function causing control chattering, thereby must deliver desired plant behavior that can only be produced via enforced sliding mode.

REFERENCES

Drakunov, S. and V. Utkin (1992). Sliding mode control in dynamic systems. *International Journal of Control* **55**(4), 1029–1037.

Edwards, C. and S. K. Spurgeon (1998). *Sliding Mode Control: Theory and Applications*. Taylor & Francis.

Franklin, G. F., J. D. Powell and M. L. Workman (1990). *Digital Control of Dynamic Systems*. Addison Wesley.

Furuta, K. (1990). Sliding mode control of a discrete system. *Systems & Control Letters* **14**, 145–152.

Lee, S.-H. and C. C. Chung (2003). Robust control using a state space disturbance observer. In: *Proc. IEEE Conference on Decision and Control*. pp. 1297–1302.

Lee, S.-H., S.-E. Baek and Y.-H. Kim (2000). Design of a dual-stage actuator control system with discrete-time sliding mode for hard disk drives. In: *Proc. IEEE Conference on Decision and Control*. pp. 3120–3125.

Lee, S.-H., S.-H. Chu and C. C. Chung (2003). Analysis and design of servomechanism and its application to disk drives. *IEEE Trans. Control Systems Technology* **11**(2), 233–241.

Milosavljevic, C. (1985). General conditions for the existence of a quasisliding mode on the switching hyperplane in discrete variable structure systems. *Automation and Remote Control* **46**(3), 307–314.

Pieper, J. K. and B. W. Surgenor (1992). Optimal discrete time sliding mode control with application. In: *Proc. IEEE Conference on Control Applications*. pp. 916–921.

Sarpturk, S. Z., Y. Isetefanopulos and O. Kaynak (1987). On the stability of discrete-time sliding mode control systems. *IEEE Trans. Automatic Control* **AC-32**(10), 930–932.

Spurgeon, S. (1992). Hyperplane design techniques for discrete-time variable structure control systems. *International Journal of Control* **55**, 445–456.

Su, W.-C., S. Drakunov and Ü. Özgüner (1996). *Robust Control via Variable Structure and Lyapunov Techniques*. Chap. Implementation of Variable Structure Control for Sampled-Data Systems, pp. 87–106. Springer-Verlag.

Utkin, V. I. (1992). *Sliding Modes in Control and Optimization*. Springer-Verlag.

Young, K. D., V. I. Utkin and Ü. Özgüner (1999). A control engineer's guide to sliding mode control. *IEEE Trans. Control Systems Technology* **7**(3), 328–342.