OPTIMAL CONTROL OF A CONTINUOUS-FLOW FAILURE PRONE MANUFACTURING SYSTEM

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Abstract: In this article the problem of inventory management of a single-stage single-product and two-machine-state continuous-flow manufacturing system with constant demand is considered. The machine is subject to operation-dependent failures. All the random variables are exponentially distributed. The goal is to obtain, in this case, the optimal policy which minimizes the discounted cost function. It is then proved that the optimal control is of hedging point type and that the value of the hedging point is nonnegative. This value is estimated by simulation.

Keywords: manufacturing systems, optimal control, stochastic failure.

1. INTRODUCTION

This paper considers a continuous-flow model of a failure-prone manufacturing system. The system produces a single product and it is characterized by two-machine-state (up and down). The objective is to determine, in this case, an optimum production control policy, which minimizes the discounted cost function. The problem of optimal control and design of manufacturing system has been studied in many works.

First of all, Kimenia and Gershwin formulated the production control of a manufacturing system with stochastic capacity (Kimemia and Gershwin, 1983). They determined that the optimum production policy has a special structure called hedging point in which a nonnegative production surplus of part-types should be maintained at times of excess capacity in order to hedge against future capacity shortages caused by machine failures. A very interesting result is the concept of hedging point that actually controls the throughput.

Moreover, Akella and Kumar found an exact value of the optimum hedging point for the case involving a single-stage single-product and two-machine-state system with constant demand rate described by continuous-flow with time-dependant machine failures, i.e., a machine can fail even if it is forced down (Akella and Kumar, 1986). Bielecki and Kumar assumed that the hedging point is known (Bielecki and Kumar, 1986) . Their result shows that for an unreliable manufacturing system under a continuous time, a zero hedging point or a zero-inventory policy can actually be optimal.

Two types of models are considered in the literature: continuous flow models and discrete flow models. Discrete flow models are often considered more realistic for discrete manufacturing but the discrete processing of parts makes the performance analysis difficult especially when simulation is used (Mourani, and al., 1983, and, Song and Sun, 2001). Continuous flow models (see Hu, and al., 1994, Sharifnia, 1988, Glasserman, 1995, Hu, 1995, Perkins and Srikant, 1998, Veatch and Caramanis, 1999, and Xie, 1989), offer a good approximation of material flows and makes the performance analysis more efficient without the need to track each individual part.

The failure model is chosen to be operations-dependant failure model (ODF), i.e. the machine can only fail when it is working. This model is the more realistic, 79% of the failures are ODF (Buzacott and Hanifin, 1978), but another possible failure model exists, called time-dependant failure. In this case the machine can fail even if it is forced down.

Our objective in that case is to find the optimal control policy which must be applied to the production speed of the machine when it is up in order to minimizes the discounted cost function and satisfy the constant demand by time unit.

This article is organized as follows. Section 2 addresses the elementary manufacturing system and the continuous-flow model and formulates the problem. Section 3 presents the discounting cost function and shows that the optimal policy is of hedging point type and that the hedging point is nonnegative. The value of the hedging point is given by simulation in section 4. We conclude the paper in section 5.

2. CONTINUOUS-FLOW MODEL

In this paper a single-stage single-product manufacturing system is considered. It is composed by a buffer, denoted by B, a machine M, and a constant demand D (Fig. 1).

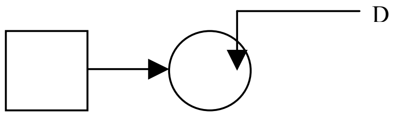

Fig. 1. Manufacturing system

2.1 Manufacturing system.

The machine is either up or down, and its state is denoted by:

$$\alpha(t)=\begin{cases} 1 & \text{if the machine is up} \\ 0 & \text{if the machine is down.} \end{cases}$$

All the random variables are exponentially distributed (memoryless property) with rate p et r respectively. Mean time between failures (MTBF) is equal to p^{-1} and mean time to repair (MTTR) is r^{-1}. The failure/repair process is an independent random process. It does not depend on the system parameters.
When the machine fails it can not work at all and when it is up it can work with a production speed $u(t)$ such as $U \geq u(t) \geq 0$ where U is the maximal production speed of the machine. In the case of $U \leq D$, even if the machine works always at the maximal production speed, the demand can not be satisfied. In this paper in order to satisfy the demand, we suppose that $U > D$. The machine is of a single-product type.

The failure are operations-dependant failures, i.e. the machine can only fail when it is working. This model is the more realistic but another possible failure model exists, called time-dependant failure. In this case the machine can fail even if it is forced down.

The inventory level at time t, denoted $x(t)$, is described by a continuous-flow model is given by :

$$x(t)=\int_0^t u(\sigma)d\sigma - Dt. \tag{1}$$

$x(t)$ could be negative or positive which respectively represents a backlog cost and a holding cost.

The demand, denoted by D, is supposed to be constant by time unit.

2.2 Cost function.

The cost criterion $J(x,\alpha)$, which depends on the buffer level and the machine state, is given by:

$$\lim_{t\to\infty} E[\int_0^t e^{-\beta\theta} g(x(\theta))d\theta \tag{2}$$

with $\beta > 0$ and the inventory cost $g(x)$ given by:

$$g(x) = \begin{cases} c^+ x^+ & \text{if } x \geq 0 \\ c^- x^- & \text{if } x < 0 \end{cases} \tag{3}$$

with $x+ = max(x, 0)$, $x^- = max(-x, 0)$ where c^+, c^- denote the holding cost and backlog cost respectively ($c^+ > 0$, $c^- > 0$).

Generally the inventory cost is convex and nonnegative in function of x (Fig. 2).

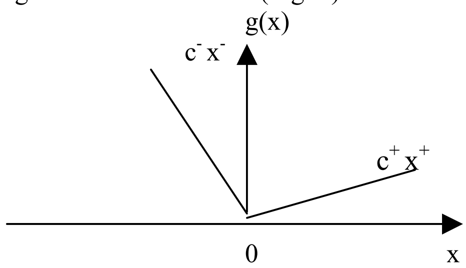

Fig. 2. Inventory cost

The mathematical study in case of discounting cost function is more complicated than average cost but is more simple to simulate.

The hedging point has been first defined by Kimemia et Gershwin (Kimemia and Gershwin, 1983). For the production control of a manufacturing system with stochastic capacity buffers, they have determined that the optimal policy has a special structure, called hedging point in which a nonnegative products surplus should be maintain in order to satisfy the demand when failures occur. Our objective is to find the optimal control policy which must be applied to the production speed $u(t)$ of the machine when it is up in order to minimizes the discounted cost function and satisfy the constant demand by time unit.

3. OPTIMAL POLICY

In a first step some hypothesis are given, thereafter the optimal policy is studied and we prove that this policy is of hedging point type and that the hedging point is nonnegative.

3.1 Hypothesis.

Hypothesis 1: there exists an optimal policy, denoted u_t, to apply to the production speed of the machine M which is stationary and deterministic: $u_t = f(x_t, \alpha_t)$.

Hypothesis 2: $J(x,\alpha)$, the discounting cost, is continuously differentiable in x, i.e. $\frac{\partial J(x,\alpha)}{\partial x}$ is continuous in x.

The proof for this last hypothesis is similar to the proof given in (Akella R., Kumar P., 1986).

In what follows, we prove that the optimal policy is of hedging point type, i.e. there exists a buffer level z^* which minimizes the discounting cost function. When the buffer level is less than z^*, the machine works at maximal production speed in order to have a stock level equal to z^*. When the buffer level is equal to z^*, the machine works in order to satisfy the demand and stay at z^*. When the buffer is greater than z^*, the machine stops working in order to come back to z^*. The hedging point policy is then given by:

$$u(t) = \begin{cases} U & if \quad x<z^* \\ D & if \quad x=z^* \\ 0 & if \quad x>z^* \end{cases} \tag{4}$$

3.2 Hedging point

The optimal policy of the production speed is stationary (Hypothesis 1) and satisfies the following HJB equations (Sage and White, 1977):

$$g(x) - D\frac{\partial J(x,0)}{\partial x} - (r+\beta)J(x,1) + rJ(x,0) = 0 \tag{5}$$

$$\inf\left\{ g(x) - \beta J(x,1) - D\frac{\partial J(x,1)}{\partial x} + u(x)\left[\frac{\partial J(x,1)}{\partial x} - p\frac{J(x,0)-J(x,1)}{U}\right]\right\} = 0 \tag{6}$$

From equations (4) and (6) we can note that $u(x)$ depends on the evolution of $\eta(x)$:

$$\eta(x) = \frac{\partial J(x,1)}{\partial x} - p\frac{J(x,0)-J(x,1)}{U} \tag{7}$$

Indeed, in order to satisfy equation (5), we can obtain the following relation between $\eta(x)$ and $u(x)$:

$$u(x) = \begin{cases} 0 & if \quad \eta(x)>0 \\ U & if \quad \eta(x)<0 \\ unspecified & if \quad \eta(x)=0 \end{cases} \tag{8}$$

Suppose that $\eta(x)$ have the following behavior (Fig. 3):

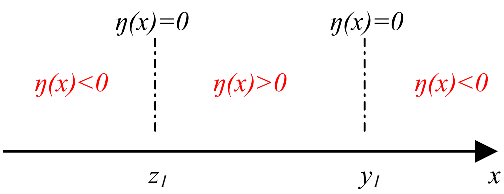

Fig. 3. Behavior of $\eta(x)$

From Figure 3, we obtain that $u(x) = D$ when $\eta(x)=0$, so we have for $u(x)$:

$$u(x) = \begin{cases} 0 & if \quad \eta(x)>0 \\ U & if \quad \eta(x)<0 \\ D & if \quad \eta(x)=0 \end{cases} \tag{9}$$

We will show in what follows that there exists a buffer level called hedging point, denoted by z^*, which minimizes the cost function and assures that the production speed follows equation (4).

3.3 Theoretical results.

Theorem 1:

$$J(x,0) \geq J(x,1) \qquad \forall x \tag{10}$$

The proof is given in Annexe1.

Indeed, it is better to start with an up-state machine than a down-state machine.

Theorem 2: The optimal policy applied to the production speed of the machine is of hedging point type.

There exists only one z, i.e. in Figure 3 z_1 is the hedging point and y_1 does not exist.

In this case we obtain the following relation between z_1 and $\eta(x)$:

$$\begin{cases} \eta(x) > 0 & si \quad x > z_1 \\ \eta(x) = 0 & si \quad x = z_1 \\ \eta(x) < 0 & si \quad x < z_1 \end{cases}$$

By equation (9), we obtain the following relation between the production speed $u(t)$ and z_1:

$$u(t) = \begin{cases} U & si \quad x < z_1 \\ D & si \quad x = z_1 \\ 0 & si \quad x > z_1 \end{cases}$$

The proof of this theorem is given in Annexe 1.

Theorem 3: The hedging point is nonnegative.

By equation (4) we know that if we can prove that when a backlog occurs in our system, the machine will produce at the maximal production speed $u^*(x)=U \quad \forall\, x<0$. Then, we know that when the buffer is null, the production speed could be equal to the demand rate D or to the maximal production speed U, i.e. the hedging point is null or greater than 0. So, we can conclude intuitivly that the hedging point is nonnegative. The proof is given in Annexe 1.

We did not found the exact value of the hedging point so we propose to estimate it by simulation in the following section.

4. NUMERICAL RESULTS

In order to simplify the simulation we have digitalized the continuous-flow model which allows us to estimate the value of the hedging point. Indeed, Xie gives us an example for the discrete-flow model (Xie, 1989). The equations are detailed in Annexe 2.

By equation (3) we obtain the following dynamical equations (Annexe 2):

$$J(x, 0) = g(x)*\Delta t+(1-r*\Delta t)J(x-D*\Delta t, 0)e^{-\beta\Delta t}+r*\Delta tJ(x, 1)e^{-\beta\Delta t}$$

$$J(x, 1) = g(x)* \Delta t+(1-\Delta t *p*u/U) e^{-\beta\Delta t}J(x+(u-D) *\Delta t, 1)+ \Delta t *p*u/UJ(x,0) e^{-\beta\Delta t}$$

Then, with a C program and with only one varying-parameter at each simulation, we obtain the following results.

Table 1 Results when the failure rate p varies

Failure rate p	Optimal buffer level $z*$	Minimum cost $J*$
0,01	35	925,48
0,02	43	1143,66
0,03	49	1282,25
0,04	52	1388,80
0,05	56	1477,95
$r=0,05$ $\beta=0,05$ $c^+=1$ $c^-=250$ $\Delta t=1$ $D=1$ $U=3$		

Table 2 Results when the repair rate r varies

Repair rate r	Optimal buffer level $z*$	Minimum cost $J*$
0,06	31	814,17
0,07	27	722,68
0,08	26	648,10
0,09	22	582,65
0,1	20	480,94
$p=0,01$ $\beta=0,05$ $c^+=1$ $c^-=250$ $\Delta t=1$ $D=1$ $U=3$		

The more important is the failure rate, the more important is the cost function. Indeed, the number of failures in this case are more important and it is then necessary to have an important number of products in the buffer in order to limitate the backlogs. We have the same results when the repair rate is small.

Table 3**:** Results when the discount factor β varies

Discount factor β	Optimal buffer level $z*$	Minimum cost $J*$
0,06	31	679,23
0,07	27	517,53
0,08	24	405,84
0,09	22	325,63
0,1	19	266,03
$p=0,01$ $r=0,05$ $c^+=1$ $c^-=250$ $\Delta t=1$ $D=1$ $U=3$		

We can note that the more important is the discount factor the more smaller is the hedging point.

Table 4**:** Results when the demand rate D varies

Demand rate D	Optimal buffer level $z*$	Minimum cost $J*$
1	3	80,18
10	200	5298,59
20	560	14885,39
30	990	25964,21
$p=0,01$ $r=0,05$ $\beta=0,05$ $c^+=1$ $c^-=250$ $\Delta t=1$ $U=100$		

The more important is the demand rate, the more important is the buffer level z* to satisfy this demand.

To conclude, we can note that the hedging point depends on the failure rate p, the repair rate r, the demand D, the backlog cost c^-, the holding cost c^+, the discount factor β and the maximal production speed U.

5. CONCLUSION

This paper considers a continuous-flow model of a failure-prone manufacturing system. The system produces a single product and it is characterized by two-machine-state (up and down). The machine failures are defined to be operation-dependant failures, i.e. they depends on the production volume and the machine can only fail when it works. All the random variables are exponentially distributed.). The demand is constant by time unit. The objective is to determine, in this case, an optimum production control policy, which minimizes the discounted cost function.

We have proved that the optimal policy is of hedging point type and the hedging point is nonnegative. The exact value of this hedging point is then obtained by simulation. In order to simplify the simulation, we have digitalized the continuous-flow model and obtain discrete equations.

As a perspective of our work, it would be very interesting to find the exact value of the hedging point in our case. It would be also important to generalize these results to other manufacturing systems such as assembling/disassembling systems and to study the case of stochastic demand.

ANNEXES

Annexe 1: Proof of theorems

Proof of theorem 1

Compare two systems:

- a system denoted $S^{x,0}$ which starts with a failure machine and an initial buffer level equal to x,
- another system denoted $S^{x,1}$ which starts with an up-state machine and the same initial stock level as system $S^{x,0}$.

Define $t^{x,0}$ as the time repair for $S^{x,0}$.

We suppose that system $S^{x,1}$ does not product $(u = 0)$ until $t^{x,0}$.

We couple these both systems until $t^{x,0}$, i.e. the production speed of the machine and the buffer are the same in $(0, t^{x,0})$. We have:

$$\begin{cases} x^{S^{x,0}}(t^{x,0}) = x^{S^{x,1}}(t^{x,0}) \\ \alpha^{S^{x,0}}(t^{x,0}) = \alpha^{S^{x,1}}(t^{x,0}) = 1 \end{cases} \quad (11)$$

So for this periode of time the inventory cost is the same for the both systems, consequently we have:

$$J^{S^{x,0}}(x,0) = J^{S^{x,1}}(x,1)$$

It is obvious that $u = 0$ is not the optimal policy for the production speed unless if the holding is very important, so we obtain:

$$J^{S^{x,0}}(x,0) = J^{S^{x,1}}(x,1) \geq J^{u^*}(x,1)$$

where $J^{u^*}(x,1)$ is the cost function for the system with optimal control policy. Q.E.D

Proof of theorem 2

By definition we have: $\lim_{x \to z_1} \eta(x) = 0$ and $\lim_{x \to y_1} \eta(x) = 0$.

Suppose that $z_1 < x < y_1$, and by equation (9) we know that $u = 0$. So the cost function $J(x,\alpha)$ could be evaluate with s the time where $x = z_1$:

$$J(x,1) = \int_0^s c^+(x - Dt)e^{-\beta t}dt + e^{-\beta s}J(z_1,1)$$

$$J(x,0) = \int_0^s c^+(x - Dt)e^{-\beta t}dt + e^{-\beta s}e^{-rs}J(z_1,1) + e^{-\beta s}(1 - e^{-rs})J(z_1,0)$$

Thereafter we calculate $\eta(x)$ by equation (7):

$$J(x,0) - J(x,1) = e^{-\beta s}(e^{-rs} - 1)[J(z_1,0) - J(z_1,1)]$$

$$\frac{\partial J(x,1)}{\partial x} = c^+ z_1 e^{-\beta s}/D + c^+(1 - e^{-\beta s})/\beta - \beta e^{-\beta s}J(z_1,1)$$

$$\eta(x) = \frac{c^+}{\beta} + e^{-\beta s}\left\{\frac{c^+ z_1}{D} - \frac{c^+}{\beta} - \beta J(z_1,1) + \frac{p}{U}(1 - e^{-rs})[J(z_1,0) - J(z_1,1)]\right\} \quad (12)$$

Equation (12) could be written as:

$$\eta(x)e^{\beta s} = \frac{e^{\beta s}.c^+}{\beta} + \left\{\frac{c^+ z_1}{D} - \frac{c^+}{\beta} - \beta J(z_1,1) + \frac{p}{U}(1 - e^{-rs})[J(z_1,0) - J(z_1,1)]\right\} \quad (13)$$

By Fig. 3 we know that $\eta(x)\ e^{\beta s} > 0\ \forall\ z_1 < x < y_1$ with $\lim_{x \to Z_1} \eta(x) = 0$ and $\lim_{x \to Y_1} \eta(x) = 0$, consequently we have:

$$\left.\frac{\partial \eta(x)e^{\beta s}}{\partial s}\right|_{\substack{s=0 \\ x=z_1}} \geq 0$$

$\forall x\ J(x,0) \geq J(x,1)$ (**Theorem 1**), then we obtain:

$$\frac{d^2\eta(x)e^{\beta s}}{d^2 s} = \frac{p}{U}[J(z_1,0) - J(z_1,1)]\, r^2 e^{-rs} + c^+\beta e^{\beta s} > 0 \qquad \forall s$$

So $\frac{\partial \eta(x)e^{\beta s}}{\partial s} > 0 \quad \forall s$.

We know that $\eta(x)\ e^{\beta s} > 0$ and $\eta(x)$ increase in $x\ \forall\ z_1 < x < y_1$ with $\lim_{x \to z_{11}} \eta(x) = 0$.

So y_1 does not exist. Q.E.D.

Proof of theorem 3

We will prove this result by contradiction. In a first step, suppose that the optimal policy is $u^*(x_0)=0\ \forall\ x_0<0$.

Compare two policies: $u^*(x_0)=0\ \forall\ x_0<0$ and the hedging point policy u^0 with a null hedging point:

$$u_0 = \begin{cases} 0 & if\ \ x>0 \\ D & if\ \ x=0 \\ U & if\ \ x<0 \end{cases}$$

The both systems have the same initial state $(x_0,1)$. Thereafter, define the following parameter, TP to be the total production made during a period of time

For the system $u^*(x_0)=0\ \forall\ x_0<0$, we know that the machine does not fail because failures are operation-dependant failures.

For the hedging point policy system we know that $u^0(t=0) = U$ because $x_0<0$.

Suppose that the machine of the hedging point policy fails for the first time at t_1. We note that during $(0, t_1)$ $u^0 > u^*=0$, so we have:

$$TP^{u0}(t_1) > TP^{u^*}(t_1) = 0$$

This machine is repaired at t_2. We see that between (t_1, t_2) the production is the same for the both systems. Consequently, we obtain the following result:

$$TP^{u0}(t_2) > TP^{u^*}(t_2) = 0\ \forall t\ in\ (0, t_2).$$

By recurrence it is obvious that even for an important number of failures n, we have the following result: $TP^{u0} > TP^{u^*} = 0$.

Define $S^{u^z}{}_{c^-}(t)$ as the buffer level for the backlog and u^0 coming from $(x_0,1)$ and $S^{u^*}{}_{c^-}(t)$ the buffer level for u^*. $S^{u^z}{}_{c+}(t)$ is the buffer level for the

holding and u^0 issue from $(x_0,1)$ and $S^{u^*}{}_{c^+}(t)$ is the buffer level for u*.
From the last results, we have :

$$S^{u^z}{}_{c^-}(t) > S^{u^*}{}_{c^-}(t) \quad \forall t .$$

$$S^{u^z}{}_{c+}(t) = S^{u^*}{}_{c^+}(t) = 0 \quad \forall t$$

So we obtain: $J^{u^z} < J^{u^*}$.
Consequently, the optimal policy could not be $u^*=0$, so the production speed should not be null when the buffer is in backlog. As the optimal policy is of hedging point type, we have the following optimal policy for the backlog:

$$\forall x < 0 \,,\, u^*=U$$

So we know that for our system the hedging point is nonnegative. Q.E.D.

Annexe 2 : Dynamical equations used in the simulation

We could write the discounted cost function as:

$$J(x,\alpha)=E[\int_0^{\Delta t} g(x(t))e^{-\beta t}dt] \\ +E[e^{-\beta\Delta t}]*E[\int_0^{\infty}\beta g(x(t'))dt']$$

where Δt is a very little period of time.
We have:

$$J(x,0)=\int_0^{\Delta t} g(x(t))e^{-\beta t}dt] \\ +(1\text{-r}*\Delta t)\text{J}(x\text{-d}*\Delta t,0)e^{-\beta\Delta t}+\text{r}*\Delta t J(x',1)e^{-\beta\Delta t}$$

Proof:
$x-U^*\Delta t \le x(t) \le x+U^*\Delta t$. Consequently $|x(t)-x| \le U^*\Delta t$
so $|g(x(t))-g(x)| \le c^- * U^*\Delta t$ with $c^- > c^+$

$$\int_0^{\Delta t} g(x(t))e^{-\beta t}dt - \int_0^{\Delta t} g(x)e^{-\beta t}dt$$
$$\le \int_0^{\Delta t} \left| g(x(t))e^{-\beta t} - g(x)e^{-\beta t} \right| dt$$
$$\le \int_0^{\Delta t} c^- * U^* \Delta t e^{-\beta t} dt$$
$$= \frac{-c^- * U^* \Delta t}{\beta} * e^{-\beta\Delta t} \Big|_0^{\Delta t}$$
$$=0(\Delta t)$$

consequently :

$$\int_0^{\Delta t} g(x(t))e^{-\beta t}dt = \int_0^{\Delta t} g(x)e^{-\beta t}dt + 0(\Delta t)$$

$$= \int_0^{\Delta t} g(x)*(1+0(\Delta t))\,dt + 0(\Delta t)$$
$$=g(x)*\Delta t+0(\Delta t)$$

$|J(x',1)-J(x,1)| =$

$$\int_x^{x'} \frac{\partial J(x,0)}{\partial x}dx = \left|\frac{\partial J(x'',0)}{\partial x}\right|_{x=x''} *(x'-x) = 0(\Delta t)$$

so we obtain:

$$J(x,0)=g(x)*\Delta t+(1-r*\Delta t)J(x-d*\Delta t,0)e^{-\beta\Delta t} \\ +r*\Delta t*J(x,1)e^{-\beta\Delta t}+0(\Delta t).$$

By a similar process, we obtain:

$$J(x, 1) = g(x).\ \Delta t+(1-\Delta t\ .p.u/U)\ e^{-\beta\Delta t} * \\ J(x+(u-D)\ *\Delta t\ ,1)+\ \Delta t\ *p*u/UJ(x,0)\ e^{-\beta\Delta t}$$

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