ADAPTIVE GAIN SLIDING MODE CONTROL FOR MULTIMACHINE POWER SYSTEMS

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Abstract: In this paper a new excitation control is developed for multimachine power systems that improves the transient stability. The design is based on the sliding mode technique which is applied on an equivalent model of the system obtained by using the backstepping technique. However, the conventional “sign” method with constant gains is replaced by the boundary layer sliding mode technique with time-varying gains, which are on-line selected in accordance to a suitable adaptation law. Stability analysis and extensive simulation results on a three-machine power system, indicate that the proposed control scheme ensures uniformly ultimately boundedness of all the system variables, reduction of chattering and fast response of the system.

Keywords: Multimachine, power system stabilizers, sliding-mode control, adaptive control.

1. INTRODUCTION

Several linear or nonlinear control techniques have been applied to the excitation control of power systems in order to improve transient stability. Conventional excitation controllers are mainly designed by using linear control theory. Especially for the case of a single machine to infinite-bus power system a method that extensively has been used is based on the linearization about an operating point and the design of linear excitation controllers (Anderson, 1971; Yu, 1983). Main disadvantages of this design such as lack of reliability and robustness are well-known. Nonlinear control techniques have been proposed mainly based on the feedback linearization technique (Chapman, *et al.*, 1993). Feedback linearization is recently enhanced by using robust control designs such as H_∞ control and L_2 disturbance attenuation (Wang, *et al.*, 1998; Xi, and Cheng, 2000; Wang, *et al.*, 2003).

In recent years new approaches have been proposed for power stability designs based on advanced nonlinear schemes such as fuzzy logic control (El-Metwally, and Malik, 1995), adaptive control (Psillakis, and Alexandridis, 2005) and neuro-control (Liu, *et al.*, 2003). Combinations of the above techniques are also proposed (Mrad, *et al.*, 2000) in order to exploit the advantages of each method under the cost of the increase of complexity. Among others the sliding-mode control technique has been applied on power systems providing rather simple control schemes (Matthews, *et al.*, 1986; Pourboghrat, *et al.*, 2004). However, the well-known performance disadvantages of this technique such as chattering or standard response decays etc., remain some of the most significant drawbacks.

In this paper, increasing the complexity within reason, we propose a modified sliding mode control scheme that overcomes the above significant drawbacks. Particularly, we consider a multimachine power system wherein each machine is represented by its third order nonlinear dynamic model and the transmission net is described by the admittance matrix. On this model the well-known backstepping technique (Krstic, *et al.*, 1995) is used in order to obtain the most possible partially linear form of the system. On this form we use the most simplified feedback linearization scheme in order to obtain a local feedback control law while all the other nonlinearities that are dependent from locally unmeasurable variables or variables that are not states are left on an unknown nonlinear term.

Consequently we apply the proposed nonlinear feedback controller where the conventional "sign" method with constant gains is replaced by the boundary layer sliding mode technique with time-varying gains, which are on-line selected in accordance to a suitable adaptation law.

By this control design we prove that the third error variable of each generator model is driven in finite-time in a neighbourhood of the origin of arbitrary small dimensions. As soon as this happens, the other two error variables insert in finite time in a circle around the origin of arbitrary small radius (Theorem 1). The boundedness of all signals is proved. The adaptation mechanism used belongs to the class of direct adaptive algorithms in the sense that it guarantees the uniform ultimate boundness (UUB) of the error variables while the estimated parameter errors remain bounded. Furthermore, as it is proved (Theorem 2) the power angle deviations converge to an even smaller region as time increases. This is important for the selection of the design constants since it leads to significantly smaller values for the control gains. Simulation results after a symmetrical three-phase short circuit fault on a two machine-infinite bus test system demonstrate the effectiveness of the proposed scheme.

2. DYNAMIC MODEL

The classical third-order single-axis dynamic generator model is used for the design of the excitation controller, whereas differential equations that represent dynamics with very short time constants have been neglected as pointed out in Psillakis, and Alexandridis (2005). In general for an *n*-generator power system, the dynamic model of the *i*-th generator is

$$\dot{\delta}_i(t) = \omega_i(t) - \omega_0 \tag{1}$$

$$\dot{\omega}_i(t) = -\frac{D_i}{M_i}\left(\omega_i(t) - \omega_0\right) + \frac{\omega_0}{M_i}\left(P_{mi} - P_{ei}(t)\right) \tag{2}$$

$$\dot{E}'_{qi}(t) = \frac{1}{T'_{d0i}}\left(E_{fi}(t) - E_{qi}(t)\right) \tag{3}$$

where

$$E_{qi}(t) = E'_{qi}(t) + (x_{di} - x'_{di})I_{di}(t) \tag{4}$$

$$E_{fi}(t) = k_{ci}u_{fi}(t) \tag{5}$$

$$I_{qi}(t) = \sum_{j=1}^{n} E'_{qj}\left(B_{ij}\sin\delta_{ij}(t) + G_{ij}\cos\delta_{ij}(t)\right) \tag{6}$$

$$I_{di}(t) = \sum_{j=1}^{n} E'_{qj}(t)\left(G_{ij}\sin\delta_{ij}(t) - B_{ij}\cos\delta_{ij}(t)\right) \tag{7}$$

$$P_{ei}(t) = E'_{qi}(t)I_{qi}(t) \tag{8}$$

$$Q_{ei} = E'_{qi}I_{di}(t) \tag{9}$$

$$E_{qi}(t) = x_{adi}I_{fi}(t) \tag{10}$$

$$V_{tqi}(t) = E'_{qi}(t) - x'_{di}I_{di}(t) \tag{11}$$

$$V_{tdi}(t) = x'_{di}I_{qi}(t) \tag{12}$$

$$V_{ti}(t) = \sqrt{V_{tqi}{}^2(t) + V_{tdi}{}^2(t)}. \tag{13}$$

The symbols used in the above equations are explained in the Appendix.
In order to obtain a partially linear system we use the backstepping technique as explained in the next.

2.1 Backstepping design.
Introducing the first error variable

$$z_{i1} = \Delta\delta_i \tag{14}$$

and viewing $\Delta\omega_i$ as a virtual control, we define the second error variable

$$z_{i2} = \Delta\omega_i - \alpha_{i1}(\Delta\delta_i) \tag{15}$$

where α_{i1} a function to be designed.
Consider the first candidate Lyapunov function

$$V_1 = \frac{1}{2}\sum_{i=1}^{n} z_{i1}^2$$

For the choice

$$\alpha_{i1}(\Delta\delta_i) = -c_{i1}z_{i1} = -c_{i1}\Delta\delta_i\ , \quad c_{i1} > 0$$

and taking into account that

$$\dot{z}_{i1} = \Delta\omega_i$$

we have

$$\dot{V}_1 = -\sum_{i=1}^{n} c_{i1}z_{i1}^2 + \sum_{i=1}^{n} z_{i1}z_{i2}$$

For the second error variable the dynamics are

$$\dot{z}_{i2} = -\frac{D_i}{M_i}\Delta\omega_i - \frac{\omega_0}{M_i}\Delta P_{ei} - \frac{\partial\alpha_{i1}}{\partial\Delta\delta_i}\Delta\omega_i$$

Viewing ΔP_{ei} as a virtual control, we introduce the third error variable

$$z_{i3} = \Delta P_{ei} - \alpha_{i2}(\Delta\delta_i, \Delta\omega_i) \tag{16}$$

where α_{i2} to be designed.
For the Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{n} z_{i2}^2$$

we have

$$\dot{V}_2 = -\sum_{i=1}^{n} c_{i1}z_{i1}^2 - \sum_{i=1}^{n}\frac{\omega_0}{M_i}z_{i2}z_{i3} + \\ + \sum_{i=1}^{n} z_{i2}\left[z_{i1} - \left(\frac{D_i}{M_i} + \frac{\partial\alpha_{i1}}{\partial\Delta\delta_i}\right)\Delta\omega_i - \frac{\omega_0}{M_i}\alpha_{i2}\right]$$

So if we take

$$\alpha_{i2}(\Delta\delta_i, \Delta\omega_i) = \frac{M_i}{\omega_0}\left[z_{i1} + c_{i2}z_{i2} - \left(\frac{D_i}{M_i} + \frac{\partial\alpha_{i1}}{\partial\Delta\delta_i}\right)\Delta\omega_i\right]$$

$$= \frac{M_i}{\omega_0}(1 + c_{i1}c_{i2})\Delta\delta_i + \frac{M_i}{\omega_0}\left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right)\Delta\omega_i$$

where $c_{i2} > 0$, then

$$\dot{V}_2 = -\sum_{i=1}^{n} c_{i1}z_{i1}^2 - \sum_{i=1}^{n} c_{i2}z_{i2}^2 - \sum_{i=1}^{n}\frac{\omega_0}{M_i}z_{i2}z_{i3}$$

For the third error variable it holds true that

$$\dot{z}_{i3} = \tilde{f}_i(t) + \frac{1}{T'_{d0i}}I_{qi}u_{fi}(t) - \frac{\partial\alpha_{i2}}{\partial\Delta\delta_i}\Delta\omega_i - \\ - \frac{\partial\alpha_{i2}}{\partial\Delta\omega_i}\left[-\frac{D_i}{M_i}\Delta\omega_i - \frac{\omega_0}{M_i}\Delta P_{ei}\right]$$

where

$$\tilde{f}_i(t) = E'_{qi}(t)\dot{I}_{qi}(t) - \frac{1}{T'_{d0i}}\left[E'_{qi}(t) + (x_{di} - x'_{di})I_{di}(t)\right]I_{qi}(t)$$

is a term that contains all the complex nonlinearities of the system. For the Lyapunov function

$$V_3 = V_2 + \frac{1}{2}\sum_{i=1}^{n} z_{i3}^2$$

its time derivative results in

$$\dot{V}_3 = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 + \sum_{i=1}^{n} z_{i3}\left[\tilde{f}_i(t) + \frac{1}{T'_{d0i}} I_{qi} u_{fi}(t) - \right.$$
$$\left. -\frac{\partial \alpha_{i2}}{\partial \Delta\delta_i}\Delta\omega_i + \frac{\partial \alpha_{i2}}{\partial \Delta\omega_i}\left(\frac{D_i}{M_i}\Delta\omega_i + \frac{\omega_0}{M_i}\Delta P_{ei}\right) - \frac{\omega_0}{M_i} z_{i2} \right]$$

Selecting now the excitation input

$$E_{fi}(t) = \frac{T'_{d0i}}{I_{qi}}\left[-c_{i3} z_{i3} + \frac{\partial \alpha_{i2}}{\partial \Delta\delta_i}\Delta\omega_i - \right.$$
$$\left. -\frac{\partial \alpha_{i2}}{\partial \Delta\omega_i}\left(\frac{D_i}{M_i}\Delta\omega_i + \frac{\omega_0}{M_i}\Delta P_{ei}\right) + v_i \right], \quad k_{ci} = 1$$

i.e. selecting the excitation control law

$$E_{fi}(t) = \frac{T'_{d0i}}{I_{qi}}\left(k_{i1}\Delta\delta_i + k_{i2}\Delta\omega_i - k_{i3}\Delta P_{ei} + v_i \right) \qquad (17)$$

where the constant gains are given by

$$K_{i1} = \frac{\omega_0}{M_i} c_{i1} + \frac{M_i}{\omega_0} c_{i3}\left(1 + c_{i1} c_{i2}\right),$$

$$K_{i2} = \frac{M_i}{\omega_0}\left[\left(c_{i3} - \frac{D_i}{M_i}\right)\left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right) + c_{i1} c_{i2} + 1 \right] + \frac{\omega_0}{M_i}$$

$$K_{i3} = c_{i1} + c_{i2} + c_{i3} - \frac{D_i}{M_i} \qquad (18)$$

and $v_i = v_i(t)$ is an arbitrary external input, we have

$$\dot{V}_3 = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} c_{i3} z_{i3}^2 +$$
$$-\sum_{i=1}^{n} \frac{\omega_0}{M_i} z_{i2} z_{i3} + \sum_{i=1}^{n} z_{i3}\left[\tilde{f}_i(t) + v_i(t) \right]$$

At this point one can easily arrive at the following thoughts: the unknown term $\tilde{f}_i(t)$ can be considered to be bounded (that is always the case since the machine voltages and currents and their rates cannot take infinite values) , i.e. there exists an unknown positive constant $\bar{M}_i > 0$ such that

$$\left| \tilde{f}_i(t) \right| \le \bar{M}_i < \infty$$

Therefore, a choice of a discontinuous external input

$$v_i(t) = -K_{0i}\,\mathrm{sgn}\left(z_{i3}\right)$$

with $K_{0i} \ge \bar{M}_i \ge 0$ together with a suitable selection of the coefficients c_{i1}, c_{i2}, c_{i3}, ensures asymptotic stability of the closed-loop system. The z_{i3}-dynamics then takes the following form

$$\dot{z}_{i3} = -c_{i3} z_{i3} + \tilde{f}_i - K_{0i}\,\mathrm{sgn}\left(z_{i3}\right)$$

This is a sliding-mode control law which converges in finite time to the surface

$$z_{i3} = \Delta P_{ei} - \frac{M_i}{\omega_0}\left(1 + c_{i1} c_{i2}\right)\Delta\delta_i -$$
$$-\frac{M_i}{\omega_0}\left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right)\Delta\omega_i = 0$$

However, the use of such a controller yields several drawbacks as explained in the Introduction.

Instead, in the next section, a direct adaptive gain sliding-mode controller with boundary layer $v_i(t)$ is designed that ensures finite-time uniform ultimate boundedness for all the error variables, while provides best response performance without chattering on the sliding surface.

3. CONTROL DESIGN & STABILITY ANALYSIS

The proposed control law is selected to be

$$v_i(t) = -K_{0i}\, sat_{\epsilon_i}\left(z_{i3}\right) = \begin{cases} -K_{0i}\,\mathrm{sgn}\left(z_{i3}\right) & when\ \left|z_{i3}\right| \ge \epsilon_i \\ -K_{0i} z_{i3}/\epsilon_i & when\ \left|z_{i3}\right| < \epsilon_i \end{cases}$$

This is sliding-mode boundary layer control that reduces the chattering effect (Slotine, 1984).

Instead of a constant K_{0i} a time varying gain $K_i(t)$ is used such that

$$K_i(t) = K_{0i} + \Delta K_i(t) \qquad (19)$$

i.e.

$$v_i(t) = -K_i(t)\, sat_{\epsilon_i}\left(z_{i3}\right) \qquad (20)$$

where $\Delta K_i(t)$ is given by the update law

$$\Delta\dot{K}_i = \begin{cases} \gamma_i \left|z_{i3}\right| & when\ \left|z_{i3}\right| \ge \epsilon_i \\ -\gamma_i\left(\epsilon_i - \left|z_{i3}\right|\right)\Delta K_i & when\ \left|z_{i3}\right| < \epsilon_i \end{cases} \qquad (21)$$

$$\Delta K_i(t_0) = 0$$

The adaptation law is selected in such a way that when $\left|z_{i3}\right| \ge \epsilon_i$ the gain increases continuously to drive fast the system inside the desired stability area. However, the choice of the adaptation law for $\left|z_{i3}\right| < \epsilon_i$ is made to reset K_i at the initial value K_{0i} after the fault is removed.

Introducing now the bound estimation error

$$\tilde{K}_i(t) = K_i(t) - \bar{M}_i$$

and choosing a Lyapunov function candidate as

$$V = V_3 + \sum_{i=1}^{n} \frac{\tilde{K}_i^2}{2\gamma_i}$$

we have for the control law (20)

$$\dot{V} = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} c_{i3} z_{i3}^2 +$$
$$+\sum_{i=1}^{n} z_{i3}\left[\tilde{f}_{i1} - K_i(t)\, sat_{\epsilon_i}\left(z_{i3}\right) \right] + \sum_{i=1}^{n} \frac{\tilde{K}_i \dot{K}_i}{\gamma_i}$$

which sequentially yields

$$\dot{V} \le \begin{cases} \sum_{i=1}^{n} \frac{\tilde{K}_i \Delta\dot{K}_i}{\gamma_i} - \sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} c_{i3} z_{i3}^2 + \\ +\sum_{i=1}^{n} \left|z_{i3}\right|\left[M_i - K_i(t)\right], \quad when\ \ \left|z_{i3}\right| \ge \epsilon_i \\ +\sum_{i=1}^{n} \frac{\tilde{K}_i \Delta\dot{K}_i}{\gamma_i} - \sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} c_{i3} z_{i3}^2 + \\ +\sum_{i=1}^{n} \left|z_{i3}\right|\left[M_i - K_i(t)\frac{\left|z_{i3}\right|}{\epsilon_i} \right], \quad when\ \left|z_{i3}\right| < \epsilon_i \end{cases}$$

$$\dot{V} \le \begin{cases} -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} c_{i3} z_{i3}^2 + \\ +\sum_{i=1}^{n} \tilde{K}_i \left[-\left|z_{i3}\right| + \frac{\Delta\dot{K}_i}{\gamma_i} \right], \quad when \quad \left|z_{i3}\right| \ge \epsilon_i \\ -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} \left[c_{i3} + \frac{K_i(t)}{\epsilon_i} \right] z_{i3}^2 + \\ +\sum_{i=1}^{n} M_i \left|z_{i3}\right| + \sum_{i=1}^{n} \frac{\tilde{K}_i \Delta\dot{K}_i}{\gamma_i}, \quad when \left|z_{i3}\right| < \epsilon_i \end{cases}$$

So we have proven that

$$\dot{V} \le -\sum_{i=1}^{n} c_{i3} \epsilon_i^2 < 0 \quad when \left|z_{i3}\right| \ge \epsilon_i$$

which means that there exists $T \le t_0 + \frac{V(t_0)}{\sum_{i=1}^{n} c_{i3} \epsilon_i^2}$ such that $\left|z_{i3}(t)\right| \le \epsilon_i,\ i = 1,2,\ldots,n \quad \forall t \ge T$. From the form of V one can easily see that all the signals $z_{ij}(t),\ K_i(t),\ i = 1,2,\ldots,n,\quad j = 1,2,3$ remain bounded in the time interval $[t_0, T]$. Since $\left|z_{i3}(t)\right| \le \epsilon_i \quad \forall t \ge T$ it holds true that

$$-\gamma_i \epsilon_i\ \Delta K_i \le \frac{d\Delta K_i}{dt} \le 0$$

$$0 \le K_i(T) e^{-\gamma_i \epsilon_i (t-T)} \le K_i(t) \le K_i(T)$$

i.e. the estimates $K_i(t)$ remain bounded for every $t \ge t_0$. Let the nonnegative function

$$V_{i1} = \frac{z_{i1}^2}{2} + \frac{z_{i2}^2}{2}$$

The dynamics for the z_{i1}, z_{i2} variables are given by

$$\begin{cases} \dot{z}_{i1} = z_{i2} - c_{i1} z_{i1} \\ \dot{z}_{i2} = -z_{i1} - c_{i2} z_{i2} - \frac{\omega_0}{M_i} z_{i3} \end{cases}$$

and the time derivative of V_{i1} for $t \ge T$ is

$$\begin{aligned} \dot{V}_{i1} &= -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 - \frac{\omega_0}{M_i} z_{i2} z_{i3} \\ &\le -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 + \frac{\omega_0}{M_i} \epsilon_i \left|z_{i2}\right| \\ &\le -c_{i1} z_{i1}^2 - c_{i2}(1-\varepsilon_i) z_{i2}^2 - \\ &\quad -\varepsilon_i c_{i2} \left(\left|z_{i2}\right| - \frac{\omega_0 \epsilon_i}{2\varepsilon_i c_{i2} M_i} \right)^2 + \frac{\omega_0^2 \epsilon_i^2}{4\varepsilon_i c_{i2} M_i^2} \end{aligned}$$

Defining $m_i = \min\{c_{i1}, c_{i2}(1-\varepsilon_i)\}$ we have that

$$\dot{V}_{i1} \le -m_i \left(z_{i1}^2 + z_{i2}^2 \right) + \frac{\omega_0^2 \epsilon_i^2}{4\varepsilon_i c_{i2} M_i^2} = -2m_i V_{i1} + \frac{\omega_0^2 \epsilon_i^2}{4\varepsilon_i c_{i2} M_i^2}$$

$$\dot{V}_{i1} \le -2m_i \left(V_{i1} - \frac{\omega_0^2 \epsilon_i^2}{8m_i \varepsilon_i c_{i2} M_i^2} \right)$$

Using the comparison principle (Lakshmikantham, and Leela, 1969) we have

$$V_{i1}(t) - \frac{\omega_0^2 \epsilon_i^2}{8m_i \varepsilon_i c_{i2} M_i^2} \le \left[V_{i1}(T) - \frac{\omega_0^2 \epsilon_i^2}{8m_i \varepsilon_i c_{i2} M_i^2} \right] e^{-2m_i(t-T)}$$

$$V_{i1}(t) \le V_{i1}(T) e^{-2m_i(t-T)} + \frac{\omega_0^2 \epsilon_i^2}{8m_i \varepsilon_i c_{i2} M_i^2}$$

Therefore, there exists a T_{i1}

$$T_{i1} = \max\left\{ T, T + \frac{1}{m_i} \ln\left[\frac{V_{i1}^{1/2}(T)}{\frac{\omega_0 \epsilon_i}{2M_i \sqrt{2m_i \varepsilon_i c_{i2}}}} \right] \right\}$$

so that for $t \ge T_{i1}$ it holds true that

$$V_{i1}(t) \le \frac{\omega_0^2 \epsilon_i^2}{4m_i \varepsilon_i c_{i2} M_i^2}$$

i.e. the error variables z_{i1}, z_{i2} enter in finite-time inside the circle

$$S = \left\{ (z_{i1}, z_{i2}) \mid z_{i1}^2 + z_{i2}^2 \le \left(\frac{\omega_0 \epsilon_i}{\sqrt{2m_i \varepsilon_i c_{i2}} M_i} \right)^2 \right\}$$

with centre the origin and radius

$$R_{i1} := \frac{\omega_0 \epsilon_i}{\sqrt{2\min\{c_{i1}, (1-\varepsilon_i)c_{i2}\} \varepsilon_i c_{i2}} M_i}$$

Thus we have proved the following theorem.

Theorem 1: For the n-machine system defined by (1)-(13) and the excitation input given by (17)-(21) there exists $T_{i1} \ge T \ge t_0$ such that for every $t \ge T_{i1}$ the z_{i1}, z_{i2}, z_{i3} error variables lie inside the cylinder

$$S = \left\{ \begin{aligned} &(z_{i1}, z_{i2}, z_{i3}) \in \mathbb{R}^3 : z_{i1}^2 + z_{i2}^2 \le \left(\frac{\omega_0 \epsilon_i}{\sqrt{2m_i \varepsilon_i c_{i2}} M_i} \right)^2 \\ &\qquad\qquad and \quad \left|z_{i3}\right| \le \epsilon_i \end{aligned} \right\}$$

Theorem 1 directly gives a bound for $\Delta\delta_i = z_{i1}$

$$\left|\Delta\delta_i(t)\right| \le \frac{\omega_0 \epsilon_i}{M_i \sqrt{2m_i \varepsilon_i c_{i2}}} \quad \forall t \ge T_{i1}$$

However, as $t \to \infty$, a tighter bound for $\lim_{t\to\infty} \left|\Delta\delta_i(t)\right|$ is given by the following Theorem.

Theorem 2: For the n-machine system defined by (1)-(13) and the excitation input given by (17)-(21) the following bounds on the angle deviation limits hold

$$\lim_{t\to\infty} \left|\Delta\delta_i(t)\right| \le R_i := \frac{\omega_0 \epsilon_i}{M_i (1 + c_{i1} c_{i2})}$$

Proof: As $t \to \infty$ the system tends to its steady state wherein

$$\lim_{t\to\infty} \Delta\dot{\delta}_i(t) = \lim_{t\to\infty} \Delta\dot{\omega}_i(t) = 0$$

From (1), (2) one can directly obtain

$$\lim_{t\to\infty} \Delta\omega_i(t) = \lim_{t\to\infty} \Delta P_{ei}(t) = 0$$

and therefore from (16) we have for z_{i3}

$$\lim_{t\to\infty} z_{i3}(t) = -\frac{M_i}{\omega_0} (1 + c_{i1} c_{i2}) \lim_{t\to\infty} \Delta\delta_i(t)$$

Finally, using that $\left|z_{i3}(t)\right| \le \epsilon_i \quad \forall t \ge T$ we obtain the bound for $\left|\Delta\delta_i\right|$ as $t \to \infty$

$$\lim_{t\to\infty} \left|\Delta\delta_i(t)\right| \le \frac{\omega_0 \epsilon_i}{M_i (1 + c_{i1} c_{i2})}$$

4. CASE STUDY

The two-generator infinite bus power system that is used to demonstrate the efficiency of the proposed controller is shown in Fig. 1. The system parameters are as follows:

$x_{T1} = 0.129$ p.u., $x_{T2} = 0.11$ p.u., $x_{12} = 0.55$ p.u.,
$x_{13} = 0.53$ p.u., $x_{23} = 0.6$ p.u., $T'_{d01} = 6.9$ sec
$x_{d1} = 1.863$ p.u., $x'_{d1} = 0.257$ p.u., $D_1 = 5.0$ p.u.,
$M_1 = 8.0$ sec, $M_2 = 10.2$ sec, $D_2 = 3.0$ p.u.,
$x_{d2} = 2.36$ p.u., $x'_{d2} = 0.319$ p.u., $T'_{d02} = 7.96$ sec

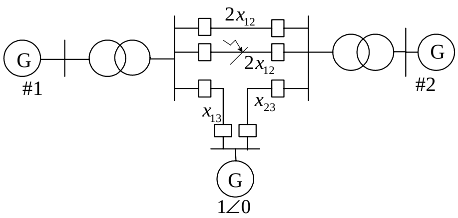

Fig. 1: Two machine infinite bus test system.

For a more accurate evaluation of the proposed controller, we take into account in the simulation the physical limits of the excitation voltage which are:

$$\left|k_{c1}u_{f1}\right| \le 5.0\,p.u., \qquad \left|k_{c2}u_{f2}\right| \le 5.0\,p.u.$$

The following case is simulated.

Permanent serious fault: A symmetrical three phase short circuit fault occurs on one of the transmission lines between Generator #1 and Generator #2 at $t = 5.1$ second. The fault is removed by opening the brakers of the faulted line at $t = 5.25$ second and the system is restored at $t = 6$ seconds. If we use λ to represent the fraction of the fault, simulations are made for $\lambda = 0.6$ i.e. for a fault near the middle of the line and towards Generator #2. The operating point considered in the simulation is:

$$\delta_{10} = 40^0, \quad V_{t10} = 0.808, \quad P_{m10} = 0.7;$$
$$\delta_{20} = 30^0, \quad V_{t20} = 0.843, \quad P_{m20} = 0.6.$$

The controllers' parameters are

$$c_{11} = 5, \quad c_{12} = 12, \quad c_{13} = 100, \quad \gamma_1 = 30,$$
$$c_{21} = 5, \quad c_{22} = 15, \quad c_{23} = 100, \quad \gamma_2 = 20,$$
$$\epsilon_1 = 0.005, \quad \epsilon_2 = 0.01, \quad K_{01} = K_{02} = 1$$

The simulation results are given in Figures 2-8.

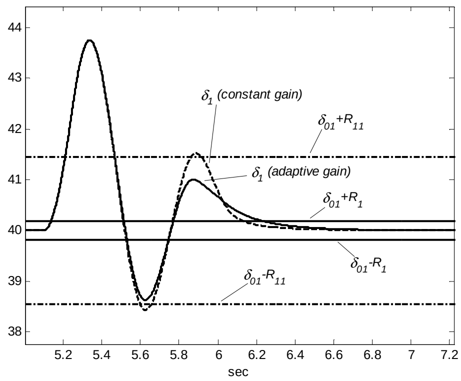

Fig. 2. Power angle deviations and their bounds for generator #1 (in degrees)

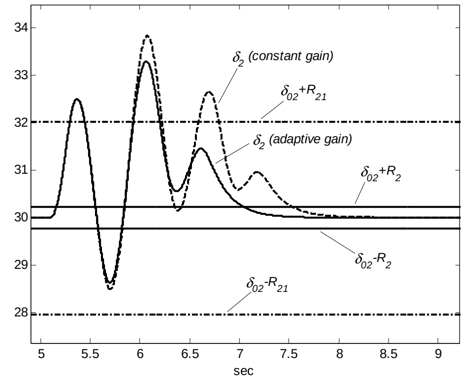

Fig. 3. Power angle deviations and their bounds for generator #2 (in degrees).

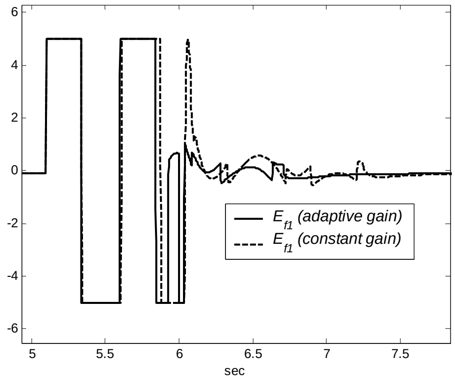

Fig. 4. Excitation voltages (in p.u.) for generator #1

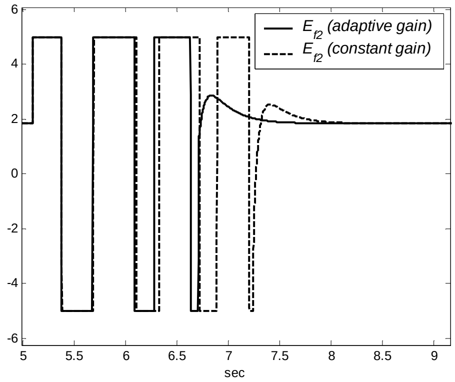

Fig. 5. Excitation voltages (in p.u.) for generator #2.

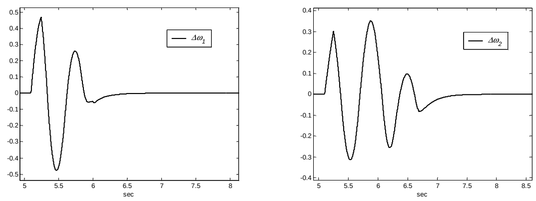

Fig. 6. Nominal frequency deviations (in rad/sec) for generators #1 and #2.

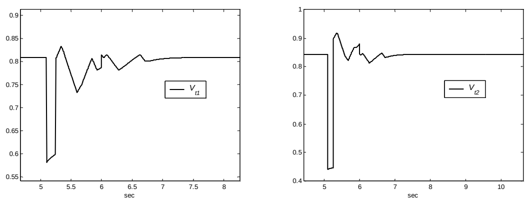

Fig. 7.Terminal voltages (in p.u.) for generators #1 and #2

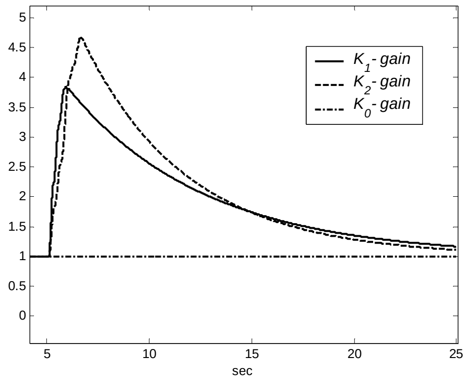

Fig. 8. Sliding mode controller gains for generators #1 and #2.

The response of the system appears to be very satisfactory since the system maintains stability after the transient period. Comparisons between the two control schemes, i.e. the boundary layer sliding mode scheme with constant and adaptive gains respectively, clearly shows the superiority of the latter since the system returns to the nominal state significantly faster.

5. CONCLUSIONS

The proposed controller is completely decentralized with a simple structure given by (17) and (18) where the nonlinear control term v_i is calculated easily from the adaptation sliding mode control law given by (19)-(21). As it is shown by an extensive analysis this control scheme ensures stability while permits the selection of the control parameters in a desired way (in accordance to the desired region width R_{i1}). The simulation results confirm the theoretical analysis and verify the effectiveness of the control scheme.

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APPENDIX

$\delta_i(t)$: power angle, in radian; $\omega_i(t)$: rotor speed, in rad/sec; ω_0: synchronous speed, in rad/sec; P_{mi}: mechanical input power, in p.u; $P_{ei}(t)$: active electrical power, in p.u.; D_i: damping constant, in p.u.; M_i: inertia coefficient, in sec; $E'_{qi}(t)$: transient EMF in the q-axis in p.u.; $E_{qi}(t)$: EMF in the q-axis, in p.u.; $E_{fi}(t)$: equivalent EMF in excitation coil, in p.u.; T'_{d0i}: d-axis transient short circuit time constant, in sec; $I_{fi}(t)$: excitation current, in p.u.; $I_{qi}(t)$: q-axis current, in p.u.; $I_{di}(t)$: d-axis current, in p.u.; $Q_{ei}(t)$: reactive electrical power, in p.u.; $V_{ti}(t)$: generator terminal voltage, in p.u.; k_{ci}: gain of generator excitation amplifier, in p.u.; $u_{fi}(t)$: input of the SCR amplifier, in p.u.; x'_{di}: d-axis transient reactance, in p.u.; x_{di}: d-axis reactance, in p.u.; x_{adi}: mutual reactance between the excitation coil and the stator coil, in p.u.; $Y_{ij} = G_{ij} + jB_{ij}$: the *i*th row and *j*th column element of nodal admittance matrix, in p.u.; $\Delta\delta_i(t) = \delta_i(t) - \delta_{i0}$; $\Delta\omega_i(t) = \omega_i(t) - \omega_0$; $\Delta P_{ei}(t) = P_{ei}(t) - P_{mi}$.