ON THE DETECTION OF UNKNOWN INPUT IN POSITIONAL CONTROL PROBLEMS WITH NOISY MEASUREMENTS

Josef SHINAR ,[*,1] **József BOKOR** ,[**,2]
Balázs KULCSÁR [**,3]

[*] *Technion, Israel Institute of Technology,*
Haifa 3200 Israel
aer4301@aerodyne.technion.ac.il
[**] *Systems and Control Laboratory,*
Computer Automation Research Institute,
Hungarian Academy of Sciences
H-1518, Budapest, P.O.Box 63.
bokor@sztaki.hu, kulcsar@sztaki.hu

Abstract: The paper examines the conditions for isolating the unknown input detection from the effects of the measurements noise in the important family of positional control problems. The study is motivated by the need of improving the homing performance of interceptor missiles against randomly maneuvering targets. The required isolation is possible if, in addition to noisy relative position measurements, noise-free measurement of the line of slight rate or the relative lateral velocity is available. Although with noisy measurement of line of slight rate the isolation of the input is not possible, the detection filter designed for the noise-free case succeeds to provide a satisfactory estimate of target acceleration, which is needed for an improved homing performance.

Keywords: : positional control, detection filter,invariant subspaces

1. INTRODUCTION

Positional control problems are extensively treated in the classical control literature, in particular in the Russian (Pontryagin, 1972; Krasovskii and Subotin, 1988). Such problems have a terminal payoff function depending on the distance between two objects at the final time. The mathematical model consists of the equations of motion controlled by accelerations. In real systems, there is a certain dynamics between the actual and the commanded value of the acceleration. Mostly these problems were analyzed assuming perfect information. In reality, however, the controls are based on available noise corrupted measurements. The available measurements of each object are its own state variables and the relative position and velocity of the other. Relative acceleration (or the acceleration of the other object) cannot be measured. It has to be reconstructed by an estimator.

For a linear process with zero-mean Gaussian white noise the Kalman filter (Kalman, 1960) is the optimal estimator in the sense of minimum variance, if its design is based on the correct dynamic system model, which includes also the (deterministic) input. Unknown inputs can be

[1] Professor Emeritus, Faculty of Aerospace Engineering
[2] Professor
[3] Research fellow

considered as a stochastic process and are approximated by the output of a (linear) "shaping filter" driven by Gaussian white noise (Zarchan, 1979). The estimation process has inherent dynamics, creating a time delay of the information on the estimated variables. Such a delay leads to control performance deterioration.

In this paper the positional control problem is a simple zero-sum pursuit-evasion game. It assumes planar motion with bounded controls and linear first-order control dynamics (Shinar, 1981). The problem of the pursuer is to find a feedback control, based on the available measurements, which minimizes the final distance against any admissible bounded control of the evader, which is considered as a disturbance input.

Both analysis and simulations (Shinar and Steinberg, 1977; Shinar and Zarkh, 1994) showed that the worst disturbance has a "bang-bang" structure (a "jump" process with infinitely fast dynamics). If the timing of the "jump" is known, the input of the estimator model can be "tuned" to represent a correct dynamic model. Moreover, even if the "jump" occurs shortly after the time anticipated by the estimator, the estimated state variables will have rather small errors. Consequently, a set of "tuned" estimators can provide good estimates, if the time of the "jump" is detected without a significant delay (Shinar et al, 2004.). Therefore, there is a need for fast detection of the unknown disturbance input (the "jump") with a small false alarm rate.

From the various options to detect an unknown input, in this paper the detection filter approach (Massoumnia, 1986) was selected for reasons detailed in the sequel. The objective of this paper is to develop a detection filter that isolates the unknown (disturbance) input from the effect of the measurement noise and to test it in a generic pursuit-evasion example with noise corrupted measurements.

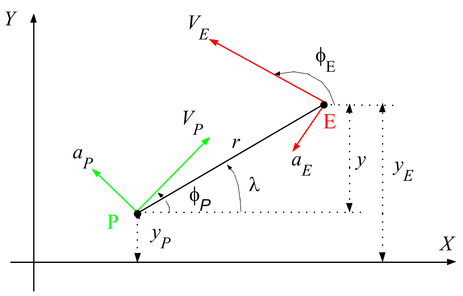


Fig. 1. Interception geometry

2. DYNAMIC MODEL

Let us consider a positional control problem in the horizontal plane with two independent controllers and noisy measurements, representing an endgame of intercepting a maneuverable target (evader) by a guided missile (pursuer). The relative geometry is depicted if Fig. 1. The X-axis of the coordinate system is parallel to the initial line of sight connecting the pursuer and the evader. The equations of planar motion, assuming constant velocities (V_E, V_P) and bounded lateral accelerations (a_E, a_P) are:

$$\dot{r} = V_E\cos(\phi_E - \lambda) - V_P\cos(\phi_P - \lambda) \qquad (1)$$
$$r(0) = r_0$$
$$r\dot{\lambda} = V_E\sin(\phi_E - \lambda) - V_P\sin(\phi_P - \lambda) \qquad (2)$$
$$\lambda(0) = 0$$
$$\dot{\phi}_E = a_E/V_E \quad \phi_E(0) = \phi_{E_0} \qquad (3)$$
$$\dot{\phi}_P = a_P/V_P \quad \phi_P(0) = \phi_{P_0} \qquad (4)$$

If the angles ϕ_E and ϕ_P are near to the nominal values of a *collision course* defined by

$$V_P\sin(\phi_{P_{col}}) = V_E\sin(\phi_{E_0}) \qquad (5)$$

and also the current line of sight angle $\lambda(t)$ is small, Eqs. (1) and (4) can be linearized. In this case

$$\dot{x} \approx \dot{r}_{col} = V_E\cos(\phi_{E_0}) - V_P\cos(\phi_{P_{col}})$$
$$\triangleq -V_c = \text{const.} \quad x(0) = r_0 \qquad (6)$$

From Eq. (6) one obtains the final time

$$t_f = r_0/V_c \qquad (7)$$

and its integration yields

$$x(t) = V_c t_{go} \qquad (8)$$

where

$$t_{go} = t_f - t. \qquad (9)$$

Due to the linearization the state vector in the equations of relative motion normal to the initial line of sight is

$$\mathbf{X^T} = (x_1, x_2, x_3, x_4) = (y, \dot{y}, \bar{a}_E, \bar{a}_P) \qquad (10)$$

where

$$y(t) \triangleq y_E(t) - y_P(t) \qquad (11)$$

and $\bar{a}_E, \bar{a}_P$ are the respective accelerations normal to the initial line of sight. If the initial conditions of the engagement are near to a “head-on” geometry, i. e. the angles ϕ_{P_0} and $(\pi - \phi_{E_0})$ are small, then $\bar{a}_E \approx a_E$, $\bar{a}_P \approx a_P$ and $V_c \approx V_E + V_P$. The corresponding equations of motion and the respective initial conditions are

$$\dot{x}_1 = x_2, \quad x_1(0) = 0, \qquad (12)$$
$$\dot{x}_2 = x_3 - x_4, \quad x_2(0) = V_E\phi_{E_0} - V_P\phi_{P_0}, \qquad (13)$$
$$\dot{x}_3 = (a_E^c - x_3)/\tau_E, \quad x_3(0) = 0, \qquad (14)$$
$$\dot{x}_4 = (a_P^c - x_4)/\tau_P, \quad x_4(0) = 0, \qquad (15)$$

where a_E^c and a_P^c are the respective commanded lateral accelerations of the evader and the pursuer, expressed by

$$a_E^c = a_E^{max}\mathbf{v}; \qquad |\mathbf{v}| \leq 1 \qquad (16)$$
$$a_P^c = a_P^{max}\mathbf{u}; \qquad |\mathbf{u}| \leq 1. \qquad (17)$$

Eqs. (14) and (15) represent the simplest (first-order) control dynamics of the evader and the pursuer with time constants τ_E and τ_P, respectively.

The problem involves two non-dimensional parameters of physical significance: the pursuer/evader maximum maneuverability ratio

$$\eta \triangleq \mathbf{a}_{\mathbf{P}}^{\mathbf{max}}/\mathbf{a}_{\mathbf{E}}^{\mathbf{max}} \tag{18}$$

and the ratio of the evader/pursuer time constants

$$\varepsilon \triangleq \tau_E/\tau_P. \tag{19}$$

The pursuit-evasion game based on Eqs. (12)–(19) with the terminal pay-off function

$$J = |x_1(t_f)| \tag{20}$$

was solved assuming perfect information (**?**). The optimal strategies of the players are of the "bang-bang" type

$$\mathbf{u}^* = \mathbf{v}^* = \operatorname{sign}\{Z\}, \quad \forall Z \neq 0, \tag{21}$$

where Z is the *zero effort miss distance*, expressed explicitly by

$$Z = x_1 + x_2 t_{go} - \Delta Z_P + \Delta Z_E \tag{22}$$

with

$$\Delta Z_P = x_3(\tau_P)^2[\exp(-\theta_P) + \theta_P - 1] \tag{23}$$

$$\Delta Z_E = x_4(\tau_E)^2[\exp(-\theta_E) + \theta_E - 1] \tag{24}$$

while $\theta_P = t_{go}/\tau_P$ and $\theta_E = t_{go}/\tau_E$.

The perfect information game solution indicates that from all initial conditions of practical importance zero miss distance is guaranteed if $\eta\varepsilon \geq 1$. For guidance law implementation one needs also the time-to-go. The available measurements are: range r, the range rate $\dot{r}$ and own acceleration ($a_p = x4$), all measured with good accuracy. From the measurements of the range and the range rate one obtains

$$t_{go} = \frac{r}{|\dot{r}|} = \frac{r}{V_c} \tag{25}$$

Implementation of the optimal missile guidance law, denoted as DGL/1, requires the knowledge of the zero effort miss distance, which includes the lateral acceleration of the target. This variable cannot be measured, it has to be estimated based on the available noise corrupted line of sight angle λ and line of sight rate $\dot{\lambda}$. These noisy measurements can be represented by:

$$y_1 = r\lambda + \xi_1 \tag{26}$$

$$y_2 = r\dot{\lambda} - \frac{x_1}{t_{go}} + \xi_2 \tag{27}$$

where ξ_1 and ξ_2 are supposed to be normally distributed random signals with known standard deviations proportional to r. The state estimator using the measurement inputs has its inherent dynamics that introduces some delay between the estimated and the true state variables. If the pursuer uses DGL/1, the evader can take advantage of the estimation delay and achieve a large miss distance by adequate optimal maneuvering (Glizer and Shinar, 2001.).

In the last years several efforts were made (Shinar and Shima, 2002; Shima et al, 2002; Shima et al, 2003) towards achieving robust homing performance with respect to random target maneuvers and to reduce the guaranteed miss distance. In a recent paper (Shinar et al, 2004.) a new integrated logic based estimation/guidance algorithm is based on a set of innovative concepts, was proposed. The new algorithm uses separate estimator elements for the different tasks of target model identification, proper state estimation and "jump" detection. The crucial element for the successful application of the new algorithm is the existence of a sufficiently fast "jump" detector, which is the topic of this paper.

3. UNKNOWN INPUT DETECTION

There are various options to detect an unknown input in a dynamic system. The most promising approaches are the design of an unknown input observer(UIO) (Chen and Patton, 1999), a system inversion for the unknown input (Szigeti *et al.*, 2002), detection filter design for a complete isolation of the unknown input and the noise effect in the filter output error space (Hammouri *et al.*, 1999; Massoumnia, 1986) or the parity space approach (Gertler, 1998).

For this particular problem we can come to the following conclusions.

For a separation of the unknown input detection from the sensor noise on the relative position measurement one needs at least two sensors. The use of the pursuer acceleration measurement x_4 does not make the isolation solvable.

The design of an UIO is not possible since the relative degree of the system w.r.t. the unknown invader input is $r > 1$.

System inversion (Szigeti *et al.*, 2002) suffers from the same problem as above since it requires the use of $3rd$-order derivatives of the noisy output (position) measurement.

Parity space approach can be considered as special case of detection filter design if assigning all filter poles to the origin (i.e. we design an open loop or MA filter). This might not be suitable for model uncertainties, thus we will focus on the possibility to find a detection filter for an independent detection of the effect of evader input to the output (position) noise. This will be investigated in the sequel.

The detection problem for LTI systems can be formulated as follows. Given a dynamic system:

$$\dot{x} = Ax + Bu + L\nu$$
$$y = C(t)x + e_i\mu,$$

where e_i is a unit vector and ν, μ are the unknown input and an output noise, respectively. The goal is to design an input observer (or detection filter) such that its output reconstructs the unknown input and this is not effected by the sensor noise μ.

In order to formulate the problem more precisely, rewrite first the above system equations as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + L_1\nu(t) + L_2\mu(t) \\ y(t) &= C(t)x(t), \end{aligned} \tag{28}$$

where $L_1 = L$ and L_2 is the so called pseudo actuator direction representing the sensor noise effect in measurement. In the LTI case one can show that $L_2 = [Af_i, f_i]$, where $Cf_i = e_i$.

The detection of the unknown command ν will be performed by designing a filter that is sensitive to the signal associated with the L_1 direction and insensitive to the signal associated with the L_2 direction. More precisely, one has to design a filter with output denoted e.g. by r_ν, such that if $\nu \neq 0$ then $r_\nu \neq 0$ and if $\nu = 0$ then $\lim_{t\to\infty} ||r_\nu(t)|| = 0$, i.e., the filter is stable. This problem is called in the fault detection literature as the fundamental problem of residual generation (FPRG).

In the solution of this problem a central role is played by the (C,A)-invariant and by the unobservability subspaces (Basile and Marro, 1987).

It is known for LTI systems that a subspace $\mathcal{W}$ is (C,A)-invariant if $A(\mathcal{W} \cap KerC) \subset \mathcal{W}$. This is equivalent to the existence of a matrix G such that $(A + GC)\mathcal{W} \subset \mathcal{W}$. A (C,A)-unobservability subspace $\mathcal{S}$ is a subspace such that there exist matrices G and H with the property that $(A + GC)\mathcal{S} \subset \mathcal{S}$, i.e., $\mathcal{S}$ is (C,A)-invariant, and $\mathcal{S} \subset KerHC$. The family of (C,A)-unobservability subspaces containing a given set $\mathcal{L}$ has a minimal element $\mathcal{S}^*$.

Denote by $\mathcal{L}_i = ImL_i$, $i = 1, 2$, and denote by $\mathcal{W}^*$ the smallest (C,A)-invariant subspace over $\mathcal{L}_2$, i.e. the reachabilty subspace of μ. Denote by $\mathcal{S}^*$ the smallest unobservability subspace containing $\mathcal{W}^*$. Then for LTI systems one has the following result.

Proposition 1. If $\mathcal{S}^* \cap \mathcal{L}_1 = 0$, then $\nu(t)$ can be reconstructed from the output of the detection filter

$$\begin{aligned} \dot{w}(t) &= Nw(t) - Gy(t) + Fu(t) \\ r_\nu(t) &= Mw(t) - Hy(t), \end{aligned} \tag{29}$$

where u, y are the known input and measured output signals of the original system, $w \in Xc/\mathcal{S}^*$ is the filter state and $r_\nu \to g\nu$ where g is the steady state filter gain.

The matrices in the above filters can be obtained as follows. Denote by P the projection operator $P : \mathcal{X} \to \mathcal{X}/\mathcal{S}^*$. The state matrices can be determined as follows: H is a solution of the equation $\mathrm{Ker}\, HC = \mathrm{Ker}\, C + \mathcal{S}^*$, and M is the matrix associated to the unique solution of $MP = HC$. Design a gain matrix G_0 such that $(A + G_0C)\mathcal{S}^* \subset \mathcal{S}^*$, (i.e. G_0 makes $\mathcal{S}$ (C,A)-invariant), and denote the restriction to the factor space by $A_0 = A + G_0C|_{\mathcal{X}/\mathcal{S}^*}$. It can be shown see e.g. (?), that on this factor space one can assign the eigenvalues arbitrarily, i.e., there is a gain matrix G_1 such that $N = A_0 + G_1M$ has prescribed eigenvalues. Then set $G = PG_0 + G_1H$ and $F = PB$.

The proofs for the existence of the above constructions can be found e.g. for LTI systems in (Massoumnia, 1986), for LPV systems in (Bokor and Balas, 2004) and for input affine nonlinear systems in (Persis and Isidori, 2001), and will not be repeated here.

4. DESIGN OF THE UNKNOWN EVADER INPUT DETECTOR

The system discussed in (12-16) can formally be written as in (28), where $e_1 = [1\ 0]^T$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & b \end{bmatrix}; \quad a = -\frac{1}{\tau_E}, \quad b = -\frac{1}{\tau_P},$$

$$C = \begin{bmatrix} 1 & 0\ 0\ 0 \\ \dfrac{1}{t_f - t} & 1\ 0\ 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{a_P^{max}}{\tau_P} \end{bmatrix},$$

and

$$L_1 = [0\ 0\ \frac{a_E^{max}}{\tau_E}\ 0]^T.$$

This system is clearly an LTV one due to the time varying observation equation. It will be shown, however, that LTI methodology can still be applied when utilizing some specific properties of this system.

The pseudo actuator direction $L_2 = [1\ 0\ 0\ 0]^T$ will be chosen since this will produce the same noise effect in y_1 as the presence of μ.

In the design procedure the first step is to compute two invariant subspaces. Denote by $\mathcal{W}^*$ the minimal (C,A)-invariant subspace containing $\mathcal{L}_2$. If the dynamics is linear, this can be computed using e.g. (C,A)-invariant subspace algorithm (CAISA).

We discuss here only those design steps and results that are relevant to our specific system and lead to the the detection filter in 29.

Step 1. Since $\mathcal{L}_2$ is one-dimensional and $KerC$ is not time varying, it can be proved by using CAISA, that

$$\mathcal{W}^* = \mathcal{L}_2 + A\mathcal{L}_2 + \ldots + A^{r-1}\mathcal{L}_2,$$

where $r_i - 1$ is such that $A^{r-1}\mathcal{L}_2 \notin \operatorname{Ker} C$, i.e. r_i is the relative degree of the system w.r.to μ. For this specific case $r = 1$ and $\mathcal{W}^*_\mu = \mathcal{L}_2$. This means that the effect of μ shows up in a one dimensional subspace of the state space.

Step 2. The next step is to construct $\mathcal{S}^*$, i.e. the smallest unobservability subspace containing $\mathcal{W}^*$. Since $KerC$ is not time varying, it can be proved that $\mathcal{S}^*$ is a constant 2-dimensional subspace, spanned by the by the vectors $[1000]^T$ and $[0011]^T$.

At this point one can check that the solvability condition is satisfied, i.e. $\mathcal{S}^* \cap \mathcal{L}_1 = 0$.

Step 3. Computation of the filter gain G_0. The subspace $\mathcal{S}^*$ is $A-invariant$ resulting in that $(A+G_0C)\mathcal{S}^* \subset \mathcal{S}^*$ can be satisfied with $G_0 = 0$. This is an important special case, since this implies that we can design an LTI detection filter even if the system is a (specific) LTV one due to time varying $C(t)$.

Step 4. The canonical projection $P : \mathcal{X} \to \mathcal{X}/\mathcal{S}^*$ can be found as

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -.701 & .701 \end{bmatrix},$$

i.e. the factor space is two dimensional and P is a constant matrix. The equation $MP = HC(t)$ allows a solution with constant $M = [-1\ 0]$ and $H = [0\ \ -1]$.

For the restriction of A on the factor space we get

$$A_0 = \begin{bmatrix} 0 & -1.4 \\ 0 & -5.0 \end{bmatrix}.$$

Step 5. Since M, A_0 is observable, one can assign the poles on the factor space $\mathcal{X}/\mathcal{S}^*$ e.g. to $\{-11, -12\}$. This results in the following filter gain

$$N = \begin{bmatrix} -18 & -1.4 \\ 29.6 & -5.0 \end{bmatrix},$$

The complete filter designed to detect the unknown evader command ν is given as in Eq.30, where:

$$G = \begin{bmatrix} 0 & -18 \\ 0 & -29.6 \end{bmatrix}, F = [0\ \ -3.5]^T$$

As it can be seen, the filter output r_ν will not be effected by the sensor noise μ and will reconstruct the signal ν modulo the filter gain g , i.e. $r_\nu \to g\nu$.

Table 1. Horizontal end game parameters

Parameter	Value
Interceptor velocity	$V_P = 2300$m/sec
Target velocity	$V_E = 2700$m/sec
Interceptor lateral acceleration limit	$a_P^{\max} = 20$g
Target lateral acceleration limit	$a_E^{\max} = 10$g
Time constant of the interceptor	$\tau_P = 0.2$sec
Time constant of the target	$\tau_E = 0.2$sec
Initial range of the endgame	$R_0 = 20$km
Duration of endgame engagement	$t_f = 4$sec
Measurement noise in λ	$\sigma_\lambda = 0.1$mrad
Measurement noise in $\dot\lambda$	$\sigma_{\dot\lambda} = [0 - 0.5]\frac{\text{mrad}}{\text{sec}}$
Sampling rate	$f = 100$Hz

The detection problem can be solved by simple thresholding and the detection delay van be tuned by assigning the filter poles properly.

Repeating the above procedure it is possible to design a detection filter for the sensor noise, too.

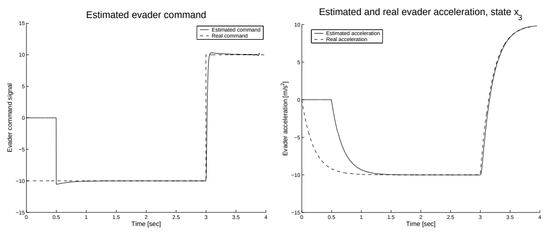
5. SIMULATION RESULTS

In this section the simulation results testing the performance of the detection filter and its effect on the homing accuracy are summarized. The target acceleration command is a "jump" process, where **v** changes it sign during the endgame. The guidance law of the missile is implemented, based on Eqs. (21)-(24), using the available information on state variables of the linear model (12)-(15). This information includes the accurate values of the time-to-go and x_4, the noise corrupted values of y_1 and y_2 as well as the value of x_3 estimated by the detection filter. The endgame parameters are summarized in Table 1.

In the examples tested in this paper the commanded target acceleration started from the normalized value of $v = -1$ at the beginning and the "jump"s from $v = -1$ to $v = +1$ may occur any time during the endgame.

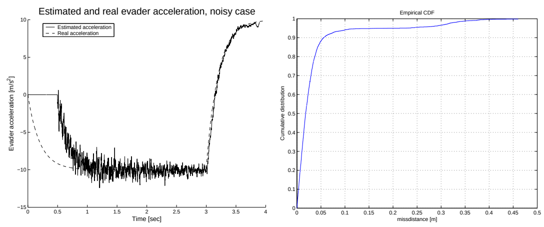
The simulations of case with $\sigma_{\dot\lambda} = 0$ demonstrated a perfect isolation of the input disturbance (the commanded target acceleration) from the measurement noise in the relative position. This isolation allows to use a fast detection filter that succeeds to identify the "jump", leading to a very small error in the estimated target acceleration, as it can be seen in Fig. 2.

Since the noise free measurement of $\dot\lambda$ is an *ideal* situation, the simulations were repeated with different levels of $\sigma_{\dot\lambda}$. The simulation results indicate the even for rather small noise levels the isolation of the input (the commanded target acceleration) from the measurement noise is not possible; therefore the "jump" cannot be identified. Nevertheless, the actual target acceleration is estimated reasonably well with a rather short delay, as it can



(a) (b)

Fig. 2. Real and estimated evader command and acceleration with nois-free line of sight rate



(a) (b)

Fig. 3. Real and estimated evader command and acceleration with noisy line of sight rate/Cumulative miss distance distribution

be seen in Fig. 3 for $\sigma_{\dot{\lambda}} = 0.1$mrad/sec. As a consequence the resulting miss distances are rather small as it can be seen in Fig. 4 that represents the cumulative probability distribution function of 4000 Monte Carlo simulation runs assuming uniformly distributed timing of the "jump".

6. CONCLUSIONS

In the course of the investigation described in this paper the following results relating to the particular dynamics of position control problems, not yet found in the technical literature, were obtained. If the only available measurement are the relative position and the pursuer's own acceleration, the unknown input cannot be isolated from the effect of the measurement noise on the relative position.

If an additional noise free measurement of the relative velocity (or the line of sight rate) is available, then the unknown input can be perfectly isolated. As a consequence, abrupt changes of the input can be detected extremely fast.

If the measurement of the relative velocity is also noise corrupted, the unknown input cannot be isolated and the detection of abrupt input changes is not possible.

In spite of the negative results for the detection in case of noise corrupted velocity measurement, the detection filter designed for noise free case succeeds to reconstruct the target acceleration in a rather satisfactory manner, leading to acceptable guidance accuracy. These results indicate that a detection filter can be an attractive candidate in guided missile systems.

REFERENCES

Basile, G. and G. Marro (1987). On the robust controlled invariant. *Sys. Contr. Letters* **9**, 191–195.

Bokor, J. and G. Balas (2004). Detection filter design for LPV systems - a geometric approach,*Automatica* **40**, 511–518.

Chen, J. and R.J. Patton (1999). *Robust Model-Based Fault Diagnosis for Dynamic Systems*, Kluwer Academic Publishers.

Gertler, J. (1998). *Fault Detection and Diagnoses in Engineering Systems*, Marcel Dekker, New York.

Hammouri, H., M. Kinnaert and E.H. El Yaagoubi (1999). Observer-based approach to fault detection and isolation for nonlinear systems, *IEEE Trans. Aut. Cont.* **44**(10), 1879–1884.

Krasovskii, N.N. and A.I. Subotin (1988). *Game-theoretical control problems*, Springler-Verlag, New York.

Massoumnia, M.A. (1986). A geometric approach to the synthesys of failure detection filters, *IEEE Trans. Automat. Contr.* **31**, 839–846.

Persis, C.De and A. Isidori (2001). A geometric approach to nonlinear fault detection and isolation, *IEEE Trans. Aut. Cont.* **AC-46**, 853–865.

Pontryagin, L.S. (1970). Linear differential games, *Proceedings of International Congress in Mathematics*, Nice, France.

Shinar, J. and D. Steinberg (1977). Analysis of Optimal Evasive Maneuvers Based on a Linearized Two-Dimensional Kinematic Model, *Journal of Aircraft* **14**(8), 795–802.

Shinar, J. and M. Zarkh (1994). Interception of Maneuvering Tactical Ballistic Missiles in the Atmosphere, *Proceedings of the 19th ICAS Congress*, Anaheim, CA, 1354–1363.

Shinar, J., V. Turetsky and Y. Oshman (2004). New logic based estimation/guidance algorithm for improved homing against randomly maneuvering targets, *Proceedings of AIAA Guidance Navigation and Control Conference*, Providence, RI.

Szigeti, F., J. Bokor and A. Edelmayer (2002). Input reconstruction by means of system inversion: application to fault detection and isolation, *In Proceedings of the 15th IFAC World Congress*, Barcelona.

Zarchan, P. (1979). Representation of Realistic Evasive Maneuvers by the Use of Shaping Filters, *Journal of Guidance and Control*, **2**(1), 290–295.