EXTENDED $\mathcal{H}_\infty$ CONTROL WITH POLE PLACEMENT CONSTRAINTS VIA LMI APPROACH AND ITS APPLICATION

Yoshiyuki Ishihara * **Yuichi Chida** *

* *Shinshu University, Nagano, Japan*

Abstract: As well known, extended $\mathcal{H}_\infty$ control accepts unstable weighting functions which are included in a generalized plant, so it has wider availability than standard $\mathcal{H}_\infty$ control. On the other hand, the root-clustering problem for $\mathcal{H}_\infty$ control has been considered by Chilali and Gahinet and the solution is derived. But it is not so easy to apply the solution to extended $\mathcal{H}_\infty$ control, because the formulation of the root-clustering controllers is discordant with that of extended $\mathcal{H}_\infty$ controllers. Therefore, we can not simply combine the root-clustering conditions with extended $\mathcal{H}_\infty$ control. In this paper, we propose a LMI based design method for this control problem by using modified formulation of controllers of the root-clustering problem derived by Chilali and Gahinet. These types of control problems appear in design problems of a vibration isolation controller which attenuates vibrations forced by narrow-band frequency disturbances. A controller for a simple vibration isolation experimental system is designed by the proposed method and performance of a designed controller is verified by experimental results.

Keywords: Extended $\mathcal{H}_\infty$ Control, Pole Placement Constraints, Vibration Isolation Control, LMIs, Frequency Shaping

1. INTRODUCTION

As well known, extended $\mathcal{H}_\infty$ control accepts unstable weighting functions which are included in a generalized plant, so it has wider availability than standard $\mathcal{H}_\infty$ control (Mita, *et al.*, 1997, Liu, *et al.*, 1997). A controller design of extended $\mathcal{H}_\infty$ control via LMI approach is also proposed by Hirata, *et al.* (2000). On the other hand, if we specify locations of closed-loop poles for improvement of transient response, additional constraints on closed-loop pole locations should be considered. The root-clustering problem has been considered by Chilali and Gahinet (1996) and the solution is derived. But it is not so easy to apply the solution to extended $\mathcal{H}_\infty$ control, because the formulation of the root-clustering controllers is discordant with that of extended $\mathcal{H}_\infty$ controllers. Therefore, we can not simply combine the root-clustering conditions with extended $\mathcal{H}_\infty$ control. In this paper, we propose a LMI based design method for this control problem by using modified formulation of controllers of the root-clustering solution derived by Chilali and Gahinet.

These types of control problems appear in design problems of a vibration isolation controller which attenuates vibrations forced by narrow-band frequency disturbance (Chida, *et al.*, 2004, Sievers, 1988). For example, a vibration isolation controller design problem concerning a rotating machine in space is described in Otsuki, *et al.* (2000) or Chida and Ishihara (2004). In the paper by Chida and Ishihara (2004), a vibration isolation controller is designed by frequency shaping method based on $\mathcal{H}_\infty$ control. In such case, we specify a weighting function such that it has steep peak gain at the disturbance frequency in order to improve vibration isolation performance. Consequently, some poles of the weighting function are located close to the imaginary axis. In this situation, if we inflict additional pole placement constraints, some poles of weighting functions become unstable as a result of the pole constraints. Therefore, extended $\mathcal{H}_\infty$ control method with pole placement constraints is available for such control problems. In this paper, a controller for a simple vibration isolation experimental system is designed by the proposed method and performance of a designed controller is verified by experimental results.

2. ANOTHER SOLUTION FOR $\mathcal{H}_\infty$ CONTROL WITH POLE PLACEMENT CONSTRAINTS

2.1 Solvability Conditions

First of all, we derive a $\mathcal{H}_\infty$ controller with pole placement constraints by a different formulation from that proposed by Chilali and Gahinet (1996). The following augmented system is considered.

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s) \begin{bmatrix} w \\ u \end{bmatrix} \tag{1}$$

$$G(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] \tag{2}$$

Where $G(s)$ is a generalized plant. It is assumed two assumptions; A) (A, B_2) is stabilizable and B) (A, C_2) is detectable. The $\mathcal{H}_\infty$ control problem is defined to find an output feedback control $u = K(s)y$ which satisfies the internal stability and the $\mathcal{H}_\infty$ norm condition on the closed-loop transfer function from w to z; $G_{zw} \in \mathcal{BH}_\infty$. A controller $K(s)$ is described as $K(s) = \{A_K, B_K, C_K, 0\}$, then $G_{zw}(s) = \{A_{cl}, B_{cl}, C_{cl}, 0\}$ is represented as follows.

$$G_{zw} = \left[\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl} & 0 \end{array}\right] = \left[\begin{array}{cc|c} A & B_2C_K & B_1 \\ B_KC_2 & A_K & B_KD_{21} \\ \hline C_1 & D_{12}C_K & 0 \end{array}\right] \tag{3}$$

The following two results are well known.
[Lemma 1](Bounded Real Lemma, Iwasaki (1997)) For a generalized system Eq.(2), the following two conditions are equivalent.
(i) A controller $K(s)$ exists such that the closed-loop system (G_{22}, K) is internally stable and $\|G_{zw}\|_\infty < 1$.
(ii) A matrix $X_{cl} > 0$ which satisfies the following inequality exists.

$$A_{cl}X_{cl} + X_{cl}A_{cl}^T + X_{cl}C_{cl}^TC_{cl}X_{cl} + B_{cl}B_{cl}^T < 0 \tag{4}$$

[Lemma 2](Chilali and Gahinet (1996))
A necessary and sufficient condition such that all poles of a closed-loop system are located in specified LMI region is that there exists a matrix $X_{\mathcal{D}} > 0$ which satisfies the following inequality.

$$\left[\alpha_{kl}X_{\mathcal{D}} + \beta_{kl}A_{cl}X_{\mathcal{D}} + \beta_{lk}X_{\mathcal{D}}A_{cl}^T\right]_{1\le k,l\le m} < 0 \tag{5}$$

Where, α_{kl} and β_{kl} are parameters for specifying a LMI stability region $\mathcal{D}$.

Constraints on pole locations are represented by LMI stability region $\mathcal{D}$ defined by Chilali and Gahinet. An example of the LMI region is specified by the hatched area shown in Fig.2.

The following theorem provides another solution of standard $\mathcal{H}_\infty$ control with pole placement constraints. The formulation of controllers is different from that of the result proposed by Chilali and Gahinet.

[Theorem 1] **1.** It is assumed that assumptions A) and B) are satisfied. A design problem of $\mathcal{H}_\infty$ control with pole placement constraints defined by LMI region $\mathcal{D}$ is solvable if the following four conditions 1) − 4) are satisfied and there exist matrices F, L, $X > 0$ and $Y > 0$.

1) $$A_FX + XA_F^T + XC_F^TC_FX + B_1B_1^T < 0, \quad X > 0 \tag{6}$$

2) $$YA_L + A_L^TY + C_1^TC_1 + YB_LB_L^TY < 0, \quad Y > 0 \tag{7}$$

3) $$\rho(Y^{-1}X^{-1}) < 1 \tag{8}$$

4) $$\left[\alpha_{kl}\begin{pmatrix} X & I \\ I & Y \end{pmatrix} + \beta_{kl}\Phi + \beta_{lk}\Phi^T\right]_{1\le k,l\le m} < 0 \tag{9}$$

Where, α_{kl} and β_{kl} are parameters for specifying a LMI region $\mathcal{D}$ and

$$\Phi := \begin{bmatrix} A_FX & A \\ -(C_1^TC_FX + YB_LB_1^T + A^T) & YA_L \end{bmatrix}. \tag{10}$$

Also, A_F and A_L are defined as the following.

$$A_F := A + B_2F, \qquad C_F := C_1 + D_{12}F \tag{11}$$

$$A_L := A + LC_2, \qquad B_L := B_1 + LD_{21} \tag{12}$$

2. When the solvability conditions are held, one of the $\mathcal{H}_\infty$ controllers is obtained by the following formula.

$$K(s) = \{A_K, B_K, C_K, 0\} \tag{13}$$

Where,

$$A_K = (A + LC_2)XZ^{-1} + B_2C_K + B_LB_1^TZ^{-1} + Y^{-1}(A^T + C_1^TC_FX)Z^{-1} \tag{14}$$

$$B_K = -L \tag{15}$$

$$C_K = FXZ^{-1} \tag{16}$$

and Z is a symmetric matrix which defined as the following equation.

$$Z := X - Y^{-1} \tag{17}$$

2.2 Proof of Theorem 1

In this paper, we consider a common solution of Eq.(4) and Eq.(5), $X_{\mathcal{D}} = X_{cl}$ [1]. Then, according to Lemma 1 and Lemma 2, the solvability conditions are described as the following two inequalities which have a common solution $X_{cl} > 0$.

$$\Theta := A_{cl}X_{cl} + X_{cl}A_{cl}^T + X_{cl}C_{cl}^T C_{cl}X_{cl} + B_{cl}B_{cl}^T < 0 \quad (18)$$

$$\Gamma := [\alpha_{kl}X_{cl} + \beta_{kl}A_{cl}X_{cl} + \beta_{lk}X_{cl}A_{cl}^T]_{1\le k,l\le m} < 0 \quad (19)$$

Hereafter, we show if Eqs.(6)–(17) are satisfied, then Eq.(18) and Eq.(19) are satisfied. X_{cl} is represented by using matrices X and Z without loss of generality, and Y is defined as follows.

$$X_{cl} := \begin{bmatrix} X & Z \\ Z & Z \end{bmatrix}, \quad Y := (X - Z)^{-1} \quad (20)$$

We can confirm the equivalence between $X_{cl} > 0$ and $X > 0$, $Y > 0$, and $\rho(Y^{-1}X^{-1}) < 1$ by using the schur complement. We define a matrix T as

$$T := \begin{bmatrix} I & 0 \\ Y & -Y \end{bmatrix}. \quad (21)$$

Since T is a nonsingular matrix, $\Theta < 0 \Leftrightarrow \hat{\Theta} := T\Theta T^T < 0$ and $\Gamma < 0 \Leftrightarrow \hat{\Gamma} := T\Gamma T^T < 0$ are satisfied. By substituting A_{cl}, B_{cl} and C_{cl} of Eq.(3) into Eq.(18), $\hat{\Theta}$ is calculated as follows.

$$\hat{\Theta} = \begin{bmatrix} \hat{\Theta}_{11} & \hat{\Theta}_{21}^T \\ \hat{\Theta}_{21} & \hat{\Theta}_{22} \end{bmatrix} < 0 \quad (22)$$

$$\hat{\Theta}_{11} = (A + B_2F)X + X(A + B_2F)^T + B_1B_1^T + X(C_1 + D_{12}F)^T(C_1 + D_{12}F)X \quad (23)$$

$$\hat{\Theta}_{22} = Y(A + LC_2) + (A + LC_2)^T Y + C_1^T C_1 + Y(B_1 + LD_{21})(B_1 + LD_{21})^T Y \quad (24)$$

$$\hat{\Theta}_{21} = Y(A - B_KC_2)X + Y(B_2C_K - A_K)Z + A^T + C_1^T C_1 X + YB_1B_1^T + C_1^T D_{12}C_K Z - YB_KD_{21}B_1^T \quad (25)$$

Where, $F := C_K Z X^{-1}$ and $L := -B_K$. On the other hand, $\hat{\Gamma}$ is calculated as the following.

$$\hat{\Gamma} = \left[\alpha_{kl}\begin{pmatrix} X & I \\ I & Y \end{pmatrix} + \beta_{kl}\begin{pmatrix} A_F X & A \\ \Psi & YA_L^T \end{pmatrix} + \beta_{lk}\begin{pmatrix} A_F X & A \\ \Psi & YA_L^T \end{pmatrix}^T\right]_{1\le k,l\le m} < 0 \quad (26)$$

$$\Psi := Y(A - B_KC_2)X + Y(B_2C_K - A_K)Z \quad (27)$$

If we specify the matrices A_K, B_K, C_K as Eqs.(14)–(16), then $\hat{\Theta}_{21} = 0$ is satisfied. It is verified equivalence between $\hat{\Theta}_{11} < 0$ and Eq.(6), and also $\hat{\Theta}_{22} < 0$ and Eq.(7), respectively. Also, since A_K is specified as Eq.(14), Ψ is described as the following equation.

$$\Psi = Y(A - B_KC_2)X + Y(B_2C_K - A_K)Z = -(C_1^T C_F X + YB_LB_1^T + A^T) \quad (28)$$

So, Eq.(9) satisfies $\Gamma < 0$. Therefore, if Eqs.(6)–(17) are satisfied, $\hat{\Theta} < 0$ and $\hat{\Gamma} < 0$ are satisfied by $X_{cl} > 0$ of Eq.(20).

[1] If the problem is considered within the restricted solutions such as $X_{\mathcal{D}} = X_{cl}$, Theorem 1 becomes necessary and sufficient conditions.

3. EXTENDED $\mathcal{H}_\infty$ CONTROL

We assume that weighting functions are connected to outputs of $G(s)$, and an augmented system shown in Fig.1 is considered. $W_z(s)$ indicates a weighting function. The extended $\mathcal{H}_\infty$ control problem is defined that output feedback $u = K(s)y$ by a controller $K(s)$ stabilizes (G_{22}, K) and satisfies the $\mathcal{H}_\infty$ norm condition $G_{zw} \in \mathcal{BH}_\infty$. In this paper, additional constraints on closed-loop pole locations are considered. If we specify a LMI stability region $\mathcal{D}$ such as the hatched area in Fig.2, outside poles of the stability region become undetectable modes even if $W_z(s)$ is stable. In such cases, extended $\mathcal{H}_\infty$ control can be applied instead of standard $\mathcal{H}_\infty$ control. Extended $\mathcal{H}_\infty$ control doesn't require stability of $G_{zw}(s)$ but (G_{22}, K). One of the solution of extended $\mathcal{H}_\infty$ control by LMI approach is derived by Hirata, *et al.* (2000). The following assumptions are introduced in Hirata, *et al.* (2000).

- A1') (A, C_2) is detectable except for the modes corresponding to eigenvalues of A_{pz}. Where, A_{pz} is a matrix which includes all of the unstable eigenvalues of $W_z(s) := \{A_z, B_z, C_z, D_z\}$.
- A2') (A, B_2) is stabilizable.

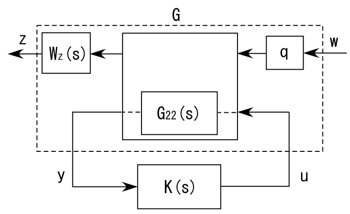

Fig.1 Structure of Augmented System

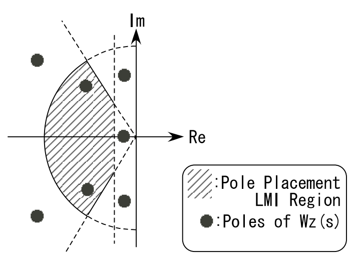

Fig.2 Pole Placement Region and Eig. of A_{pz}

B') Let $\mathcal{U}_0$ a set of full row rank matrices U_0 for which there exists a matrix L satisfying the following equations.

$$U_0(A + LC_2) = A_{pz}U_0, \quad (29)$$
$$U_0(B_1 + LD_{21}) = 0, \quad (30)$$
$$\text{and} \quad \sigma[(A + LC_2)^T | R^n/\mathrm{Im}U_0^T] \subset C^- \quad (31)$$

Let $\mathcal{L}(U_0)$ be a set of matrices L satisfying Eqs.(29)–(31) for a fixed $U_0 \in \mathcal{U}_0$.

C') $[C_2, D_{21}]$ has full row rank and $[B_2^T, D_{12}^T]^T$ has full column rank.

The assumption B') represents that A_{pz} becomes uncontrollable modes of G_{zw}, and it is equivalent to a condition that the modes become invariant zeros of a system $\{A, B_1, C_2, D_{21}\}$.

4. SOLVABILITY CONDITION OF EXTENDED $\mathcal{H}_\infty$ CONTROL WITH POLE PLACEMENT CONSTRAINTS

[Theorem 2] **1.** For a control system shown in Fig.1, it is assumed that the assumptions A'), B') and C') are satisfied. Then, the extended $\mathcal{H}_\infty$ control problem with pole placement constraints is solvable if there exist $U_0 \in \mathcal{U}_0$, F, L, $X > 0$ and $Y > 0$ which satisfy the following conditions 1) – 4).

$$1)\ A_F X + XA_F^T + XC_F^T C_F X + B_1 B_1^T < 0, \quad X > 0 \quad (32)$$

$$2)\ U_1(YA_L + A_L^T Y + C_1^T C_1 + YB_L B_L^T Y)U_1^T < 0, \quad U_0 Y = 0, \quad U_1 Y U_1^T > 0 \quad (33)$$

$$3)\ \left[\alpha_{kl}\begin{pmatrix} X & U_1^T \\ U_1 & U_1 Y U_1^T \end{pmatrix} + \beta_{kl}\Omega + \beta_{lk}\Omega^T\right]_{1\le k,l\le m} < 0 \quad (34)$$

$$\Omega := \begin{bmatrix} A_F X & AU_1^T \\ -U_1(C_1^T C_F X + YB_L B_1^T + A^T) & U_1 Y A_L U_1^T \end{bmatrix} \quad (35)$$

Where, α_{kl} and β_{kl} are parameters for specifying a LMI region $\mathcal{D}$.

$$4)\ \rho(Y^\dagger X^{-1}) < 1 \quad (36)$$

Where, U_1 is an arbitrary matrix such that it makes $[U_0^T, U_1^T]$ be a non-singular matrix and $\mathrm{Im}U_1^T = \ker U_0$ is satisfied. The definition of A_F and A_L are the same as Eqs.(11) and (12).

2. If the solvability conditions are satisfied, a controller is obtained by equations which is derived by substituting the following $Y^\dagger$ and Z into Y^{-1} and Z of Eqs.(14)–(16), respectively.

$$Y^\dagger := U_1^\dagger\{(U_1^\dagger)^T Y U_1^\dagger\}^{-1}(U_1^\dagger)^T \quad (37)$$
$$Z := X - Y^\dagger \quad (38)$$

Where, $U_1^\dagger$ represents pseudo-inverse of U_1.

5. LMI CONDITIONS

LMI conditions are derived based on Theorem 2. We define $U_L := U_0 L$, then U_0 which satisfies Eqs.(29)–(31) are determined according to the following condition.

$$[U_0\ U_L]\begin{bmatrix} A & B_1 \\ C_2 & D_{21}\end{bmatrix} = A_{pz}[U_0\ 0] \quad (39)$$

If $\mathrm{Im}[U_0, U_L]^T$ is uniquely determined, the conditions of Theorem 2 can be described by LMI conditions. We define the following matrices by using an arbitrary matrix U_1 which makes $[U_0^T, U_1^T]$ be a non-singular matrix.

$$\left.\begin{array}{l} A_U := U_1 A U_1^\dagger,\ C_{1U} := C_1 U_1^\dagger,\ C_{2U} := C_2 U_1^\dagger \\ B_{1U} := U_1 B_1,\ L_U := U_1 L \\ Y_U := (U_1^\dagger)^T Y U_1^\dagger > 0,\ N := Y_U L_U \end{array}\right\} \quad (40)$$

[Theorem 3] If $\mathrm{Im}[U_0, U_L]^T$ is uniquely determined, the solvability conditions of Theorem 2 are equivalent to existence conditions of $X > 0$, $Y_U > 0$, M, and N which satisfy the following LMIs.

$$\begin{bmatrix} AX + XA^T + B_2 M + M^T B_2^T + B_1 B_1^T & XC_1^T + M^T D_{12}^T \\ C_1 X + D_{12} M & -I \end{bmatrix} < 0 \quad (41)$$

$$\begin{bmatrix} Y_U A_U + A_U^T Y_U + NC_{2U} + C_{2U}^T N^T + C_{1U}^T C_{1U} & Y_U B_{1U} + ND_{21} \\ B_{1U}^T Y_U + D_{21}^T N^T & -I \end{bmatrix} < 0 \quad (42)$$

$$\left[\alpha_{kl}\begin{pmatrix} X & U_1^\dagger \\ (U_1^\dagger)^T & Y_U \end{pmatrix} + \beta_{kl}\Xi + \beta_{lk}\Xi^T\right]_{1\le k,l\le m} < 0 \quad (43)$$

$$\begin{bmatrix} X & U_1^\dagger \\ (U_1^\dagger)^T & Y_U \end{bmatrix} > 0 \quad (44)$$

Where,

$$\Xi := \begin{bmatrix} AX + B_2 M & AU_1^\dagger \\ \begin{pmatrix} C_{1U}^T C_1 X + Y_U B_{1U} B_1^T + C_{1U}^T D_{12} M \\ + ND_{21} B_1^T + (AU_1^\dagger)^T \end{pmatrix} & Y_U A_U + NC_{2U} \end{bmatrix} \quad (45)$$

(Proof) The conditions are derived by defining $M := FX$ and Eq.(40), and by some calculations of Eqs.(32)–(35).

If the LMI conditions are solvable, an extended $\mathcal{H}_\infty$ controller whose poles are located in specified LMI region is obtained by the following procedure.

Step 1 Obtain U_0 which satisfies Eq.(39), and calculate U_1 by $\mathrm{Im}U_1^T = \ker U_0$. U_L is derived such that it satisfies Eq.(39).
Step 2 Obtain $X > 0$, $Y_U > 0$, M, and N by solving Eqs.(41)–(44).
Step 3 F, L and Y which satisfy Eqs.(32)–(36) are obtained by using X, Y_U, M and N as follows.

$$Y = U_1^T Y_U U_1 \tag{46}$$

$$F = MX^{-1}, \quad L = \begin{bmatrix} U_0 \\ U_1 \end{bmatrix}^{-1} \begin{bmatrix} U_L \\ Y_U^{-1} N \end{bmatrix} \tag{47}$$

Step 4 A controller is derived by **2.** of Theorem 2 by using X, F, L and Y obtained in Step 3, and Z by Eq.(38).

6. APPLICATION TO VIBRATION ISOLATION CONTROLLER DESIGN

6.1 Controlled System

Controlled system is a simple experimental system shown in Fig.3. This system is a rotational type vibration system composed of 2-inertias and springs. Control torque, f, and vibration disturbance, d, are applied to the system by two DC motors. Rotational angles of inertias are detected by rotary encoders which are mounted into the axis of the system. The system is stabilized by feedback control $f = K(s)(\dot{\theta}_1 - \dot{\theta}_2)$. Hereafter, it is assumed that frequency of the disturbances in the rough is known (Otsuki, *et al.*, 2000), and we assumed that d is a sinusoidal disturbance of 3[Hz]. The Bode plot of the plant is shown in Fig.4. Control objectives are the followings.

Controller Design Objectives

1. Apply appropriate damping effect to two vibration modes of 0.65[Hz] and 9.13[Hz].
2. Isolate the influence of narrow-band frequency disturbance, d, of 3[Hz] to the angle θ_2.
3. Locate closed-loop poles to an appropriate LMI stability region.

6.2 Design Procedure

An augmented system for a $\mathcal{H}_\infty$ control design is specified in Fig.5. In Fig.5, $u = f$, $y_1 = \theta_2$, and $y_2 = \dot{\theta}_1 - \dot{\theta}_2$. Since the transfer function from d to z_1 corresponds to vibration isolation performance, the performance is directly shaped by specifying weighting function $W_n(s)$. Specified weighting functions are described as follows and the gain plots are shown in Fig.6. It is noticed that $W_n(s)$ has poles close to the imaginary axis.

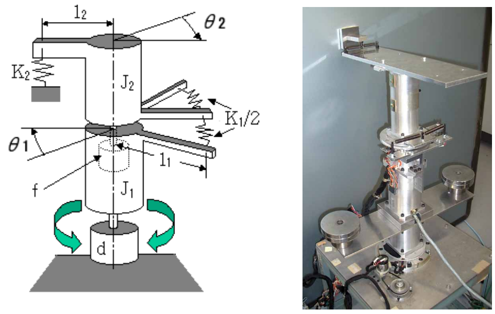

Fig.3 Experimental System

$$\left.\begin{aligned}
W_r(s) &= \frac{3.0 \cdot 10^3 (s+10)^2}{(s+1000)(s+1005)} \cdot \frac{1/3s + 10}{1/0.06s + 1000} \\
W_n(s) &= \frac{0.281(s+0.1)^2}{(s+100)(s+101)} \prod_{i=1}^{3} \frac{s^2 + 2\xi_i\omega_i s + \omega_i^2}{s^2 + 2\varsigma_i\omega_i s + \omega_i^2} \\
&\xi_1 = 7.0, \quad \varsigma_1 = 0.004, \quad \omega_1 = 3.00 \cdot 2\pi \\
&\xi_2 = 100.0, \quad \varsigma_2 = 0.5, \quad \omega_2 = 0.50 \cdot 2\pi \\
&\xi_3 = 50.0, \quad \varsigma_3 = 1.75, \quad \omega_3 = 9.0 \cdot 2\pi \\
\varepsilon &= 10^{-5}
\end{aligned}\right\} \tag{48}$$

U_0, U_L is obtained as follows. We transform the matrix A of the augmented system to the diagonal canonical form by using a transformation matrix T. Then, a matrix U_0 of Eq.(39) is represented as follows.

$$U_0(T^{-1}AT) + U_L(C_2T) = A_{pz}U_0 \tag{49}$$

$$U_0(T^{-1}B_1) + U_L D_{21} = 0 \tag{50}$$

We obtain $U_L = -U_0 T^{-1} B_1 D_{21}^{\dagger}$ by Eq.(50). Substituting it into Eq.(49) and by some calculations, we notice that $U_0 = (0, I)$ satisfies Eqs.(49)–(50). U_L is obtained by substituting U_0 into Eq.(50).

Specified LMI region for pole locations is shown in Fig.7. Where, we specify as $\phi = \pi/2.25$. α_{kl} and

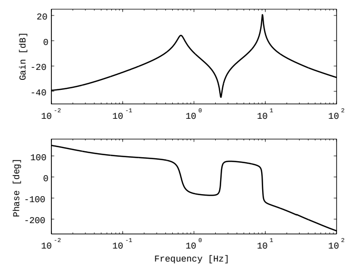

Fig.4 Bode Plot of Controlled System

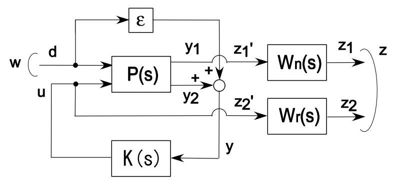

Fig.5 Augmented System

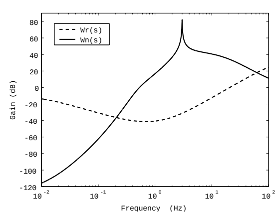

Fig.6 Gain of Weightings

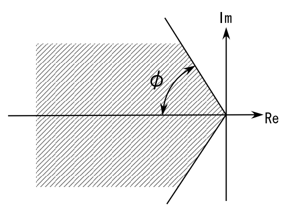

Fig7 LMI Region

β_{kl} of Eq.(43) are specified as $\alpha_{kl} = 0$ $(k, l = 1, 2)$, $\beta_{11} = \sin\phi$, $\beta_{12} = \cos\phi$, $\beta_{21} = -\cos\phi$, and $\beta_{22} = \sin\phi$.

6.3 Performance of Designed Controller

LMI conditions of Theorem 3 are feasible and a controller is derived. Transfer functions of closed-loop systems are indicated in Fig.8. These plots are obtained by numerical simulations based on a plant model. An experimental result of time response when a sinusoidal vibration disturbance is applied to the system is shown in Fig.9. In Fig.9, the solid line indicates the proposed LMI controller and the dashed line denotes the controller by Chida and Ishihara (2004), it was the champion result except the proposed LMI method. The LMI controller provides superior vibration isolation performance as expected. Damping effect for the second order vibration mode of the plant is improved compared with the dashed line. Maps of closed-loop pole locations are shown in Fig.10 and Fig.11. According to Fig.10, it is noticed that the controller by Chida and Ishihara (2004) includes insufficient poles whose damping ratios are close to zero (indicated by the arrows). On the other hand, the proposed LMI controller provides sufficient damping ratios.

7. CONCLUSIONS

In this paper, we derive solvability conditions for extended $\mathcal{H}_\infty$ control with pole placement constraints and a formulation of a controller. The proposed method is applied to a vibration isolation controller design for an experimental system. The performance of the proposed controller is verified by control experiments.

REFERENCES

Chida, Y. and Y. Ishihara (2004). Vibration Isolation Controller Design by Frequency Shaping with Pole Placement Constraints, *2004 IEEE CCA*, pp.412-417.

Chida, Y., Y. Ishihara and T. Okina (2004). Vibration Isolation Controller Design by Frequency Shaping, *the 10th IFAC/IFORS/IFIP Symposium on LSS*, pp. 186-191.

Otsuki, F., H. Uematsu, Y. Nakamura, Y. Chida, O. Nishimura, K. Ohtomi & M. Tanaka (2000). Vibration Isolation Control of Centrifuge Rotor, *Proc. of the 5th MOVIC*, pp. 415-420.

Sievers, L.A. and A.H. von Flotow (1988). Linear Control Design for Active Vibration Isolation of Narrow Band Disturbances, *Proc. of the 27th CDC*, pp. 1032-1037.

Liu, K.Z. and M. Hirata & T. Sato (1997). All Solutions to the $\mathcal{H}_\infty$ Control Synthesis Problem with Unstable Weights, *Proc. of the 36th IEEE CDC San Diego*, pp. 4641-4646.

Mita, T., X. Xin, K. Tomiyama and B.D.O. Anderson (1997). Extended $\mathcal{H}_\infty$ control -unified solutions to $\mathcal{H}_\infty$ servo and estimation problems (in Japanese), *Trans. of SICE*, **33**. pp. 656-664.

Hirata, M., K.Z.Liu, T.Saito and K.Nonami (2000). A solvability condition of an extended $\mathcal{H}_\infty$ control problem using Riccati inequalities, *Int. J. Control*, Vol.73, No.4, pp. 265-275.

Chilali, M. and P.Gahinet (1996). $\mathcal{H}_\infty$ Design with Pole Placement Constraints: an LMI Approach, *IEEE Trans. Aut. Contr.*, Vol.41, No.3, pp. 358-367.

Iwasaki, T. (1997). *LMI and Control*(in Japanese), Sho-ko-do.

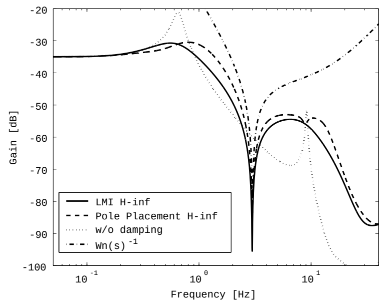

Fig.8 Gain from d to θ_2

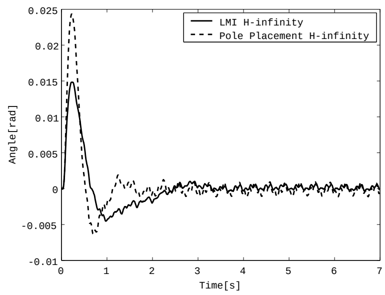

Fig.9 Time Response

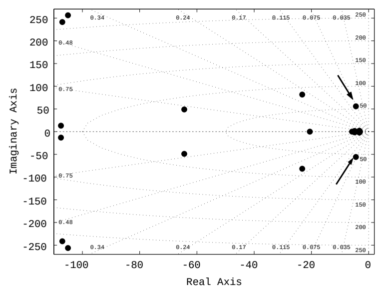

Fig.10 Pole Map (Chida and Ishihara, 2004)

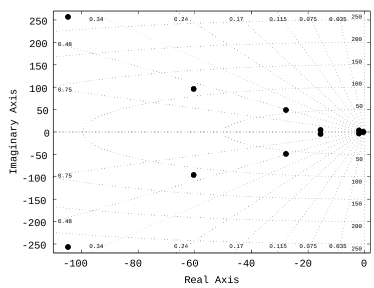

Fig.11 Pole Map (proposed LMI $\mathcal{H}_\infty$)