COORDINATING CONTROL OF MOTION OF REDUNDANT MANIPULATORS

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Abstract: The paper discusses problems of spatial motion of lightweight dexterous manipulators with respect to complex environment represented in the form of holonomic restrictions. The relevant control problem implies maintaining a compact configuration of the kinematic chain and stabilization of the robot principal line with respect to desired smooth time-varying curves. The solutions are based on the approaches of the theory of nonlinear multi-input/multi-output (MIMO) control and are reduced to output coordination and output stabilization of a nonlinear MIMO plant.

Keywords: robotics, redundancy, dexterous manipulations, nonlinear control

I. INTRODUCTION

Redundant number of degrees of freedom (DOF) of robotic manipulators is a necessary condition for their dexterity and versatility. Dexterous multi-link robots are able to perform nontrivial locomotion tasks such as penetrating into hard to come domains of the operational zone, perfect obstacle avoidance, suitable approaching external objects, motion along complex curvilinear trajectories and so on (Seraji, 1989; Murray et al., 1993; Miroshnik and Nikiforov, 1994; Caccavale and Siciliano, 2001; Furuta, 2002). Execution of these tasks and the necessity for fruitful utilization of the extra DOF in the course of the robot motion induce certain difficulties of control referred to the redundancy problem. The latter is caused by uncertainty of the robot configuration, as well as the required control actions, when the number of controlled DOF is greater than the dimension of the Cartesian space. The most difficult control problems are connected with spatial motion of hyper-redundant robotic systems (articulated and snake-like robots, *variable geometry truss* manipulators, or VGT-robots, etc.; Chirikjian, 1995, Miroshnik *et al.*, 2003) and time variations of the given trajectory. [1]

A natural way to overcome the uncertainty of the redundant robot's control is to impose additional constrains on the robot motion (Seraji, 1989; Miroshnik and Nikiforov, 1994-1997; Fradkov, et al., 1999; Miroshnik, et al., 2001; Mirosh-

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nik, et al., 2003; Chevallereau, et al., 2003). These constrains written as holonomic relations

$$\varphi_i(x_1, \ldots, x_j, \ldots) \;=\; 0$$

of certain robot variables x_j can define, for instance, conditions of the identity or proportionality of some joint coordinates and their rates, a desired orientation of the links in the Cartesian space, etc. The most evident relations of the Cartesian coordinates are given by the analytical description of end-point trajectory itself or by the trajectories of other specific points of the robot kinematic chain.

The fulfillment of the additional constrains provides the coordinated motion of the links of the redundant mechanism and maintaining its desired configuration in the course of the end-point displacement along a given trajectory. Such a kind of the robot behavior can be arranged by means of tracking control when the coordinating relations are involved into the procedure of calculating the inverse kinematics (Whitney, 1969; De Luca and Oriolo, 1996). However the tracking control strategy has certain disadvantages concerning complexity of the on-line calculation of the inverse kinematics and the precise interpolation of the desired trajectories, as well as decreased dynamic quality of tracking the task-oriented coordinates (Fradkov, et al., 1999). For hyper-redundant manipulators, the relevant solutions were proposed by using "infinite-DOF concept" (Chirikjian, 1995), which is not so efficient when the number of robot links is not very large.

A direct solution to the robot control problems with *time-invariant* holonomic restrictions based on *coordinating control principle* was given in (Miroshnik and Nikiforov, 1995-1996; Fradkov, et al., 1999, Miroshnik, et al., 2003). This implies introducing the necessary number of the *task-oriented coordinates*

$$\varepsilon_i \;=\; \varphi_i(x_1, \ldots, x_j, \ldots),$$

characterizing deviations from the desired restricting relations. In this way, the multi-dimensional control task is reduced to a set of simple stabilization problems solved by using nonlinear control techniques (Miroshnik and Nikiforov, 1996-1997; Fradkov, et al., 1999).

This paper discusses problems of maintaining the required configuration of the multi-link robot, when moving in complex environment. The problem is reduced to stabilization of the robot central line about a given *time-varying goal trajectory* and an appropriate longitudinal displacement of the central points, providing variation of the length of the robot body. The solution is based on approaches of the theory of nonlinear and MIMO control (Fradkov, et al., 1999; Miroshnik 2004). The main problems (connected with system redundancy, coordination of the motions of different link of the robot and the necessity to provide the compact time-varying configuration of the robot chain) are reduced to problems of the output coordination and stabilization of a nonlinear multi-input/multi-output plant with respect to spatial (planar) time-varying attractors.

2. ROBOT MODELS AND PROBLEM STATEMENT

We restrict our consideration to a planar multi-link robot consisting of standard 1-DOF links with rotational joints (Fig. 1). Kinematics of the j-th link is described by the equation

$$\begin{aligned} y^j \;&=\; y^{j-1} + \\ &+\; 0.5\xi T^T(\alpha^{j-1})z + 0.5\xi T^T(\alpha^j)z, \quad (1) \end{aligned}$$

where $y^j = \mathrm{col}(y_1^j, y_2^j) \in \mathcal{Y} \subset \mathbb{R}^2$ is the vector of Cartesian coordinates of a principal (central) point P^j and α^j the angular attitude of the jth link, ξ is the length of the link, $T(\alpha^j)$ is an orthogonal matrix ($T^T = T^{-1}$), $z = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$.

A light-weight planar robot with standard rotational joints is an m-degree-of-freedom spatial kinematic mechanism described by the equations

$$\dot{q} \;=\; Bu, \qquad (2)$$

$$y^j \;=\; c^j(q), \quad j = 1, \ldots, m, \qquad (3)$$

where $q = \{q_j\} \in \mathcal{Q} \subset \mathbb{R}^m$ is the vectors of joint (generalized) coordinates, $u = \{u_j\} \in \mathbb{R}^m$ is the

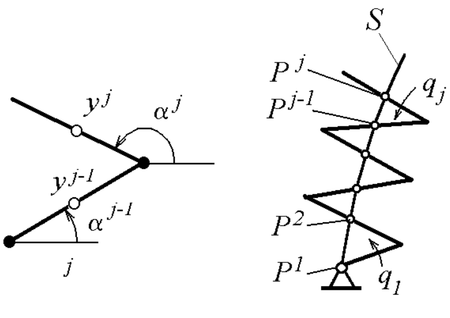

Fig. 1. Manipulator configuration

vector of control variables, B is an invertable matrix,

$$c^1(q) = 0, \ c^j(q) = c^{j-1}(q) + + 0.5\xi T^T(\alpha^{j-1})z + 0.5\xi T^T(\alpha^j)z, \quad (4)$$

$j = 2, \ldots, m$, and

$$\alpha_j = n_j^T q, \ \ n_j^T = \mid 1...10...0 \mid, \quad (5)$$

$j = 1, \ldots, m$. Differentiating equation (3) with respect to time and substituting (2), one can obtain the robot model in the Cartesian space

$$\dot{y}^j = C^j(q)B\, u, \quad (6)$$

where the matrices $C^j(q) = \partial c^j/\partial q$ are found as

$$C^1(q) = 0, \ C^j(q) = C^{j-1}(q) + +0.5\xi T^T(\alpha^{j-1})rn_{j-1}^T + 0.5\xi T^T(\alpha^j)rn_j^T, (7)$$

$j = 2, \ldots, m$, $r = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$.

Consider the motion of the robot in the Cartesian space $\mathcal{Y}$ with respect to a prescribed smooth time-varying curve $\mathcal{S}$ given by the equation

$$\varphi(y,t) = 0, \quad (8)$$

while the path length is defined as

$$s = \psi(y,t). \quad (9)$$

Henceforward we make use of the orthonormalized description of the curve (see Fradkov, et al., 1999; Miroshnik, 2004), for which the smooth functions φ and ψ are assumed to be such that, on the curve $\mathcal{S}$, the Jacobian matrix $J(y) = \begin{vmatrix} \partial\psi/\partial y \\ \partial\varphi/\partial y \end{vmatrix}$ of the mapping (ψ, φ) is orthogonal, or

$$J(y)|_{y \in S} = T(\alpha^*). \quad (10)$$

Here the matrix $T(\alpha^*)$ is associated with a movable (*Frenet*) frame of the curve, $\alpha^* = \alpha^*(y)$ is the angle of its orientation, and

$$T(\alpha^*) = \begin{vmatrix} \tau_1^T(\alpha^*) \\ \tau_2^T(\alpha^*) \end{vmatrix} = \begin{vmatrix} \cos\ \alpha^* & \sin\ \alpha^* \\ -\sin\ \alpha^* & \cos\ \alpha^* \end{vmatrix}. \quad (11)$$

We study the trajectory problem as that of keeping up the motion of all central points P^j along the curve S^* and providing a desired longitudinal velocity $\dot{s}$ of the end-point P^m. For, we introduce $m-1$ relations

$$\varphi(y^j, t) = 0, \ \ j = 2, \ldots, m, \quad (12)$$

and define the longitudinal motion of the end-point P^m as

$$s^m = \psi(y^m, t). \quad (13)$$

Then the control problem is stated as finding the control u such that, in time, equations (12) hold and

$$\dot{s}^m = v^*(t). \quad (14)$$

3. CONTROL DESIGN

Define violations of the conditions (12) associated with orthogonal deviations from the curve $\mathcal{S}$ by using the variables

$$e^j = \varphi(y^j, t), \ \ j = 2, \ldots, m. \quad (15)$$

Equations (13) and (15) introduce a coordinate change (transformation to the robot task-oriented variables, Fradkov, et al., 1999). The control problem is reduced to (asymptotic) eliminating the deviations e^j and maintaining relation (14).

To derive the task-oriented model, we differentiate equations (13), (15) and, after the substitution of (6), find

$$\begin{vmatrix} \dot{s}^m \\ \dot{e}^m \end{vmatrix} = T(\alpha^*(y^m))C^m(q)Bu + \begin{vmatrix} \sigma \\ \epsilon^m \end{vmatrix}, (16)$$

$$\dot{e}^j = \tau_2(\alpha^*(y^j))C^j(q)Bu + \epsilon^j, \quad (17)$$

$j = 1, \ldots, m-1$, where

$$\sigma(y,t) = \frac{\partial\psi^m}{\partial t}, \quad \epsilon^j(y,t) = \frac{\partial\varphi^j}{\partial t}, \quad (18)$$

$j = 1, \ldots, m$.

Let us now transform the control variables according to the expressions

$$T(\alpha^*(y^m))\ C^m(q)B\ u = \begin{vmatrix} u_s \\ u_e^m \end{vmatrix}, \quad (19)$$

$$\tau_2^T(\alpha^*(y^j))\ C^j(q)B\ u = u_e^j, \quad (20)$$

where u_s is the longitudinal control and u_e^j are the transversal (or error) controls. Then equations (16)-(17) take the form

$$\dot{s}^m = u_s + \sigma, \quad (21)$$

$$\dot{e}^j = u_e^j + \epsilon^j, \ j = 1, \ldots, m. \quad (22)$$

Choosing

$$u_s = v^* - \sigma, \quad (23)$$

$$u_e^j = -\ Ke^j - \epsilon_j, \ \ j = 1, \ldots, m, \quad (24)$$

where $K > 0$, we provide the required velocity v^* of the longitudinal motion (see (14)) and asymptotic zeroing of the errors e^j.

To obtain the resulting control of the robot, we have to find an inverse control transformation, i.e. to solve the m equations

$$\begin{aligned} \tau_1^T(\alpha^*(y^m))\; C^m(q)B\; u &= u_s, \\ \tau_2^T(\alpha^*(y^m))\; C^m(q)B\; u &= u_e^m, \\ \tau_2^T(\alpha^*(y^j))\; C^j(q)B\; u &= u_e^j,\; j = 1, \ldots, m-1, \end{aligned} \tag{25}$$

with respect to m control variables u^j. The transformation is reduced to the inversion of the matrix

$$C(q, y^2, \ldots, y^m) = \begin{vmatrix} \tau_1^T(\alpha^*(y^m))\; C^m B \\ \tau_2^T(\alpha^*(y^m))\; C^m B \\ \tau_2^T(\alpha^*(y^{m-1}))\; C^{m-1} B \\ \cdots \\ \tau_2^T(\alpha^*(y^2))\; C^2 B \end{vmatrix} \tag{26}$$

and takes the form

$$u \;=\; C^{-1}(q, y^2, \ldots, y^m) \begin{vmatrix} u_s \\ u_e \end{vmatrix}, \tag{27}$$

where $u_e = \{u_e^j\}$, $j = 2, \ldots, m$.

Thus, the resulting control law is represented by the local controllers (23)-(24) and the transformation (27).

4. SIMULATION RESULTS

In order to illustrate the efficiency of the proposed control strategy, simulations of multi-link manipulators were conducted by using the special software package RSim-2D. The problem consists in proportional motion of the robot end point along time-varying straight lines and circles. Additionally, it was required to provide well-coordinated motion of the other links that insured the compactness and required configuration of the kinematic chain.

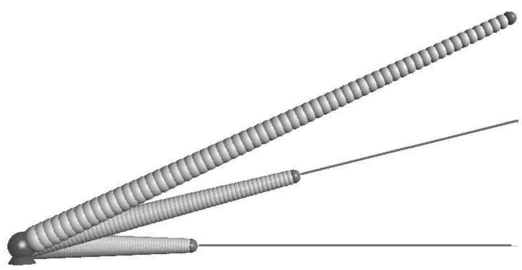

Fig. 2. Motion along moving straight line

The simulation results are illustrated by Fig. 2–3. Different stages the 64-link robot motion along a moving straight line are shown in Fig. 2, and its displacement on a circle are represented in Fig. 3.

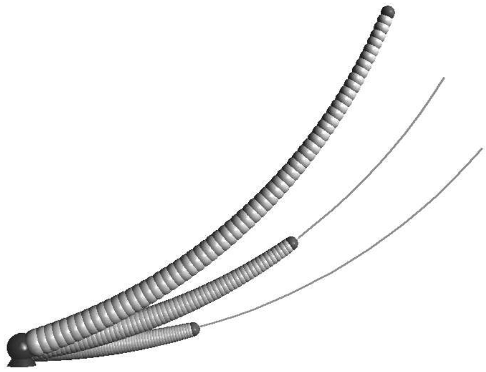

Fig. 3. Motion along moving arc

The phases (a)–(c) of a more complex technological task of the insertion of the 50-link robot body into the hole in a bounded operation zone are shown in the Fig. 4.

The simulation confirms the perfectly coordinated behavior of the dexterous robots and compact configuration of their chains in the course of the complex trajectory motion.

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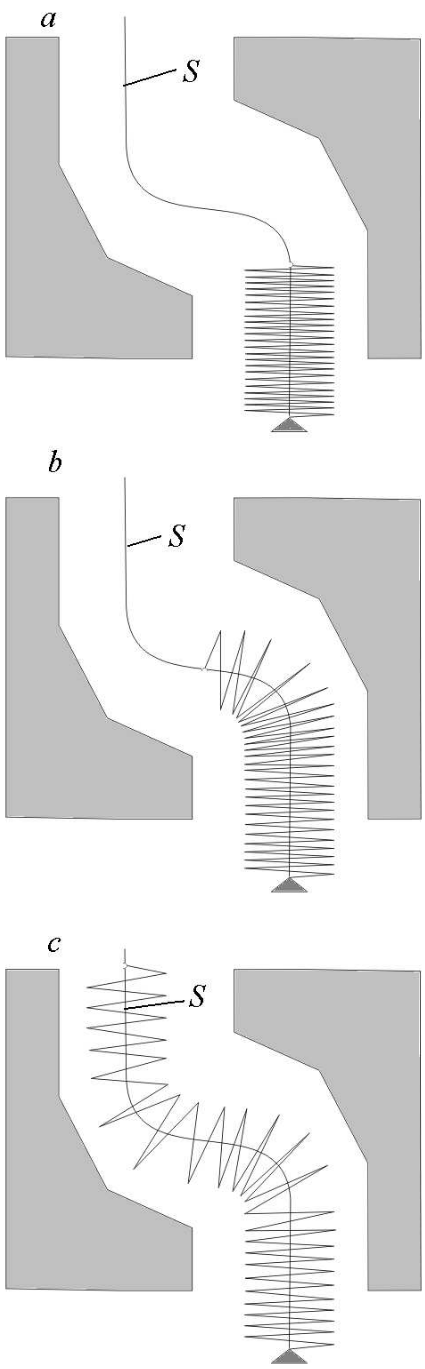

Fig. 4. Penetrating into the hole

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