ADAPTIVE NEURAL-FUZZY CONTROL OF UNCERTAIN NONHOLONOMIC SYSTEMS

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Abstract: An adaptive neural-fuzzy controller is presented in this paper for mechanical systems with nonholonomic constraints in the presence of uncertainties about plant parameters. The controller is designed based on a reduced model. The neural-fuzzy (NF) controller is constructed in order to eliminate the need for dynamic modeling and error prone process in obtaining the regressor matrix. The proposed controller guarantees that the system motion asymptotically converges to the desired manifold. Numerical simulation are conducted to verify the effectiveness of the proposed method.

Keywords: Adaptive control, Neural-fuzzy, Nonholonomic, Constrained systems

1. INTRODUCTION

In recent years, much attention has been devoted to the problem of controlling nonholonomic systems. Many mechanical systems (such as wheeled mobile robots, tractor-trailer systems, free-floating space robots, underwater vehicles, etc.) are subjected to nonholonomic velocity constraints (Kolmanovsky and McClamroch, 1995)(Campion *et al.*, 1991). Due to Brockett's theorem (Brockett, 1983), it is well known that nonholonomic systems with restricted mobility cannot be stabilized to a desired configuration via differentiable, or even continuous, pure-state feedback (Bloch *et al.*, 1992). Instead, by using decomposition transformations and nonlinear feedbacks, conditions for smooth asymptotic stabilization to an equilibrium manifold can be established (Campion *et al.*, 1991). This makes the stabilization problem of nonholonomic systems one of the most challenging topics in control theory and applications.

Up to now, most research work on controller design for nonholonomic systems has being focused on the kinematic control problem, where the systems are represented by their kinematic models and velocity acts as the control input. In practice, however, it is more realistic to formulate the nonholonomic system control problem at the dynamic level, where the torque and force are taken as the control inputs. Different researchers have investigated this problem. Several results have been published in recent years wherein motion control design of nonholonomic mechanical systems has been successfully treated (Bloch *et al.*, 1992)(You and Chen, 1993). In these designs, the dynamic models were assumed to be perfect, and exactly known. All these methods depend on the exact cancelation of the robot dynamics to achieve the control objective.

In real applications, however, perfect cancelation of the robot dynamics is rarely possible. Recently, the control problem for nonholonomic mechanical systems with parametric uncertainties have been studied (Su and Stepanenko, 1995)(Chang and Chen, 2000)(Ge *et al.*, 2001)(Wang *et al.*, 2004).

In order to cope with highly uncertain nonlinear systems, as an alternative, adaptive approximation based control is presented to solve the problem. NF is a neural network-based fuzzy logic control and decision system, and is suitable for online nonlinear systems identification and control (Lin and Lee, 1991). It brings the learning abilities of the neural networks to automate and realize the design of fuzzy logic control systems. Therefore, in contrast to the pure neural networks or fuzzy logic, the NF possesses both of their advantages. In this paper, the parameterized NF approximator is expressed as a series of the commonly used Radial Basis Function (RBF) because of its nice approximation properties, i.e., $y_{NF} = \sum_j w_j \phi(\sigma_j, \|\mathbf{z} - c_j\|)$, where w_j is the connection weight, and c_j and σ_j are the center and width respectively, which determine the shape of function ϕ. In RBF approximation, tuning of the parameters c_j and σ_j is not a trivial task since they both appear nonlinearly. In this paper, a novel parameter updating law of w_j, c_j and σ_j derived based on Lyapunov synthesis is presented to design stable NF controller guaranteeing the closed-loop stability for a class of the nonholonomic constrained systems. It is shown that the system motion converges to the desired manifold asymptotically. In real implementation, the center and width of the NF approximator can be fixed easily by choosing the updating gain to zero according to different system configuration and requirements.

2. LINEARLY PARAMETERIZED NEURO-FUZZY SYSTEMS

Fuzzy systems are rule-based systems. A typical format of a fuzzy system consists of a collection of fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. The purpose of fuzzifier is to provide scale mapping of the crisp input corresponding to the linguistic forms as labeled by a fuzzy set. Fuzzy inference engine is the kernel of fuzzy system and uses the fuzzy IF-THEN rules to determine a mapping from input universe to output universe based on fuzzy logic policies. The TSK-type fuzzy rule (Takagi and Sugeno, 1985) used here is in the following form:

$$R^l : IF\ z_1\ is\ F_1^l\ AND\ z_2\ is\ F_2^l \cdots\ AND$$
$$z_{n_z}\ is\ F_{n_z}^l, THEN\ y^l = k_0^l + k_1^l z_1 + \cdots + k_{n_z}^l z_{n_z}$$

where F_i^l $(i = 1, 2, \dots, n_z)$ are fuzzy sets, k_j^l $(j = 0, 1, \dots, n_z)$ are real-valued parameters, $z = [z_1, z_2, \dots, z_{n_z}]^T$ is the system input, and y^l is the fuzzy system output due to the l-th rule R^l $(l = 1, 2, \dots, N)$. In this paper, the zero-order TSK-fuzzy system is chosen, i.e., $y^l = k_0^l$. Finally, defuzzifier is used as the scale mapping of the linguistic value into a corresponding crisp output value.

The NF is a multilayer feedforward network that integrates the TSK-type fuzzy system and RBF neural network into a connectionist structure. It consists of four layers as shown in Figure 1.

Layer 1: This layer is the input layer, whose nodes just transmit the input variables z to the next layer directly.

Layer 2: This layer is the membership function layer that receives the signals from the input layer and calculates the membership of the input variables. The membership function chosen in this paper is the Gaussian membership function as described by:

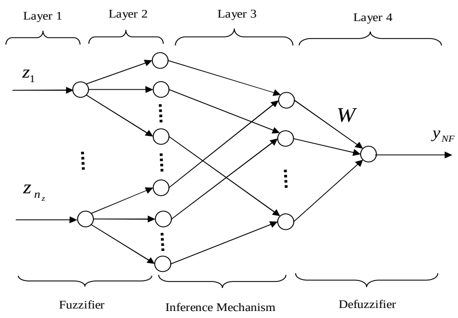

Fig. 1. The structure of NF system

$$\mu_{li}(z_i, c_{li}, \sigma_{li}) = \exp\left(-\frac{(z_i - c_{li})^2}{\sigma_{li}^2}\right) \qquad (1)$$

where $i = 1, 2, \cdots, n_z$, and $l = 1, 2, \cdots, N$.

Layer 3: This layer is the rule layer. The number of the nodes in this layer represents the number of fuzzy rules. It computes the fired strength of a rule as following

$$\kappa_l(z, c_l, \sigma_l) = \prod_{i=1}^{n_z} \mu_{li},\ l = 1, 2, \cdots, N \qquad (2)$$

where $c_l = [c_{l1}, \cdots, c_{ln_z}]^T$ and $\sigma_l = [\sigma_{l1}, \cdots, \sigma_{ln_z}]^T$.

Layer 4: This layer is the output layer, where defuzzification of the TSK-type NF system is performed. The output of the whole NF system is given by:

$$\begin{aligned} y_{NF} &= \sum_{l=1}^{N} \phi_l y^l \\ &= \sum_{l=1}^{N} \left[\frac{\prod_{i=1}^{n_z} \exp\left(-\frac{(z_i - c_{li})^2}{\sigma_{li}^2}\right)}{\sum_{l=1}^{N} \prod_{i=1}^{n_z} \exp\left(-\frac{(z_i - c_{li})^2}{\sigma_{li}^2}\right)} \right] k_0^l \\ &= W^T \Phi(z, c, \sigma) \end{aligned} \qquad (3)$$

where

$$\phi_l(z, c_l, \sigma_l) = \kappa_l / \sum_{l=1}^{N} \kappa_l \qquad (4)$$

$$\Phi(z, c, \sigma) = [\phi_1, \phi_2, \cdots, \phi_N]^T \qquad (5)$$

$$W = [k_0^1, k_0^2, \cdots, k_0^N]^T \qquad (6)$$

with $c = [c_1^T, \cdots, c_N^T]^T$ and $\sigma = [\sigma_1^T, \cdots, \sigma_N^T]^T$.

It has been proved by the universal approximation theorem (Wang, 1992) that, if the number of fuzzy rules N is sufficiently large, $W^T\Phi(z, c, \sigma)$ can approximate any continuous function, $g(z)$, to any arbitrary accuracy over a compact set $\Omega_z \subset \mathbf{R}^{n_z}$, i.e.

$$g(z) = W^{*T}\Phi(z, c^*, \sigma^*) + \varepsilon_g(z),\ \forall z \in \Omega_z \subset \mathbf{R}^{n_z}$$

where $\varepsilon_g(z)$ is the approximation error which is bounded over the compact set Ω_z and W^*, c^* and σ^* are defined as the values of W, c and σ that minimize $|\varepsilon_g|$ for all $z \in \Omega_z \subset \mathbf{R}^{n_z}$.

Let $\hat{W}$, $\hat{c}$ and $\hat{\sigma}$ be the estimates of W^*, c^* and σ^*, the following results are useful to characterize the approximation error of the NF system.

Assumption 1. On the compact set Ω_z, the ideal NF parameter vectors W^*, c^*, σ^* are bounded by

$$\|W^*\| \leq w_{max}, \quad \|c^*\| \leq c_{max}, \quad \|\sigma^*\| \leq \sigma_{max}$$

with w_{max}, c_{max} and σ_{max} being positive constants and the approximation error is bounded by $|\varepsilon_g(z)| \leq \varepsilon_g^*$.

Lemma 1. (Jia *et al.*, 2004) NF approximation error can be expressed as

$$\begin{aligned}&\hat{W}^T\Phi(z,\hat{c},\hat{\sigma}) - W^{*T}\Phi(z,c^*,\sigma^*) = \\ &\tilde{W}^T(\hat{\Phi} - \hat{\Phi}'_c\hat{c} - \hat{\Phi}'_\sigma\hat{\sigma}) + \hat{W}^T(\hat{\Phi}'_c\tilde{c} + \hat{\Phi}'_\sigma\tilde{\sigma}) + d_u \quad (7)\end{aligned}$$

where $\hat{\Phi} = \Phi(z,\hat{c},\hat{\sigma})$, $\tilde{W} = \hat{W} - W^*$, $\tilde{c} = \hat{c} - c^*$ and $\tilde{\sigma} = \hat{\sigma} - \sigma^*$ are defined to be the estimation errors, $\hat{\Phi}'_c = [\hat{\phi}_{c1}', \hat{\phi}_{c2}', \cdots, \hat{\phi}_{cN}']^T \in \mathbf{R}^{N\times(Nn_z)}$ and $\hat{\Phi}'_\sigma = [\hat{\phi}_{\sigma 1}', \hat{\phi}_{\sigma 2}', \cdots, \hat{\phi}_{\sigma N}']^T \in \mathbf{R}^{N\times(Nn_z)}$, where

$$\hat{\phi}_{ci}' = \left[\frac{\partial\phi_i}{\partial c_{11}}, \ldots, \frac{\partial\phi_i}{\partial c_{1n_z}}, \ldots, \frac{\partial\phi_i}{\partial c_{N1}}, \ldots, \frac{\partial\phi_i}{\partial c_{Nn_z}}\right]$$

$$\hat{\phi}_{\sigma i}' = \left[\frac{\partial\phi_i}{\partial \sigma_{11}}, \ldots, \frac{\partial\phi_i}{\partial \sigma_{1n_z}}, \ldots, \frac{\partial\phi_i}{\partial \sigma_{N1}}, \ldots, \frac{\partial\phi_i}{\partial \sigma_{Nn_z}}\right]$$

for $c = \hat{c}$, $\sigma = \hat{\sigma}$, $i = 1, 2, \ldots, N$, and d_u is bounded by

$$\begin{aligned}|d_u| \leq\; & c_{max}\|\hat{W}^T\hat{\Phi}'_c\| + \sigma_{max}\|\hat{W}^T\hat{\Phi}'_\sigma\| \\ & + w_{max}(1 + \|\hat{\Phi}'_c\hat{c}\| + \|\hat{\Phi}'_\sigma\hat{\sigma}\|) \quad (8)\end{aligned}$$

3. SYSTEM DESCRIPTIONS

According to the Euler-Lagrangian formulation, the joint-space dynamics of an n-dimensional constrained mechanical system can be described as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\tau + f \quad (9)$$

where $q = [q_1, \ \ldots, \ q_n]^T \in R^n$ denotes the vector of generalized coordinates; $\tau \in R^r$ is the vector of generalized control input force; $f \in R^n$ denotes the vector of constraint forces; $D(q) \in R^{n\times n}$ is the symmetric bounded positive definite inertia matrix; $C(\dot{q}, q)\dot{q} \in R^n$ denotes the Centripetal and Coriolis torques; $G(q) \in R^n$ is the gravitational torque vector; $B \in R^{n\times r}$ is a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of the system.

Property 1. : Matrix $(\dot{D} - 2C)$ is skew-symmetric if all the elements of matrix $C(q,\dot{q})$ are defined in the Christoffel form (Ge *et al.*, 1998).

When the system is subjected to nonholonomic constraint, the m nonintegrable and independent velocity constraints can be expressed as

$$J(q)\dot{q} = 0 \quad (10)$$

where $J : R^n \to R^{m\times n}$ is the kinematic constraint matrix which is assume to have full rank m.

The constraint (10) is referred to the classical nonholonomic constraint when it is not integrable. In the paper, constraint (10) is assumed to be completely nonholonomic and exactly known. The effect of the constraints can be viewed as restricting the dynamics on the manifold Ω_{nh} as

$$\Omega_{nh} = \{(q,\dot{q}) | J(q)\dot{q} = 0\}$$

It is noted that since the nonholonomic constraint (10) is nonintegrable, there is no explicit restriction on the values of the configuration variables.

Based on the nonholonomic constraint (10), the generalized constraint forces in mechanical system (9) can be given by

$$f = J^T(q)\lambda$$

where $\lambda \in R^m$ is known as friction force on the contact point between the rigid body and environmental surfaces.

Denote the kinematic constraint matrix $J(q)$ as

$$J^T(q) = [J_1(q), \ \ldots, \ J_m(q)]$$

where $J_1(q), \ \ldots, \ J_m(q)$ are smooth n-dimensional covector fields on R. Assume that the annihilator of the co-distribution spanned by the covector fields $J_1(q), \ \ldots, \ J_m(q)$ is an $(n-m)$-dimensional smooth nonsingular distribution Δ on R. This distribution Δ is spanned by a set of $(n-m)$ smooth and linearly independent vector fields $r_1(q), \ \ldots, \ r_{n-m}(q)$, i.e., $\Delta = span\{r_1(q), \ \ldots, \ r_{n-m}(q)\}$. Then the following relations are satisfied

$$R^T(q)J^T(q) = 0 \quad (11)$$

where $R(q) = [r_1(q), \ \ldots, \ r_{n-m}(q)] \in R^{n\times(n-m)}$. Constraints (10) and (11) imply the existence of vector $\dot{x} \in R^{n-m}$, such that

$$\dot{q} = R(q)\dot{x} \quad (12)$$

Differentiating (12), we obtain

$$\ddot{q} = R\ddot{x} + \dot{R}\dot{x}$$

The dynamic equation (9), which satisfies the nonholonomic constraint (10), can be rewritten in terms of the internal state variable $\dot{x}$ as

$$\begin{aligned}& D(q)R(q)\ddot{x} + C_2(q,\dot{q})\dot{x} + G(q) \\ &= B(q)\tau + J^T(q)\lambda \quad (13)\end{aligned}$$

where $C_2(q,\dot{q}) = D(q)\dot{R}(q) + C(q,\dot{q})R(q)$.

Property 2. : Matrix $D_R(q)$ is symmetric and positive-definite.

Property 3. : $N_R = \dot{D}_R(q) - 2C_R(q,\dot{q})$ is skew-symmetric.

Property 4. : $D(q)$, $G(q)$, $J(q)$ and $R(q)$ are bounded and continuous if x is bounded and uniformly continuous. $C(q,\dot{q})$ and $\dot{R}(q)$ are bounded if $\dot{x}$ is bounded. $C(q,\dot{q})$ and $\dot{R}(q)$ are uniformly continuous if $\dot{x}$ is uniformly continuous. (Chang and Chen, 2000)

Remark 1. System (13) is the so called reduced form of nonholonomic mechanical system. Since constraint (10) has been embedded into the dynamic equation (9), (13) is suitable for the subsequent controller design. It should be noted that reduced state space is $(2n-m)$ dimensional (Su and Stepanenko, 1995). The system is described by the n-vector q and $(n-m)$-vector $\dot{x}$ which represents the system internal states.

4. ADAPTIVE NEURAL-FUZZY CONTROL DESIGN

In this section, adaptive neural-fuzzy control for mechanical systems with nonholonomic constraints subject to plant uncertainties and external disturbances is considered.

Consider the constrained dynamic equation (9) together with m independent nonholonomic constraints (10). For simplicity of design, the following assumptions required through out this section.

Assumption 2. : The matrix $R^T(q)B(q)$ is of full rank, which guarantees all $n-m$ degrees of freedom can be (independently) actuated.

The above assumption always holds for a large class of nonholonomic mechanical systems such as nonholonomic Caplygin systems (which include a vertical wheel rolling without slipping on a plane surface, a mobile wheeled robot moving on a horizontal plane, and a knife edge moving in point contact on a plane surface, etc. In these systems, the internal state $\dot{x}(q)$ and variable $x(q)$ possess practical physical meanings.)

By appropriate selecting a set of $(n-m)$ vector of variables $x(q)$ and $\dot{x}(q)$, the control objective can be specified as: given a desired x_d and $\dot{x}_d$, determine a control law such that for any $(q(0),\dot{q}(0)) \in \Omega$ then $x(q)$ and $\dot{x}$ asymptotically converge to a manifold Ω_{nhd} specified as

$$\Omega_{nhd} = \{(q,\dot{q},\lambda) | x(q) = x_d, \dot{q} = R(q)\dot{x}_d\} \quad (14)$$

The variable $x(q)$ can be thought as $(n-m)$ "output equations" of the nonholonomic system.

Assumption 3. : The desired reference trajectory $x_d(t)$ is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order.

In the following, we define $e_x = x - x_d$, $\dot{x}_r = \dot{x}_d - \rho_1 e_x$, $s = \dot{e}_x + \rho_1 e_x$, where $\dot{x}_r$ is the reference trajectory described in internal state space.

To facilitate the analysis of neural networks, the *GL* matrix and its product operator introduced in (Ge *et al.*, 1998) are used. Denote the *GL* vectors and matrices by $\{\cdot\}$, and the *GL* product operator by "$\bullet$". To avoid any possible confusion, $[\cdot]$ is used to denote the conventional vector and matrix.

For controller design, define the following new variables $\mu = Rs$, $\nu = R\dot{x}_r$. The time derivatives of ν and σ are given by $\dot{\nu} = \dot{R}\dot{x}_r + R\ddot{x}_r$, and $\dot{\mu} = \dot{R}s + R\ddot{s}$.

Consider the control law given by

$$\begin{aligned}\tau = (R^TB)^{-1}R^T[\hat{D}(q)\dot{\nu} + \hat{C}(q,\dot{q})\nu + \hat{G}(q) \\ -K_\mu\mu - K_s\text{sgn}(\mu)] \quad (15)\end{aligned}$$

where the estimates $(\hat{*})$ required in (15) be provided by NF such that

$$\hat{D}(q) = [\{\hat{W}_D\}^T \bullet \{\hat{\phi}_D\}] \quad (16)$$
$$\hat{C}(q,\dot{q}) = [\{\hat{W}_C\}^T \bullet \{\hat{\phi}_C\}] \quad (17)$$
$$\hat{G}(q) = [\{\hat{W}_G\}^T \bullet \{\hat{\phi}_G\}] \quad (18)$$

and

$$D(q) = [\{W_D^*\}^T \bullet \{\phi_D^*\}] + \epsilon_D$$
$$C(q,\dot{q}) = [\{W_C^*\}^T \bullet \{\phi_C^*\}] + \epsilon_C$$
$$G(q) = [\{W_G^*\}^T \bullet \{\phi_G^*\}] + \epsilon_G$$

where $[\{W_D^*\},\{\phi_D^*\}]$, $[\{W_C^*\},\{\phi_C^*\}]$ and $[\{W_G^*\},\{\phi_G^*\}]$ are the desired parameter and basis function pairs of the NF emulation of $D(q)$, $C(q,\dot{q})$ and $G(q)$ respectively; and ϵ_D, ϵ_C, ϵ_G are the collective NF reconstruction errors respectively.

The system dynamics is rewritten as

$$\begin{aligned}R^TDR\dot{s} = R^TB\tau - R^T(D\dot{\nu} + C\nu + G) \\ -R^TC_2s \quad (19)\end{aligned}$$

Let

$$\{\xi_{D_k}\} = \{\xi_{D_{k1}}\ \xi_{D_{k2}}\ \ldots\ \xi_{D_{kn}}\}$$
$$\{\zeta_{D_k}\} = \{\zeta_{D_{k1}}\ \zeta_{D_{k2}}\ \ldots\ \zeta_{D_{kn}}\}$$
$$\{\eta_{D_k}\} = \{\eta_{D_{k1}}\ \eta_{D_{k2}}\ \ldots\ \eta_{D_{kn}}\}$$

where $\xi_{D_{ki}} = \hat{\phi}_{D_{ki}} - \hat{\phi}'_{D_{ki},c_{D_{ki}}}\hat{c}_{D_{ki}} - \hat{\phi}'_{D_{ki},\sigma_{D_{ki}}}\hat{\sigma}_{D_{ki}}$, $\zeta_{D_{ki}} = \hat{\phi}'_{D_{ki},c_{D_{ki}}}$, and $\eta_{D_{ki}} = \hat{\phi}'_{D_{ki},\sigma_{D_{ki}}}$.

Similarly, we have

$$\{\xi_{C_k}\} = \{\xi_{C_{k1}}\ \xi_{C_{k2}}\ \ldots\ \xi_{C_{kn}}\}$$
$$\{\zeta_{C_k}\} = \{\zeta_{C_{k1}}\ \zeta_{C_{k2}}\ \ldots\ \zeta_{C_{kn}}\}$$
$$\{\eta_{C_k}\} = \{\eta_{C_{k1}}\ \eta_{C_{k2}}\ \ldots\ \eta_{C_{kn}}\}$$
$$\{\xi_{G_k}\} = \{\xi_{G_{k1}}\ \xi_{G_{k2}}\ \ldots\ \xi_{G_{kn}}\}$$
$$\{\zeta_{G_k}\} = \{\zeta_{G_{k1}}\ \zeta_{G_{k2}}\ \ldots\ \zeta_{G_{kn}}\}$$
$$\{\eta_{G_k}\} = \{\eta_{G_{k1}}\ \eta_{G_{k2}}\ \ldots\ \eta_{G_{kn}}\}$$

where $\xi_{C_{ki}} = \hat{\phi}_{C_{ki}} - \hat{\phi}'_{C_{ki},c_{C_{ki}}}\hat{c}_{C_{ki}} - \hat{\phi}'_{C_{ki},\sigma_{C_{ki}}}\hat{\sigma}_{C_{ki}}$, $\zeta_{C_{ki}} = \hat{\phi}'_{C_{ki},c_{C_{ki}}}$, $\eta_{C_{ki}} = \hat{\phi}'_{C_{ki},\sigma_{C_{ki}}}$, $\xi_{G_{ki}} = \hat{\phi}_{G_{ki}} -$

$\hat{\phi}'_{G_{ki},c_{G_{ki}}}\hat{c}_{G_{ki}} - \hat{\phi}'_{G_{ki},\sigma_{G_{ki}}}\hat{\sigma}_{G_{ki}}$, $\zeta_{G_{ki}} = \hat{\phi}'_{G_{ki},c_{G_{ki}}}$, and $\eta_{G_{ki}} = \hat{\phi}'_{G_{ki},\sigma_{G_{ki}}}$.

Using the control law (15), the closed-loop system error equation can be obtained

$$\begin{aligned}
&R^T D R\dot{s} \\
&= R^T([\{\tilde{W}_D\}^T \bullet \{\xi_D\} - \{\hat{W}_D\}^T \bullet \{\{\zeta_D\} \bullet \{\tilde{c}_D\}\} \\
&\quad -\{\hat{W}_D\}^T \bullet \{\{\eta_D\} \bullet \{\tilde{\sigma}_D\}\}]\dot{\nu} + [\{\tilde{W}_C\}^T \bullet \{\xi_C\} \\
&\quad -\{\hat{W}_C\}^T \bullet \{\{\zeta_C\} \bullet \{\tilde{c}_C\}\} \\
&\quad -\{\hat{W}_C\}^T \bullet \{\{\eta_C\} \bullet \{\tilde{\sigma}_C\}\}]\nu + [\{\tilde{W}_G\}^T \bullet \{\xi_G\} \\
&\quad -\{\hat{W}_G\}^T \bullet \{\{\zeta_G \bullet \{\tilde{c}_G\}\} - K_\mu \mu + d - C_2 s \\
&\quad -\{\hat{W}_G\}^T \bullet \{\{\eta_G \bullet \{\tilde{\sigma}_G\}\}] - K_s \mathrm{sgn}(\mu)) \qquad (20)
\end{aligned}$$

where $d = (\epsilon_D + d_D)\dot{\nu} + (\epsilon_C + d_C)\nu + \epsilon_G + d_G$.

Theorem 1. : For the closed-loop system (20), asymptotic stability, i.e., e_x and $\dot{e}_x$ asymptotically converge to zero, is achieved if K_σ is positive definite, $K_s = \mathrm{diag}[K_{si}]$, $K_{si} \geq ||d||$ and the parameter adaptation laws are given by

$$\dot{\hat{W}}_{D_k} = -\Gamma_{D_k} \bullet \{\xi_{D_k}\}\dot{\nu}\mu_k \qquad (21)$$

$$\dot{\hat{c}}_{D_k} = -\Gamma_{c_{D_k}} \bullet \{\{\zeta_{D_k}\}^T \bullet \{\hat{W}_{D_k}\}\}\dot{\nu}\mu_k \qquad (22)$$

$$\dot{\hat{\sigma}}_{D_k} = -\Gamma_{\sigma_{D_k}} \bullet \{\{\eta_{D_k}\}^T \bullet \{\hat{W}_{D_k}\}\}\dot{\nu}\mu_k \qquad (23)$$

$$\dot{\hat{W}}_{C_k} = -\Gamma_{C_k} \bullet \{\xi_{C_k}\}\nu\mu_k \qquad (24)$$

$$\dot{\hat{c}}_{C_k} = -\Gamma_{c_{C_k}} \bullet \{\{\zeta_{C_k}\}^T \bullet \{\hat{W}_{C_k}\}\}\nu\mu_k \qquad (25)$$

$$\dot{\hat{\sigma}}_{C_k} = -\Gamma_{\sigma_{C_k}} \bullet \{\{\eta_{C_k}\}^T \bullet \{\hat{W}_{C_k}\}\}\nu\mu_k \qquad (26)$$

$$\dot{\hat{W}}_{G_k} = -\Gamma_{G_k} \bullet \{\xi_{G_k}\}\mu_k \qquad (27)$$

$$\dot{\hat{c}}_{G_k} = -\Gamma_{c_{G_k}} \bullet \{\{\zeta_{G_k}\}^T \bullet \{\hat{W}_{G_k}\}\}\mu_k \qquad (28)$$

$$\dot{\hat{\sigma}}_{G_k} = -\Gamma_{\sigma_{G_k}} \bullet \{\{\eta_{G_k}\}^T \bullet \{\hat{W}_{G_k}\}\}\mu_k \qquad (29)$$

where Γ_{D_k}, $\Gamma_{c_{D_k}}$, $\Gamma_{\sigma_{D_k}}$, Γ_{C_k}, $\Gamma_{c_{C_k}}$, $\Gamma_{\sigma_{C_k}}$, Γ_{G_k}, $\Gamma_{c_{G_k}}$, and $\Gamma_{\sigma_{G_k}}$ are dimensional compatible symmetric positive definite matrices; moreover, all the closed-loop signals are bounded.

Proof:

Choose the Lyapunov function candidate

$$\begin{aligned}
V &= \frac{1}{2}s^T R^T D R s + \frac{1}{2}\sum_{i=1}^{n} \tilde{W}_{D_k}^T \Gamma_{D_k}^{-1} \tilde{W}_{D_k} \\
&\quad + \frac{1}{2}\sum_{i=1}^{n} \tilde{W}_{C_k}^T \Gamma_{C_k}^{-1} \tilde{W}_{C_k} + \frac{1}{2}\sum_{i=1}^{n} \tilde{W}_{G_k}^T \Gamma_{G_k}^{-1} \tilde{W}_{G_k} \\
&\quad + \frac{1}{2}\sum_{i=1}^{n} \tilde{c}_{D_k}^T \Gamma_{c_{D_k}}^{-1} \tilde{c}_{D_k} + \frac{1}{2}\sum_{i=1}^{n} \tilde{c}_{C_k}^T \Gamma_{c_{C_k}}^{-1} \tilde{c}_{C_k} \\
&\quad + \frac{1}{2}\sum_{i=1}^{n} \tilde{c}_{G_k}^T \Gamma_{c_{G_k}}^{-1} \tilde{c}_{G_k} + \frac{1}{2}\sum_{i=1}^{n} \tilde{\sigma}_{D_k}^T \Gamma_{\sigma_{D_k}}^{-1} \tilde{\sigma}_{D_k} \\
&\quad + \frac{1}{2}\sum_{i=1}^{n} \tilde{\sigma}_{C_k}^T \Gamma_{\sigma_{C_k}}^{-1} \tilde{\sigma}_{C_k} + \frac{1}{2}\sum_{i=1}^{n} \tilde{\sigma}_{G_k}^T \Gamma_{\sigma_{G_k}}^{-1} \tilde{\sigma}_{G_k} \qquad (30)
\end{aligned}$$

By virtue of equation (20), substituting the adaptation law (21)-(29) into the time derivative of V, and after some simple calculations using the GL operator, yields

$$\dot{V} = -s^T R^T K_\mu R s + \mu^T d - \mu^T K_s \mathrm{sgn}(\mu) \leq 0 \qquad (31)$$

where the skew-symmetric property of $\dot{D} - 2C$, and $\dot{\tilde{W}}_{D_k} = \dot{\hat{W}}_{D_k}$, $\dot{\tilde{W}}_{C_k} = \dot{\hat{W}}_{C_k}$, $\dot{\tilde{W}}_{G_k} = \dot{\hat{W}}_{G_k}$, $\dot{\tilde{c}}_{D_k} = \dot{\hat{c}}_{D_k}$, $\dot{\tilde{c}}_{C_k} = \dot{\hat{c}}_{C_k}$, $\dot{\tilde{c}}_{G_k} = \dot{\hat{c}}_{G_k}$, and $\dot{\tilde{\sigma}}_{D_k} = \dot{\hat{\sigma}}_{D_k}$, $\dot{\tilde{\sigma}}_{C_k} = \dot{\hat{\sigma}}_{C_k}$, $\dot{\tilde{\sigma}}_{G_k} = \dot{\hat{\sigma}}_{G_k}$ are used.

As $V \geq 0$ and $\dot{V} \leq 0$, $V \in L_\infty$. From the definition of V, it follows that s is bounded and $\tilde{W}_{D_k}, \tilde{W}_{C_k}, \tilde{W}_{G_k}, \tilde{c}_{D_k}, \tilde{c}_{C_k}, \tilde{c}_{G_k}, \tilde{\sigma}_{D_k}, \tilde{\sigma}_{C_k}$ and $\tilde{\sigma}_{G_k}$ are all bounded. Thus, we have all the estimates $\hat{W}_{D_k}, \hat{W}_{C_k}, \hat{W}_{G_k}$, $\hat{c}_{D_k}, \hat{c}_{C_k}, \hat{c}_{G_k}, \hat{\sigma}_{D_k}, \hat{\sigma}_{C_k}$ and $\hat{\sigma}_{G_k}$ are bounded. It can be obtained that $e_x, \dot{e}_x \in L_\infty^{n-m}$.

Integrating both sides of (31), we have

$$\int_0^t s^T R^T K_\sigma R s \leq V(0) - V(t) \leq V(0)$$

Thus, $s \in L_2^{n-m}$. From (13), we can conclude $\ddot{x} \in L_\infty^{n-m}$, which leads to $\ddot{q} \in L_\infty^n$. Since $\dot{x}, \ddot{x} \in L_\infty^{n-m}$ have been established before, we can conclude that $\dot{s} \in L_\infty^{n-m}$. Now, with $s \in L_2^{n-m}$, $\dot{s} \in L_\infty^{n-m}$, we can conclude that s asymptotically converge to zero. Hence it can be concluded that $e_x, \dot{e}_x \to 0$ as $t \to \infty$. Q.E.D.

5. SIMULATION RESULTS

A simplified model of a mobile wheeled robot moving on a horizontal plane, constituted by a rigid trolley equipped with nondeformable wheel, as given in details in (Campion *et al.*, 1991), is used to verify the validity of the proposed control approach.

The dynamic model can be expressed as (Campion *et al.*, 1991)

$$\begin{aligned}
m\ddot{x} &= \lambda \cos\theta - \frac{1}{P}(\tau_1 + \tau_2)\sin\theta \\
m\ddot{y} &= \lambda \sin\theta + \frac{1}{P}(\tau_1 + \tau_2)\cos\theta \\
I\ddot{\theta} &= \frac{L}{P}(\tau_1 - \tau_2) \qquad (32)
\end{aligned}$$

where x, y are coordinates in an inertial frame, θ is an orientation of the wheel with respect to the inertial frame, m is the mass of the robot, and I is its inertial moment around the vertical axis, P is the radius of the wheels and $2L$ the length of the front wheels, and $\tau_i, i = 1,\ 2$ is the torques provided by the motors. For simplicity, we set $P = L = 1$.

The nonholonomic constraint is written as $\dot{x}\cos\theta + \dot{y}\sin\theta = 0$. The matrix $J(q)$ is therefore defined as $J(q) = [\cos\theta \ \ \sin\theta \ \ 0]$, where $q = [x\ y\ \theta\ ]^T$. The "outputs" are chosen as $x(q) = [y \ \ \theta\ ]^T$.

In order to simulate, it can be easily derived $D(q) = \mathrm{diag}[m,\ m,\ I], C(q, \dot{q}) = 0$ and $G(q) = 0$.

In the simulation, we assume the real parameter $m = 0.5$, and $I = 0.5$. With the initial condition $q(0) = [0,\ 0.5,\ 0.785]^T$ and $\dot{q}(0) = [0,\ 0,\ 0]^T$ and the desired manifold Ω_{nhd} is chosen as $\Omega_{nhd} = \{(q, \dot{q}, \lambda)|x(q) = 0, \dot{q} = 0\}$. The proposed NF control action is calculated by (15), and the input of the NF approximator is given by $z = [y,\ \theta]$. The following design parameters are chosen for the simplest case in the simulation: each neural-fuzzy approximator contains 25 nodes, with centers c_l evenly spanned in $[-2,\ 2]$, the width $\sigma_l = 0.5$ and $\hat{W}_D = 0$. By Theorem 1, the control gain K_μ is selected as $K_\sigma = \mathrm{diag}[1]$, $\rho_1 = 1$. The adaptation gain in adaptation law (21) is chosen as $\Gamma_{D_k} = \mathrm{diag}[0.5]$. In the simulation, K_s is chosen to be zero to show the robustness of the proposed controller.

The results of the simulation are shown in Figs. 2-3. Fig. 2 shows the responses, including y, θ, $\dot{x}$, $\dot{y}$ and $\dot{\theta}$, of the simulated nonholonomic constrained robot. The torques exerted at the mobile robot are given by Fig. 3. It can be seen that all system states converge to the desired manifold Ω_{nhd} and all signals in closed-loop are bounded. These results verify the validity of the proposed algorithm.

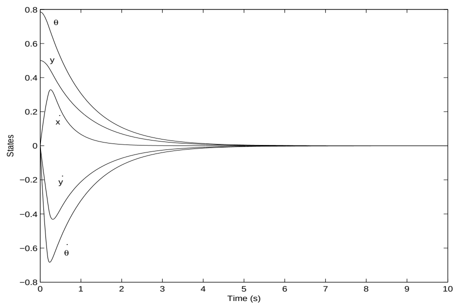

Fig. 2. Simulated nonholonomic system's responses

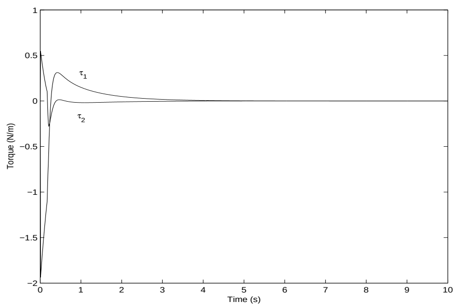

Fig. 3. Control torques of the simulated nonholonomic system

6. CONCLUSION

In this paper, the problem of control of mechanical systems with classical nonholonomic constraints subject to dynamic uncertainties is considered. An adaptive neural-fuzzy control algorithm has been designed to drive the system motion converge to the desired manifold. The proposed controllers are non-regressor based and require no information on the system dynamics. Simulation results have shown the effectiveness of the proposed controller.

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