PERFORMANCE EVALUATION AND OPTIMIZATION OF A TWO-STAGE PRODUCTION-DISTRIBUTION SYSTEM WITH BATCH ORDERS AND FINITE TRANSPORTATION TIME

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Abstract: This paper proposes an analytical method for performance evaluation and optimization of a production-distribution system composed of a warehouse supplied from an upstream manufacturing plant. Customer orders arrive randomly with random order size and the production capacity is finite. Transportation time from plant to warehouse is fixed. The analytical approach needs only the first two moments of random variables of the system to evaluate the order-to-delivery lead-time of the warehouse, the total inventory on order, the inventory holding, backlogging cost and fill rate. We use a gradient-based method to minimize the average total inventory cost subject to fill rate requirement. Numerical comparisons with simulation show that the analytical approach is very efficient. *Copyright © 2005 IFAC*

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1. INTRODUCTION

A production-distribution system can be viewed as a network of manufacturing plants, warehouses and customers through which orders and products flow. The dynamic of such a system is triggered by customer orders and it is subject to constraints such as the production capacity, inventory costs and transportation lead time. Numerous performance evaluation and optimization approaches have been developed in order to evaluate and help decision making about inventory levels and service targets. A detailed presentation of the main inventory control policies can be found in (Zipkin,2000). Due to the page limitation, we limit ourselves to a short review of papers most related to our work.

The approaches proposed in (Ettl, *et al.*, 2000) and (Liu, *et al.*,2004) are most closely related to this paper. An approach for performance evaluation and optimization of supply chains is introduced in (Ettl, *et al.*, 2000). The supply chain considered in it is composed of a set of inter-related stores and each store location is modeled by an $M^x/G/\infty$ queuing system. The authors derive analytical expressions for performance measures such as the store lead time, the

service level and the inventory cost. They proposed a constraint nonlinear optimization model to minimize the inventory cost while satisfying customer service-level requirements. The main disadvantage of this approach is that it does not consider the production capacity, which is a key constraint in production-distribution systems.

In (Liu, *et al.*,2004) the authors developed a multi-stage inventory queue systems. A job-queue decomposition approach was proposed in order to evaluate the performance of serial manufacturing and supply systems with control at every stage. An efficient procedure is proposed to minimize the inventory cost. The model is based on GI/G/1 queue, with products arriving one at a time. Batch sizes and transportation times are not considered.

The motivation of our work is to provide efficient methodologies for performance evaluation and optimization of realistic supply chains network models with features such as random batch orders, finite production capacities and transportation times. The work presented in this paper is the first step in analyzing a basic supply network with a manufacturing plant and a warehouse.

The approach that we propose deals with performance evaluation of a two-stage production-distribution system with a warehouse supplying from a production plant. Customer orders arrive at the warehouse randomly. A customer order is filled if there is enough inventory at the warehouse and is backlogged if the inventory is empty. The inventory at the warehouse is controlled by a base-stock policy and replenishment orders are sent to the production plant upon arrival of customer orders. The production capacity of the plant is finite. Transportation from plant to warehouse is performed on the basis of replenishment orders and the transportation time is constant. To the best of our knowledge, in the literature, there is no inventory policy optimization approach related to this problem.

An analytical approach is proposed for performance evaluation based on several approximation techniques that are proved efficient for most manufacturing systems: (i) two-moment approximation, (ii) markovian approximation of general processes, (iii) approximation of dependent random processes by independent ones. More specifically, the analytical approach needs only the first two moments of random variables of the system to evaluate the order-to-delivery lead time of the warehouse, the total inventory on order and the inventory holding and backlogging cost. The production plant is modeled as a $M^X/G/1$ queueing system and the transportation system is represented as a $M^X/D/\infty$ queueing system. Approximate analytical expressions are derived for estimation of the first and/or second moments of order-to-delivery lead time and inventory on order. A log-normal distribution is then used to approximate the distribution of the inventory on order and to determine the total inventory holding/backlogging cost of the warehouse. The inventory cost is then minimized using a gradient based method while respecting a given service level. Numerical comparisons with simulation show that the analytical approach is very efficient and it is not sensitive on the workload of the production plant.

The reminder of the paper is organized as follows. The two-stage production-distribution system we consider is formally defined in Section 2. The analytical approach is presented in Section 3. In Section 4, numerical results are given to compare the analytical approach and the simulation. Concluding remarks are given in Section 5.

2 THE PRODUCTION-DISTRIBUTION SYSTEM

This paper considers a two-stage production-distribution system with an upstream production plant supplying a downstream warehouse (see Figure 1). Only one type of products is considered. The warehouse faces batch customer arrival processes and generates orders to the plant. The plant processes orders from the warehouse according to its available capacity and on a FIFO basis. Any order filled at the plant is then send to warehouse and the transportation time is significant and cannot be neglected. Detailed description of the system is given in the following.
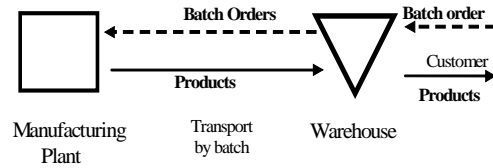

Fig. 1. The system topology

Customer orders arrive randomly according to a compound Poisson process with arrival rate λ. The quantity of each order is a random positive integer variable X and the quantities of different orders are iid random variables. Assume that the order quantity X has finite first two moments with mean μ_X and standard deviation σ_X. Upon the arrival of customer order of size X, if the on-hand inventory of the warehouse is enough, the order is totally filled. Otherwise, the customer order is only partially filled according to the on-hand inventory level. The remaining quantity is backlogged and will be filled by future product deliveries from the plant. Consequently, customer orders can be split.

The inventory of the warehouse is managed following a base-stock control policy. This policy relies on the concept of inventory position which is equal to on-hand inventory at the warehouse minus the backlogged quantity plus the total pending order quantities. The base stock policy keeps the inventory position constant at the so-called base-stock level R. As a result, whenever a customer order of size X arrives, the warehouse immediately issues a replenishment order of size X to the plant. As a result, the replenishment orders arriving at the plant also form a compound Poisson process.

The manufacturing plant is modeled by a single-server queueing system. Replenishment orders arrive according to a compound Poisson process and wait in a FIFO queue for production at the plant. The plant or server serves each batch order on a unit basis. The processing time of each product unit is a random variable τ of general distribution with mean m_τ and standard deviation σ_τ. The total service time of a replenishment order X depends on its order size. The manufacturing plant is modeled as an $M^x/G/1$ queueing system. The total time elapsed between the arrival of a replenishment order at the plant and the moment when the order is ready from delivery to the warehouse is the plant lead time, denoted as L_s. The delivery of products from plant to warehouse is made on the basis of replenishment orders. When a replenishment order of size X is ready at the plant, the batch of X product units is transported to the warehouse. The time needed to transport a batch from the plant to the warehouse is a given constant

called transportation lead time L_t. The order-to-delivery lead time of the warehouse, denoted by L, represents the time elapsed between sending an order to the plant and receiving at the warehouse the products of the order. It is defined by the sum of the plant lead time and the transport lead time, i.e. $L = L_s + L_t$.

Another important issue in inventory management is the inventory cost. Holding a unit of product in the warehouse incurs a cost of C_h per time unit. Demands waiting to be filled incur a cost of C_b per waiting time unit for each product unit of backlogged orders.

3. PERFORMANCE EVALUATION

This section addresses the performance and cost evaluation of the production-distribution system. We first evaluate the first and/or second moments of the following performance variables:

- L_s: lead-time at the plant for a typical order unit (not batch) issued by the warehouse
- N_s: total number of units waiting to be or being processed at the plant
- N_t: total number of units in transportation to the warehouse
- L: order-to-delivery lead-time for a typical order unit issued by the warehouse
- $N = N_s + N_t$: total number of units on order also called inventory on order.

These performance variables are then used in the evaluation of the inventory and backlogging cost.

The replenishment orders issued by the warehouse are not split till delivery at the warehouse. As a result, we sometimes call a replenishment order a batch when no confusion is possible.

3.1 The manufacturing plant model

As mentioned before, the plant can be modeled by a $M^X/G/1$ queuing system. As the warehouse is managed according to a base-stock policy, replenishment orders arriving at the manufacturing plant follows the same compound Poisson process as the customer orders. Let us first consider the service time T_B of a replenishment order.

$$T_B = \Sigma_{i=1}^{X} \tau_i \tag{1}$$

where X is the size of the replenishment order and τ_i is the service time of i-th units of the order. By assumption, X and τ_i are mutually independent random variables. Hence,

$$m_{TB} = E[T_B] = m_\tau m_x \tag{2}$$

where $m_\tau := E[\tau]$ and $m_x := E[X]$. By conditioning on X and by mutual independence of X and τ_i,

$$E[T_B^2] = E[E[(\Sigma_{i=1}^{X} \tau_i)^2 \mid X]]$$
$$= E[X \cdot E[\tau^2] + X(X-1)(E[\tau])^2] \tag{3}$$
$$= m_x \sigma_\tau^2 + m_\tau^2(\sigma_x^2 + m_x^2)$$

where, $\sigma_\tau^2 := Var[\tau]$ and $\sigma_x^2 := Var[X]$.

Combining relations (2) and (3),

$$\sigma_{TB}^2 := Var(T_B) = m_x \sigma_\tau^2 + m_\tau^2 \sigma_x^2 \tag{4}$$

Consider first replenishment orders as basic customers of the queueing system. The $M^X/G/1$ queuing system then becomes a $M/G/1$ queuing system with T_B as service time. The mean number N_B of batches in the plant can be computed by the Pollazek-Khinchin formula (Cassandras,1993):

$$E(N_B) = \frac{\rho}{1-\rho} - \frac{\rho^2}{2(1-\rho)}(1 - \sigma_{TB}^2/m_{TB}^2) \tag{5}$$

where ρ is the traffic intensity for the queuing system defined as follows :

$$\rho = \lambda m_{TB} \tag{6}$$

From (2)-(6):

$$E(N_B) = \frac{\lambda(2m_x m_\tau - m_x^2 m_\tau^2 \lambda + \lambda m_x \sigma_\tau^2 + \lambda \sigma_x^2 m_\tau^2)}{2(1 - m_x m_\tau \lambda)} \tag{7}$$

By Little's law, the mean of the total time L_B a replenishment order spends at the plant is :

$$E[L_B] = E[N_B]/\lambda \tag{8}$$

Note that $E[L_B]$ is the average time a batch spends at the plant and $E[L_s]$ is a average time a product unit ordered by customers spends at the plant. These two concepts are different. Although the time of a product unit in a particular customer order (or batch) of size X spends in the pant is exactly the same as the time the related batch spends in the plant, i.e. $L_s = L_B$, however $E[L_B]$ and $E[L_s]$ differ according to the distribution of the order quantity X. Fortunately, according to large number of numerical experiences, $E[L_B]$ and $E[L_s]$ are very close for most realistic distribution of X. For this reason, the following approximation is used:

$$E[L_s] \approx E[L_B] \tag{9}$$

Apply Little's law to product units, we have:

$$E[N_s] = \lambda E[X] \cdot E[L_s] \approx m_x E[N_B] \tag{10}$$

Consider now the second moment of the performance indicators. First,

$$\sigma_S^2 := Var[N_S] = E[N_S^2] - E[N_S]^2 \tag{11}$$

The second moment of the number of products at the plant is given by the following expression:

$$E[N_s^2] = E[E[(\sum_{i=1}^{N_B} X_i^2 + \sum_{\substack{i,j \\ i \neq j}} X_i X_j) \mid N_B]]$$

where X_i is the number of units of replenishment order i. In general, N_B depends on the size X_1 of the batch being served and $X_1 \mid N_B$ does not have the same distribution as X_i with $i \neq 1$. By assuming the independence of N_B and X_1 and by assuming that

$X_1 | N_B$ and X_i with $i \neq 1$ are iid random variables, the following approximation is obtained:

$$E[N_s^2] \approx E[N_B.E[X^2] + N_B(N_B-1).E^2[X]]$$
$$= E[N_B].E[X^2] + (E[N_B^2] - E[N_B]).E^2[X]] \quad (12)$$

What remains is to estimate the second moment of the queue length N_B of an M/G/1 queue. Given the service time distribution of the M/G/1 queue, generating function of the queue length of M/G/1 can be used to compute the second moment of N_B by using the third moment of service time (Cooper, et al., 1981;Buzacott, et al., 1993).

$$E[N_B^2] = \frac{2\lambda^3 E[T_B^3] + 3\lambda^2 E[T_B^2](3-2\rho)}{6(1-\rho)} + \frac{\lambda^2 \rho E[T_B^2]}{6(1-\rho)} + \frac{\lambda^4 E^2[T_B^2]}{2(1-\rho)^2} + \rho \quad (13)$$

where $E[T_B^3]$ is the third moment of the service time for one batch, and can be represented as:

$$E[T_B^3] = E[E[(\sum_{i=1}^{X} \tau_i)^3 | X]]$$
$$= E[X]E[\tau^3] + 3(E[X^2] - E[X])E[\tau]E[\tau^2]$$
$$+ (E[X^3] - 3E[X^2] + 2E[X])E^3[\tau]$$

As the purpose of this paper is to propose a performance evaluation by using only the first two moments of any random variable, the approach presented in (Cooper, et al., 1981;Buzacott, et al., 1993) can not be used directly. Instead we propose to approximate the distribution of service time for one unit by a log-normal distribution (Saporta,1990) with mean m_τ and variance σ_τ^2. Then we can evaluate the third moment of the service time for one unit $E[\tau^3]$ using the mean and variance of unit service time. This implies that the random variable $\log(\tau)$ follows a normal distribution of mean M and standard deviation S :

$$m_\tau = e^{M + S^2/2} \quad (14)$$

$$\sigma_\tau^2 = e^{S^2 + 2M}(e^{S^2} - 1) \quad (15)$$

$$E[\tau^3] = \sqrt{e^S - 1}(2 + e^{S^2})\sigma_\tau^3 + 3E[\tau^2]E[\tau] - 2E^3[\tau] \quad (16)$$

We obtain $S^2 = \ln(\sigma_\tau^2/m_\tau^2 + 1)$ by the first two formulas, and then we can represent $E[\tau^3]$ by the mean m_τ and variance σ_τ^2 of unit service time.

3.2 The transport system model

The purpose of this subsection is to evaluate the distribution of the number N_t of product units in transportation. Notice that the arrival process at the transportation system is the departure process of replenishment orders from the plant and it is in general not a Poisson process. Let us approximate the arrival process of batches at the transportation system

as a Poisson process. The transportation system can then be approximated by a $M^X/D/\infty$ queuing system (Liu, et al.,1990;Masuyama, and Takine,2002) with a fixed transportation time L_t.

As the average arrival rate of product units at the transportation system is $\lambda E[X]$ and the transportation delay is L_t. From Little's Law,

$$m_t := E[N_t] = \lambda E[X].L_t = \lambda m_x L_t \quad (17)$$

The second moment of N_t is as follows:

$$E[N_t^2] = E\left[\left(\sum_{i=1}^{N_{Bt}} X_i\right)^2\right]$$

where N_{Bt} is the number of batches in transportation. The $M^X/D/\infty$ approximation implies the mutual independence of N_{Bt} and X_i:

$$E[N_t^2] = E\left[E\left[\left(\sum_{i=1}^{N_{Bt}} X_i\right)^2 | N_{Bt}\right]\right]$$
$$= E\left[N_{Bt}E[X^2] + (N_{Bt}^2 - N_{Bt})E^2[X]\right]$$
$$= E[N_{Bt}]Var(X) + E[N_{Bt}^2]E^2[X]$$

N_{Bt} is the number of Poisson arrivals in a time interval of length L_t with $E[N_{Bt}] = \lambda L_t$ and $E[N_{Bt}^2] = (\lambda L_t)^2 + \lambda L_t$. Introducing these terms in the above equations leads to:

$$E[N_t^2] = \lambda L_t Var(X) + (\lambda L_t + (\lambda L_t)^2)E^2[X]$$

and

$$\sigma_t^2 := Var(N_t) = E[N_t^2] - E^2[N_t] = \lambda L_t(\sigma_x^2 + m_x^2) \quad (18)$$

3.3. Order to delivery lead-time and inventory on order

We are now ready to evaluate the order to delivery lead-time $L = L_s + L_t$ and the total inventory on order $N = N_s + N_t$. The following results are immediate:

$$E[L] = E[L_s] + L_t \quad (19)$$

$$m_N := E[N] = E[N_s] + E[N_t] \quad (20)$$

Notice that N_s and N_t are in general dependent. An approximation of the second moment of N is obtained by considering N_s and N_t as two independent random variables.

$$\sigma_N^2 := \sigma_s^2 + \sigma_t^2 \quad (21)$$

3.4 Average inventory cost estimation

By definition of the base stock policy, the following relation holds:

$$R = I - B + N \quad (22)$$

where R is the base stock level, I is the random inventory on hand, B is the backlogged quantities and N is the total inventory on order. Since customer

orders can be partially filled, either $I = 0$ or $B = 0$. As a result,

$$I = (R-N)^+ \qquad (23)$$
$$B = (N-R)^+ \qquad (24)$$

where $(x)^+ = \max\{x, 0\}$. Note that the net inventory level $IN = I - B = R - N$.

The total inventory cost is composed of two parts: the holding cost and the stock out penalty. The total inventory cost is:

$$C(R) = C_h E[I] + C_b E[B] \qquad (25)$$

Combining (23) – (25),

$$C(R) = \int_0^\infty g(R-x) f_N(x) dx \qquad (26)$$

where $f_N(x)$ is the pdf of the inventory on order N and

$$g(x) = \begin{cases} C_h x, & \text{if } x \geq 0 \\ -C_b x, & \text{otherwise.} \end{cases}$$

Deriving the distribution function $f_N(x)$ of N is a very difficult task. In this paper, we propose to approximate $f_N(x)$ by a log-normal distribution (Saporta,1990) with mean m_N and variance σ^2_N:

$$f_N(x) \approx \frac{1}{Sx\sqrt{2\pi}} e^{-(\ln x - M)^2/(2S^2)} \qquad (27)$$

With $S^2 = \ln(\dfrac{\sigma_N^2}{m_N^2} + 1)$ and $M = \ln m_N - S^2/2$.

The use of log-normal approximation instead of usual normal approximation is explained in Section 4 and is mainly due to the large variance of N with respect of its mean. Combining (26) and (27) and from the definition of $g(x)$,

$$C(R) = \int_{-\infty}^\infty g(R - e^{M+S.z}) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \qquad (28)$$

$$C(R) = C_h \int_{-\infty}^{Z^*} (R - e^{M+S.z}) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$
$$+ C_b \int_{Z^*}^\infty (e^{M+S.z} - R) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \qquad (29)$$

where $\mathbf{z}^*$ is the solution of equation $R - exp(M + Sz) = 0$.

3.5 Fill rate estimation

For the warehouse which directly supplies external customer demand, the fill rate is an important indicator of the required customer service level, and strongly depends on the base stock level. In this paper, we define it as the percentage of order units filled from stock without delay. Let X denote the quantity of an order of a particular customer, Because of the property of Poisson arrival see average, the inventory level observed by this customer follows the stationary distribution of X and the fill rate can be expressed as:

$$P(X \leq I) \qquad (30)$$

where I is the inventory on hand of warehouse. From Section 3.4,

$$P(X \leq I) = \sum_{i=1}^\infty P(N \leq R-i).P_x(X=i)$$

where P_x is probability density function of order quantities, From (28):

$$P(N \leq R-i) = \int_0^{R-i} f_N(N) dN = \int_{-\infty}^{z'} f(z) dz$$

where z' is the solution of equation
$$(R-i) - exp(M + Sz) = 0.$$

3.6 The optimization model

The objective of our optimization model is to minimize the total inventory cost as specified in (29) while satisfying customer service-level requirement expressed as $P(X \leq I) > \alpha$. Let us have a closer look at the objective function (29). $R - e^{M+S.z}$ is increasing in R. Let us denote $Y = R - e^{M+S.z}$, observe (27), $g(Y)$ is linear convex function of R. So

$$g(R - e^{M+S.z}) \quad \text{and} \quad \int_{-\infty}^\infty g(R - e^{M+S.z}) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

are both convex in R. Because of this property of objective function, we use gradient-based method to solve the constrained optimization problem with

$$d\frac{C(R)}{dR}$$

$$= C_h(R - e^{M+S.Z^*}) \frac{1}{\sqrt{2\pi}} e^{-z^{*2}/2} \frac{dz}{dR} + C_h \int_{-\infty}^{Z^*} \frac{1}{\sqrt{2\pi}} e^{-z^{*2}/2} dz$$

$$+ C_b(e^{M+S.Z^*} - R) \frac{1}{\sqrt{2\pi}} e^{-z^{*2}/2} \frac{dz}{dR} - C_b \int_{Z^*}^\infty \frac{1}{\sqrt{2\pi}} e^{-z^{*2}/2} dz$$

$$= C_h \int_{-\infty}^{z^*} f(z) dz - C_b \int_{z^*}^\infty f(z)$$

$$= C_h \Phi(z^*) - C_b(1 - \Phi(z^*)) \qquad (31)$$

where $f(z)$ is the pdf of standard normal distribution and $\Phi(z)$ is the distribution function of Z.

4. NUMERICAL RESULTS

The aim of this section is to validate by simulation the analytical results presented in the previous section. Numerical experimentation is conducted on one example defined as follows. The arrival rate λ of customer orders varies from 1 to 1.6. The batch size of each customer order X is an integer uniformly distributed in (Kleinrock,1975;Buzacott, and Shanthikumar,1993). The service time τ for each product unit is exponentially distributed with mean equal to 0.1. The transportation time is 3.0. Both of unit inventory holding cost C_h and unit backlogging cost C_b are 1. Base stock level is set at R = 50. In order to obtain faithful simulation results, a very long simulation time of 10,000,000 time units is used. Results of both analytical approach and the simulation are given in Tables 1 and 2. Although only the first two moments of each random variable are taken into account, the analytical approach provides very good fits moment estimation of order-

to-delivery lead-time and the total inventory on order. Even though a number of approximations are made in the analytical approach, the second moment estimation of the inventory on order N is very good with error no more than 3.5%. Finally the log-normal approximation leads to good estimation of the total inventory cost. Another interesting phenomenon is that the quality of the result seems to be rather insensitive to the traffic intensity ρ.

Note that we also tried the normal distribution approximation of the inventory on order N. The results are bad especially when the traffic intensity is high. This is mainly due to the large variance of N with respect its mean.

Table 1 Simulation vs. approximation results for production lead-time Ls and order-to-delivery time L

Arrival rate	ρ	E[Ls] (sim)	E[Ls] (analyt)	Error (%)	E[L] (sim)	E[L] (analyt)	Error (%)
1.0	0.60	1.1730	1.1750	0.1706	4.1730	4.1750	0.0480
1.3	0.78	1.9542	1.9591	0.2506	4.9542	4.9591	0.0991
1.5	0.90	4.0304	4.0500	0.4842	7.0304	7.0500	0.2790
1.6	0.96	9.6031	9.8000	2.0092	12.603	12.800	1.5623

Table 2 Simulation vs. approximation results for inventory on order and inventory cost

Arrival rate	ρ	m_N (sim)	m_N (analyt)	Error (%)	σ_N (sim)	σ_N (analyt)	Error (%)	Cost (sim)	Cost (analyt)	Error (%)
1.0	0.60	25.4324	25.0500	1.5266	14.1190	13.9651	1.1023	25.5746	26.4905	3.5811
1.3	0.78	39.1543	38.6809	1.2237	20.7695	20.6483	0.5873	19.4611	19.7240	1.3510
1.5	0.90	63.8623	63.4500	0.6498	39.3581	39.5504	0.4867	28.6332	27.6893	3.2966
1.6	0.96	121.597	122.880	1.0442	92.2559	95.6122	3.5103	77.7748	77.5252	0.3208

Table 3 Simulation vs. approximation results for fill rate and inventory cost with optimal R*

Arrival rate	ρ	Service Level (%)	optimal R*	Fill rate (sim)	Fill rate (analyt)	Error (%)	Cost (sim)	Cost (analyt)	Error (%)
1.0	0.60	90	50	0.9014	0.9092	0.8572	25.5746	26.4905	3.5811
1.3	0.78	90	71	0.8954	0.9034	0.5448	34.2482	35.1262	2.5635
1.5	0.90	90	119	0.8962	0.9020	0.7371	61.6398	62.2328	0.9621
1.6	0.96	90	241	0.8902	0.9008	1.1813	137.756	137.901	0.1056

To minimize the objective function *C(R)*, we use the conjugate gradient method [Press, *et al.*,1994], which is a standard technique for nonlinear optimization. For constraint function, we use binary search method to estimate the bound of variable *R*. Numerical results are given in table 3.

5. CONCLUSION

This paper proposed an analytical model to estimate the performances of a two stages production-distribution system characterized by i) a random client demand, ii) a finite manufacturing capacity; iii) a random batch size for orders, manufacturing and delivery quantities; iv) a constant transport time from the plant to the warehouse. Our future research work consists in extending the approach to a network of production-distribution systems with random transportation time.

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