ROBUST MIXED H_2/H_∞ CONTROL FOR SYSTEMS WITH STOCHASTIC NONLINEARITY [1]

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Abstract: In this paper, the problem of mixed H_2/H_∞ control is considered for a class of uncertain discrete-time nonlinear stochastic systems. The nonlinearities are described by statistical means of the stochastic variables, and the uncertainties are represented by deterministic norm-bounded parameter perturbations. The mixed H_2/H_∞ control problem is formulated in terms of the notion of exponentially mean-square quadratic stability, and the characterizations of both the H_2 control performance and the H_∞ robustness performance. A new technique is developed to deal with the matrix trace terms arising from the stochastic nonlinearities, and the well-known S-procedure is adopted to handle the deterministic uncertainties. A unified framework is established to solve the addressed mixed H_2/H_∞ control problem by using a linear matrix inequality (LMI) approach.

Keywords: Mixed H_2/H_∞ control; stochastic nonlinearity; deterministic uncertainty; linear matrix inequality

1. INTRODUCTION

In engineering practice, it is always welcome to design a controller that achieves multiple objectives. A typical example is the mixed H_2/H_∞ control scheme, which attempts to capture the benefits of both the H_2 control performance and the H_∞ robustness performance, simultaneously. In general, a pure H_2 controller is designed for a good measure of transient performance (Chen and Zhou, 2001), while a pure H_∞ control framework is developed for robustness with respect to disturbances and system uncertainties. Therefore, the mixed H_2/H_∞ multiobjective design framework has a better and clearer physical interpretation, and has received much attention from the control research community in the past few decades.

Since stochastic modeling has been playing a more and more important role in engineering designs, the stochastic H_∞ control problem has attracted growing research attention recently. Many research results have been available which, unfortunately, are mainly for linear stochastic systems. In (Hinrichsen and Pritchard, 1998), a stochastic bounded real lemma has been developed to solve the H_∞ control problem for stochastic linear systems with state- and control-dependent noises. The results have then been extended to the H_∞ control problem for discrete-time stochastic linear systems with state- and control-dependent noises(Bouhtouri et al., 1999). A robust stochas-

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tic H_∞ control problem has been addressed in (Ugrinovskii, 1998) to deal with the systems in the presence of stochastic uncertainty. Very recently, a stochastic mixed H_2/H_∞ control problem has been considered for the system with state-dependent noises in (Chen and Zhang, 2004), where sufficient conditions have been provided in terms of the existence of the solutions of cross-coupled Riccati equations. However, despite its importance, there are very few results on the mixed H_2/H_∞ control problem for nonlinear stochastic systems with or without parameter uncertainties, primarily because of the mathematical complexities. This situation motivates us to tackle a general class of uncertain nonlinear stochastic systems with mixed H_2/H_∞ control performance constraints.

The purpose of this paper is to develop an LMI approach to solving the mixed H_2/H_∞ control problem for a class of uncertain discrete time nonlinear stochastic systems. We aim to design a state feedback controller such that, for all admissible stochastic nonlinearities and deterministic uncertainties, the closed-loop system is exponentially mean-square quadratically stable, the H_2 control performance is achieved, and the prescribed disturbance attenuation level is guaranteed in an H_∞ sense. The nonlinearities considered in this paper, which are characterized by statistical means of the stochastic variables, are shown to be more general than many well-studied nonlinearities in the literature concerning nonlinear stochastic systems. The parameter uncertainties are assumed to be norm-bounded and enter the system matrices. A new technique is developed to deal with the matrix trace terms arising from the stochastic nonlinearities, and the well-known S-procedure is adopted to handle the deterministic uncertainties. The solution to the mixed H_2/H_∞ control problem is enforced within a unified LMI framework.

2. PROBLEM FORMULATION

Consider the following class of discrete-time systems with stochastic nonlinearities and deterministic norm-bounded parameter uncertainties:

$$\begin{aligned} x_{k+1} &= (A + H_1FE)x_k + f(x_k, u_k) + B_1w_k + B_2u_k, \\ z_{\infty k} &= L_\infty x_k, \\ z_{2k} &= L_2x_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^r$ is the control input, $z_{\infty k} \in \mathbb{R}^{p1}$ is a combination of the states to be controlled (with respect to H_∞-norm constraints), $z_{2k} \in \mathbb{R}^{p2}$ is another combination of the states to be controlled (with respect to H_2-norm constraints), $w_k \in \mathbb{R}^m$ is the process noise, which is a zero mean Gaussian white noise sequences with covariance R, and A, B_1, B_2, L_∞, L_2, H_1 and E are known real matrices with appropriate dimensions.

The matrix $F \in \mathbb{R}^{i\times j}$ represents the deterministic norm-bounded parameter uncertainties, i.e.

$$FF^T \leq I. \tag{2}$$

The deterministic uncertain matrix F is said to be admissible if it satisfies the condition (2).

The function $f(x_k, u_k)$: $\mathbb{R}^n \times \mathbb{R}^r \rightarrow \mathbb{R}^n$ is a stochastic nonlinear function of the states and control inputs, which is assumed to have the following first moment for all x_k and u_k:

$$\mathbb{E}\{f_k|x_k, u_k\} = 0, \tag{3}$$

with its covariance given by

$$\mathbb{E}\{f_kf_k^T|x_k, u_k\} = \sum_{i=1}^{q} \theta_i\theta_i^T(x_k^T\Gamma_ix_k + u_k^T\Pi_iu_k) \tag{4}$$

where θ_i $(i = 1, ..., q)$ is a known column vector, Γ_i and Π_i $(i = 1, ..., q)$ are known positive-definite matrices with appropriate dimensions.

We now consider the following state feedback controller for the system (1):

$$u_k = Kx_k, \tag{5}$$

where K is the state feedback gain to be determined.

The closed-loop system is governed as follows by substituting (5) into (1):

$$x_{k+1} = A_Kx_k + f(x_k, Kx_k) + B_1w_k, \tag{6}$$

where

$$A_K = A + B_2K + H_1FE. \tag{7}$$

Before giving our design goal, we introduce the following notion of exponentially quadratic stability in the mean-square sense for the closed-loop system (6).

Definition 1. The system (6) is said to be *exponentially mean-square quadratically stable* if, with $w_k = 0$, there exist constants $\alpha \geq 1$ and $\tau \in (0, 1)$ such that

$$\mathbb{E}\{\|x_k\|^2\} \leq \alpha\tau^k\mathbb{E}\{\|x_0\|^2\}, \quad \forall x_0 \in \mathbb{R}^n, \quad k \in \mathbb{I}^+, \tag{8}$$

for all admissible uncertainties satisfying (2).

The purpose of this paper is to seek a state feedback controller of the form (5), for the system (1), such that for all stochastic nonlinearities and all admissible deterministic uncertainties, the closed-loop system is exponentially mean-square quadratically stable, and additional H_2 control performance constraint and H_∞ robustness performance constraint are also satisfied. In other words, we aim to design a controller such that the closed-loop system satisfies the following requirements (Q1) and (Q2), simultaneously:

(Q1) For a given constant $\beta > 0$, the system (6) is exponentially mean-square quadratically stable and the following constraint is satisfied:

$$J_2 = \lim_{k\to\infty} \mathbb{E}\{\|z_{2k}\|^2\} < \beta. \quad (9)$$

(Q2) For a given $\gamma > 0$, the system (6) is exponentially mean-square quadratically stable and the following constraint is achieved:

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|z_{\infty k}\|^2\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|w_k\|^2\}, \quad (10)$$

for all nonzero w_k under zero initial condition.

3. ROBUST MIXED H_2/H_∞ ANALYSIS PROBLEM

In this section, we shall first discuss the H_2 control problem (Q1), then deal with the H_∞ control problem (Q2), and finally give the solution to the robust mixed H_2/H_∞ control problem for the system (6).

3.1 The H_2 control problem

To facilitate our discussion on the H_2 control problem (Q1), we need the following technical result.

Lemma 1. Let $V(x_k) = x_k^T P x_k$ be a Lyapunov functional where $P > 0$. If there exist positive real scalars λ, μ, ν, and $0 < \psi < 1$ such that

$$\mu\|x_k\|^2 \le V(x_k) \le \nu\|x_k\|^2, \quad (11)$$

and

$$\mathbb{E}\{V(x_{k+1})|x_k\} - V(x_k) \le \lambda - \psi V(x_k), \quad (12)$$

then the sequence x_k satisfies

$$\mathbb{E}\{\|x_k\|^2\} \le \frac{\nu}{\mu}\|x_0\|^2(1-\psi)^k + \frac{\lambda}{\mu\psi}. \quad (13)$$

According to Definition 1, we have the following theorem that provides sufficient conditions for the system (6) to be exponentially quadratically stable in the mean-square sense.

Theorem 1. Given the feedback gain matrix K. The system (6) is exponentially mean-square quadratically stable if, for all admissible uncertainties, there exists a positive definite matrix P satisfying

$$A_K^T P A_K - P + \sum_{i=1}^{q} (\Gamma_i + K^T \Pi_i K)\mathrm{tr}(\theta_i\theta_i^T P) < 0. \quad (14)$$

The stability analysis problem has been discussed in Theorem 1. Our next goal is to derive conditions for the H_2 performance constraint, (9), to be satisfied. Before proceeding, we need the following lemma.

Lemma 2. (Yaz and Yaz, 1999) If the system (6) is exponentially mean-square quadratically stable, then

$$\rho\{A_K \otimes A_K + \sum_{i=1}^{q} \mathrm{st}(\theta_i\theta_i^T)\mathrm{st}^T(\Gamma_i + K^T\Pi_i K)\} < 1, \quad (15)$$

or equivalently

$$\rho\{A_K^T \otimes A_K^T + \sum_{i=1}^{q} \mathrm{st}(\Gamma_i + K^T\Pi_i K)\mathrm{st}^T(\theta_i\theta_i^T)\} < 1, \quad (16)$$

where $\otimes$ is the Kronecker product of matrices; ρ is the spectral radius of a matrix, and st stands for the stack of a matrix that forms a vector out of the columns of the matrix.

Define the state covariance by

$$Q_k := \mathbb{E}\{x_k x_k^T\}$$

and then the Lyapunov-type equation that governs the evolution of the state covariance matrix Q_k can be derived from the system (6) and the relation (5) as follows:

$$Q_{k+1} = A_K Q_k A_K^T + \sum_{i=1}^{q} \theta_i\theta_i^T \mathrm{tr}[Q_k(\Gamma_i + K^T\Pi_i K)] + B_1 R B_1^T, \quad (17)$$

which can be rewritten (17) in the form of the stack matrix by:

$$\mathrm{st}(Q_{k+1}) = \Psi \cdot \mathrm{st}(Q_k) + \mathrm{st}(B_1 R B_1^T), \quad (18)$$

where

$$\Psi := A_K \otimes A_K + \sum_{i=1}^{q} \mathrm{st}(\theta_i\theta_i^T)\mathrm{st}^T(\Gamma_i + K^T\Pi_i K).$$

If the system (6) is exponentially mean-square quadratically stable, it follows from Lemma 2 that $\rho(\Psi) < 1$ and Q_k in (18) converges to a constant matrix Q when $k \to \infty$, i.e.

$$Q = \lim_{k\to\infty} Q_k. \quad (19)$$

Therefore, H_2 performance can be written by

$$J_2 = \lim_{k\to\infty} \mathbb{E}\{\|z_{2k}\|^2\} = \mathrm{tr}[L_2 Q L_2^T]. \quad (20)$$

In order to make sure that the H_2 performance and H_∞ performance can be tackled within the same framework by using a unified LMI approach, we will need to derive an alternative expression of the H_2 performance (20). Suppose now that there exists a matrix $\hat{P}_k > 0$ such that the following backward recursion is satisfied:

$$\hat{P}_k = A_K^T \hat{P}_{k+1} A_K + \sum_{i=1}^{q} (\Gamma_i + K^T\Pi_i K)\mathrm{tr}(\theta_i\theta_i^T \hat{P}_{k+1}) + L_2^T L_2, \quad (21)$$

which can be rearranged in terms of the stack operator as follows:

$$\mathrm{st}(\hat{P}_k) = \Phi \cdot \mathrm{st}(\hat{P}_{k+1}) + \mathrm{st}(L_2^T L_2), \tag{22}$$

where

$$\Phi := A_K^T \otimes A_K^T + \sum_{i=1}^{q} \mathrm{st}(\Gamma_i + K^T \Pi_i K)\mathrm{st}^T(\theta_i \theta_i^T).$$

If the system (6) is exponentially mean-square quadratically stable, then it follows from Lemma 2 that $\rho(\Phi) < 1$ and $\hat{P}_k$ in (22 converges to $\hat{P}$) when $k \to \infty$, i.e.

$$\hat{P} = \lim_{k\to\infty} \hat{P}_k. \tag{23}$$

Hence, in the steady state, (21) becomes:

$$\hat{P} = A_K^T \hat{P} A_K + \sum_{i=1}^{q} (\Gamma_i + K^T \Pi_i K)\mathrm{tr}(\theta_i \theta_i^T \hat{P}) + L_2^T L_2. \tag{24}$$

Summing up (21)-(24), we obtain the following result that gives an alternative to the H_2 performance, and facilitates our later consideration on the H_∞ performance constraint.

Theorem 2. If the system (6) is exponentially mean-square quadratically stable, H_2 performance can be expressed in terms of $\hat{P}$ as follows:

$$J_2 = \mathrm{tr}[RB_1^T \hat{P} B_1]. \tag{25}$$

where $\hat{P} > 0$ is the solution to (24).

Remark 1. We use (25) to compute the H_2 performance instead of (20). The reason is that the H_2 control performance and H_∞ robustness performance need to be characterized as a similar structure so that the solution to the mixed H_2/H_∞ control problem can be obtained by using a unified LMI approach. We will see in the next subsection that the structure of (25) is similar to that for the H_∞ robustness performance.

Notice that the system model in (1)-(1) involves parameter uncertainties, and hence the exact H_2 performance (25) cannot be obtained by simply solving the equation (24). One way to deal with this problem is to provide an upper bound for the H_2 performance. Suppose that there exists a positive definite matrix P such that the following matrix inequality is satisfied:

$$\triangle + L_2^T L_2 < 0. \tag{26}$$

where

$$\triangle := A_K^T P A_K - P + \sum_{i=1}^{q} (\Gamma_i + K^T \Pi_i K)\mathrm{tr}(\theta_i \theta_i^T P)$$

Before proving that the solution $P > 0$ to (26) is an upper bound for $\hat{P}$ in Theorem 3, we need the following lemma.

Lemma 3. Consider the system

$$\xi_{k+1} = M\xi_k + f(\xi_k), \tag{27}$$

where $\mathbb{E}\{f_k|\xi_k\} = 0$, and $\mathbb{E}\{f_k f_k^T|\xi_k\}=\sum_{i=1}^{q} \theta_i \theta_i^T (\xi_k^T \Xi_i \xi_k)$, θ_i $(i = 1, ..., q)$ are known column vectors, Ξ_i $(i = 1, ..., q)$ are known positive-definite matrices with appropriate dimensions. If the system (27) is exponentially mean-square stable, and there exists a symmetric matrix Y satisfying

$$M^T Y M - Y + \sum_{i=1}^{q} \Xi_i \mathrm{tr}(\theta_i \theta_i^T Y) < 0, \tag{28}$$

then $Y \geq 0$.

Now we are ready to give the upper bound for $\hat{P}$. Comparing (24) to (26), we obtain the following main result in this subsection.

Theorem 3. If there exists a positive definite matrix P satisfying (26), then the system (6) is exponentially mean-square quadratically stable,

$$\hat{P} \leq P, \tag{29}$$

and

$$\mathrm{tr}[RB_1^T \hat{P} B_1] \leq \mathrm{tr}[RB_1^T P B_1], \tag{30}$$

where $\hat{P} > 0$ satisfies (24).

The corollary given below follows immediately from Theorem 3 and (9).

Corollary 1. If there exists a positive definite matrix P satisfying (26) and $\mathrm{tr}[RB_1^T P B_1] < \beta$ where $\beta > 0$ is a given scalar, then the system (6) is exponentially mean-square quadratically stable, and (9) is satisfied for $\beta > 0$.

3.2 H_∞ control problem

Contrary to the standard H_∞ performance formulation, we shall use the expression (10) to describe the H_∞ performance of the stochastic system, where the expectation operator is utilized on both the controlled output and the disturbance input, see (Bouhtouri et al., 1999) for more details.

What we are going to do now is to derive sufficient conditions ensuring the H_∞-norm performance for the uncertain nonlinear stochastic system considered in this paper.

Theorem 4. Given a scalar $\gamma > 0$ and a feedback gain matrix K. The system (6) is exponentially mean-square quadratically stable and the H_∞-norm constraint (10) is achieved for all nonzero w_k, if there exists a positive definite matrix P satisfying

$$\begin{bmatrix} \triangle + L_\infty^T L_\infty & A_K^T P B_1 \\ B_1^T P A_K & B_1^T P B_1 - \gamma^2 I \end{bmatrix} < 0, \tag{31}$$

for all admissible uncertainties.

Up to now, the H_2 control problem and the H_∞ control problem have been considered separately. Before proceeding to the next section, we will need to discuss the mixed H_2/H_∞ analysis problem.

3.3 Robust mixed H_2/H_∞ analysis problem

According to the results obtained so far and from the conditions (14), (26) and (31), we summarize that (26) and (31) imply (14). Hence, in the mixed H_2/H_∞ design problem, (14) becomes redundant. In order to realize our design goals (Q1) and (Q2) simultaneously, it can be easily seen that the robust mixed H_2/H_∞ control problem addressed in Section 2 can be *restated* as follows.

Problem A: Design a controller (5) such that there exists a positive definite matrix P satisfying the following inequalities:

$$\mathrm{tr}[RB_1^T PB_1] < \beta, \tag{32}$$

$$\triangle + L_2^T L_2 < 0, \tag{33}$$

$$\begin{bmatrix} \triangle + L_\infty^T L_\infty & A_K^T PB_1 \\ B_1^T PA_K & B_1^T PB_1 - \gamma^2 I \end{bmatrix} < 0. \tag{34}$$

The purpose of *Problem A* is to find a controller (5) so as to ensure that (32)-(34) are satisfied for all admissible uncertainties, and subsequently the stability, the H_2 and H_∞ constraints are all achieved. Note that at this stage, such a problem is still complicated since the matrix trace terms and the uncertainty F are involved in (32)-(34). Our goal in the next section is therefore to develop an LMI approach to designing the desired controller based on (32)-(34).

4. ROBUST MIXED H_2/H_∞ CONTROLLER DESIGN

Before giving our main result, we recall the following useful lemmas.

Lemma 4. (S-procedure)(Boyd et al., 1994) Let $M = M^T$, H and E be real matrices of appropriate dimensions, with F satisfying (2), then

$$M + HFE + E^T F^T H^T < 0, \tag{35}$$

if and only if, there exists a positive scalar $\varepsilon > 0$ such that

$$M + \varepsilon HH^T + \frac{1}{\varepsilon} E^T E < 0, \tag{36}$$

or equivalently

$$\begin{bmatrix} M & \varepsilon H & E^T \\ \varepsilon H^T & -\varepsilon I & 0 \\ E & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{37}$$

In order to recast *Problem A* into a convex optimization problem, we first tackle the matrix trace terms in (32)-(34) by introducing new variables, which is actually one of the technical contributions in this paper. The following theorem presents sufficient conditions for solving *Problem A*.

Theorem 5. Given constants $\gamma > 0$, $\beta > 0$, and the feedback gain matrix K. If there exist positive definite matrix $P > 0$ and $\Theta > 0$, and positive scalars $\alpha_i > 0$ $(i = 1, \cdots, q)$ such that the following matrix inequalities

$$\mathrm{tr}(\Theta) < \beta, \tag{38}$$

$$\begin{bmatrix} -\Theta & R^{\frac{1}{2}} B_1^T \\ B_1 R^{\frac{1}{2}} & -P^{-1} \end{bmatrix} < 0, \tag{39}$$

$$\begin{bmatrix} -\alpha_i & \alpha_i \theta_i^T \\ \alpha_i \theta_i & -P^{-1} \end{bmatrix} < 0 \quad (i = 1, \cdots, q), \tag{40}$$

$$\begin{bmatrix} -P & A_K^T & \Gamma_1^{\frac{1}{2}} & \cdots & \Gamma_q^{\frac{1}{2}} & K^T & \cdots & K^T & L_2^T \\ A_K & -P^{-1} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \Gamma_1^{\frac{1}{2}} & 0 & -\alpha_1 I & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Gamma_q^{\frac{1}{2}} & 0 & 0 & \cdots & -\alpha_q I & 0 & \cdots & 0 & 0 \\ K & 0 & 0 & \cdots & 0 & -\alpha_1 \Pi_1^{-1} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ K & 0 & 0 & \cdots & 0 & 0 & \cdots & -\alpha_q \Pi_q^{-1} & 0 \\ L_2 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & -I \end{bmatrix} < 0, \tag{41}$$

$$\left[\begin{matrix} -P & 0 & A_K^T & \Gamma_1^{\frac{1}{2}} & \cdots \\ 0 & -\gamma^2 I & B_1^T & 0 & \cdots \\ A_K & B_1 & -P^{-1} & 0 & \cdots \\ \Gamma_1^{\frac{1}{2}} & 0 & 0 & -\alpha_1 I & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \Gamma_q^{\frac{1}{2}} & 0 & 0 & 0 & \cdots \\ K & 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ K & 0 & 0 & 0 & \cdots \\ L_\infty & 0 & 0 & 0 & \cdots \end{matrix}\right.$$

$$\left.\begin{matrix} \Gamma_q^{\frac{1}{2}} & K^T & \cdots & K^T & L_\infty^T & \\ 0 & 0 & \cdots & 0 & 0 & \\ 0 & 0 & \cdots & 0 & 0 & \\ 0 & 0 & \cdots & 0 & 0 & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \\ -\alpha_q I & 0 & \cdots & 0 & 0 & \\ 0 & -\alpha_1 \Pi_1^{-1} & \cdots & 0 & 0 & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \\ & 0 & 0 & \cdots & -\alpha_q \Pi_q^{-1} & 0 \\ 0 & 0 & \cdots & 0 & -I & \end{matrix}\right] < 0, \tag{42}$$

hold, then (32)-(34) are satisfied.

In the following, we will continue to "eliminate" the uncertainty F contained in (41) and (42) by using the well-known S-procedure technique, and then the desired robust mixed H_2/H_∞ controller could be obtained via an LMI approach by solving *Problem A*.

Theorem 6. Given constants $\gamma > 0$ and $\beta > 0$. If there exist positive-definite matrix $X > 0$ and $\Theta > 0$, a real matrix G, positive scalars $\alpha_i > 0$ $(i = 1, \cdots, q)$ and $\varepsilon_i > 0$ $(i = 1, 2)$ such that the following linear matrix inequalities

$$
\begin{aligned}
&\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \Theta \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T + \cdots \\
&+ \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \Theta \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T < \beta,
\end{aligned} \tag{43}
$$

$$
\begin{bmatrix} -\Theta & R^{\frac{1}{2}} B_1^T \\ B_1 R^{\frac{1}{2}} & -X \end{bmatrix} < 0, \tag{44}
$$

$$
\begin{bmatrix} -\alpha_i & \alpha_i \theta_i^T \\ \alpha_i \theta_i & -X \end{bmatrix} < 0 \quad (i = 1, \cdots, q), \tag{45}
$$

$$
\left[\begin{array}{cccccc}
-X & * & * & * & * & * \\
AX + B_2 G & -X & * & * & * & * \\
\Gamma_1^{\frac{1}{2}} X & 0 & -\alpha_1 I & * & * & * \\
\cdots & \cdots & \cdots & \cdots & * & * \\
\Gamma_q^{\frac{1}{2}} X & 0 & 0 & \cdots & -\alpha_q I & * \\
G & 0 & 0 & \cdots & 0 & -\alpha_1 \Pi_1^{-1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
G & 0 & 0 & \cdots & 0 & 0 \\
L_2 X & 0 & 0 & \cdots & 0 & 0 \\
0 & \varepsilon_1 H_1^T & 0 & \cdots & 0 & 0 \\
EX & 0 & 0 & \cdots & 0 & 0
\end{array}\right.
$$

$$
\left.\begin{array}{ccccc}
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
\cdots & * & * & * & * \\
\cdots & -\alpha_q \Pi_q^{-1} & * & * & * \\
\cdots & 0 & -I & * & * \\
\cdots & 0 & 0 & -\varepsilon_1 I & * \\
\cdots & 0 & 0 & 0 & -\varepsilon_1 I
\end{array}\right] < 0, \tag{46}
$$

$$
\left[\begin{array}{ccccccc}
-X & * & * & * & * & * & * \\
0 & -\gamma^2 I & * & * & * & * & * \\
AX + B_2 G & B_1 & -X & * & * & * & * \\
\Gamma_1^{\frac{1}{2}} X & 0 & 0 & -\alpha_1 I & * & * & * \\
\cdots & \cdots & \cdots & \cdots & \cdots & * & * \\
\Gamma_q^{\frac{1}{2}} X & 0 & 0 & 0 & \cdots & -\alpha_q I & * \\
G & 0 & 0 & 0 & \cdots & 0 & -\alpha_1 \Pi_1^{-1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
G & 0 & 0 & 0 & \cdots & 0 & 0 \\
L_\infty X & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \varepsilon_2 H_1^T & 0 & \cdots & 0 & 0 \\
EX & 0 & 0 & 0 & \cdots & 0 & 0
\end{array}\right.
$$

$$
\left.\begin{array}{ccccc}
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
\cdots & * & * & * & * \\
\cdots & -\alpha_q \Pi_q^{-1} & * & * & * \\
\cdots & 0 & -I & * & * \\
\cdots & 0 & 0 & -\varepsilon_2 I & * \\
\cdots & 0 & 0 & 0 & -\varepsilon_2 I
\end{array}\right] < 0, \tag{47}
$$

are feasible, then there exists a state feedback controller of the form (5) such that the requirements (Q1) and (Q2) are satisfied for all stochastic nonlinearities and all admissible deterministic uncertainties. Moreover, the desired controller (5) can be determined by

$$
K = G X^{-1}. \tag{48}
$$

5. CONCLUSIONS

A robust mixed H_2/H_∞ controller has been designed in this paper for a class of uncertain discrete time nonlinear stochastic systems. A key technique has been used to deal with the matrix trace terms arising from the stochastic nonlinearities, and the well-known S-procedure has been adopted to handle the deterministic uncertainties. A unified framework has been established to solve the addressed mixed H_2/H_∞ control problem, and sufficient conditions for the solvability of the mixed H_2/H_∞ control problem have been given in terms of a set of feasible LMIs. Our method can also be extended to output feedback case, and the results will appear in the near future.

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