# ACTIVE NOISE CONTROL IN A CAVITY USING INTERPOLATED MODELS.

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Abstract: This paper demonstrates a practical way for system identification using subspace and cubic spline methods to minimize the task of obtaining models using experimental frequency responses for a large number of sensor-actuator locations. These models are basically required in varied combinations for optimal actuator-sensor placement. Measuring at all the required positions is impractical and analytical modelling methods are helpless for nonideal boundary conditions. This paper also presents experimental results of active noise control, using interpolated models for an acoustic cavity. Acoustic enclosures are resonant systems which required an accurate system model to design high performance controllers. A controller using minimax LQG control method based on interpolated models provides promising results for the use of interpolated models in control design.


Keywords: Active noise control, boundary conditions, Frequency responses, Identification, Interpolation, Model approximation, State space, Subspace methods

## 1. INTRODUCTION

Reducing noise in acoustic enclosures has been a focus of active noise control research in the previous decade (Elliott, 1999), (Balachandran *et al.*, 1996), (Fuller and von Flotow, 1995) due to the rapid technology growth in affordable and practical digital signal processing (Eriksson, 1990) and (Kuo and Chen, 1958). The new technology can systematically overcome the difficulties of very high-order models, non-minimum phase behavior and uncertainties caused by finite dimensional approximations, uncertainties caused by non-uniform boundary conditions, and sound-structure dynamics of enclosure structure (Kelkar and Pota, 2000*a*).

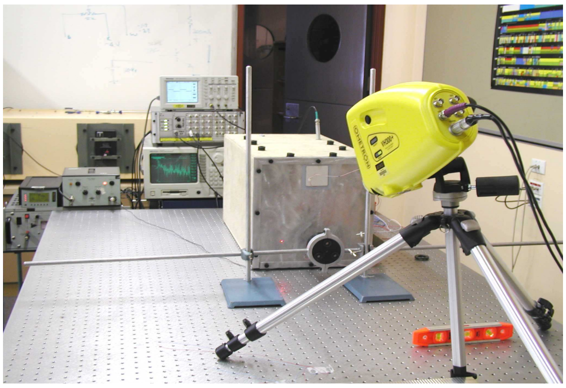

Fig. 1. The experimental acoustic cavity

The modelling and control of acoustic cavities with one wall made of flexible structure has been discussed in (Banks *et al.*, 1991), (Kim and Brennan, 1999), and (Demetriou and Fahroo, 1999). The work in (Banks *et al.*, 1991) demonstrates acoustic noise control using only structural actuators while (Demetriou and Fahroo, 1999) and (Kim and Brennan, 1999) present acoustic noise control using both structural and acoustic actuators. Only (Kim and Brennan, 1999) has experimental results which confirms with the theoretical results that the control performance of the structural actuators alone is about two times better than the performance of the acoustic actuators only.

Analytic modelling for control often results in a poor model if the system is even mildly realistic (Kelkar and Pota, 2000*b*). In modern control, on the other hand, system identification is presented in state-space (SS) models standing out as the natural way of representing multivariable systems (McKelvey *et al.*, 1996). Methods which identify SS models by means of geometrical properties of the input and output sequences without the need for an explicit parameterizations of the model set are commonly known as subspace methods (McKelvey *et al.*, 2000). Essentially, a subspace algorithm delivers no system identification difference between multi-input, multi-out (MIMO) and single-input, single-output (SISO). The algorithm delivers estimated models from frequency response in a SS basis, wherein the transfer function is insensitive to small perturbations in the matrix elements. Frequency domain methods, in addition, are suitable for building very accurate models of complex systems which permit physical interpretation of the parameters (Schoukens and Pintelon, 1991).

One of the advantages of subspace methods is that the identification of multivariable systems is as simple as for scalar systems (McKelvey *et al.*, 2000). This leads to an idea that the numerical approximation techniques should provide a possibility of system identification at any position of a plant where the necessary data is not available but using the neighbouring data instead. Cubic spline interpolation is the approximation technique used in this paper. The cubic spline approximation uses cubic polynomials between each successive pair of nodes or available data to provide a fitting curve where the desired value is expected to be on it. The experimental plant in this paper is an acoustic cavity at the Australian Defence Force Academy (ADFA) shown in Figure 1.

This paper not only presents the approximation of identified systems using SS representation for positions where no frequency responses are measured, but also shows control application of minimax LQG control method (Petersen *et al.*, 2000) using these approximated system models in comparison with the performance of the same control method using the system identification derived from real measurement of frequency responses. The experimental result of the controller using interpolated model is promising.

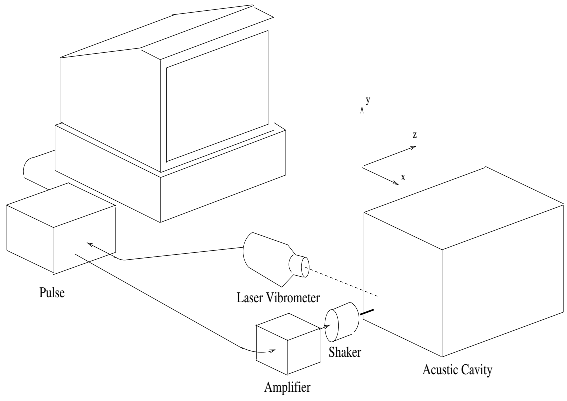


Fig. 2. System identification setup

Combinations of frequency responses of input to output at arbitrary positions are required for most of the optimal actuator-sensor placement methods. These methods are mostly based on controllability and observability Gramians for several sensor-actuator locations. Analytic modelling is quite often inappropriate because of difficulties of nonideal boundary condition. In addition, it is impractical to obtain all required models from measuring frequency response. The cubic spline interpolation method can be used to fulfil modelling work where required data is unavailable but neighbouring data is provided.

## 2. EXPERIMENT SETUP

The acoustic cavity used for experiments at ADFA has the dimensions $300\ mm \times 300\ mm \times 600\ mm$ with five sides made of timber. The sixth side is covered by an aluminium square sheet with thickness of $1\ mm$ fixed tightly at all the four edges by three screws each. The origin $(0, 0, 0)$ is at the bottom left corner of the aluminium plate, looking into it; $x$-axis is along the width of the cavity, $y$-axis is along the hight of the cavity, and $z$-axis is into the cavity.

Input to the plate is provided by a Brüel & Kjær electromagnetic shaker Type 4810 and power amplifier type 2718. An OMETRON laser-doppler-vibrometer VH300+ measures the pointwise plate vibration output. Frequency response data is collected by Brüel & Kjær PULSE multi-analyzer in the frequency range of 10–300 Hz. The set up is schematically illustrated in Figure 2.

Structure-borne sound at 50 or 100 Hz may often be treated merely as mechanical vibrations with a finite number of degrees of freedom (Cremer *et al.*, 1988). And, the size of the acoustic enclosure is usually relative small such as a cavity of $30 \times 40 \times 150 cm^3$ in (Kim and Brennan, 1999). Hence the model-order of the dynamics in cavity acoustic noise control experiments is generally not very high. Furthermore, many successful experiments of noise control in reverberant environment with diverse methods (Elliott *et al.*, 1990), (Fuller and von Flotow, 1995), (Kim and Brennan, 1999), (Poh *et al.*, 1996) are concentrated in low range of frequency only. For this reason, modelling is concentrated in the frequency range of 10–300 Hz.

For Multi-input Single-output (MISO) system, the sensor location is fixed while the actuator position changes along the $x$-direction with a fixed $y$-value or along $y$-direction with a fixed $x$-value. The different actuator positions are assigned as $N_{i,j}$. Similarly for SIMO systems the sensor measures at different positions along $x$- or $y$-direction while the actuator is at a fixed position. The different sensor positions are assigned as $N_{l,m}$.

## 3. SYSTEM MODELLING AND IDENTIFICATION

### *3.1 State-space Representation*

A system can be described as a SS representation (Eykhoff, 1974) in time and frequency domain as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad s\mathbf{X}(s) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t), \quad \mathbf{Y}(s) = \mathbf{C}\mathbf{X}(s) + \mathbf{D}\mathbf{U}(s)$$

where coefficients

**A** is system characteristic $n \times n$ matrix,
**B** is input or distribution $n \times p$ matrix,
**C** is output or measurement $q \times n$ matrix,
**D** is input-output $q \times p$ matrix,
with $n$ = system order, $p$ = input number, and $q$ = output number.

It can be seen that matrices **B**, **C** and **D** depend only on the input and output locations. Many optimal sensor-actuator location methods need models for a large number of sensor-actuator position combinations. In most cases it is impractical to obtain all these models from experimental data and system identification techniques. Moreover, when the structure of the system doesn't satisfy ideal boundary conditions, it becomes difficult to obtain analytical expressions for matrices **B**, **C** and **D** as functions of sensor-actuator location.

### *3.2 Interpolation procedure for identification*

In this paper transfer functions corresponding to different sensor-actuator locations are obtained from interpolation. The data for interpolation is

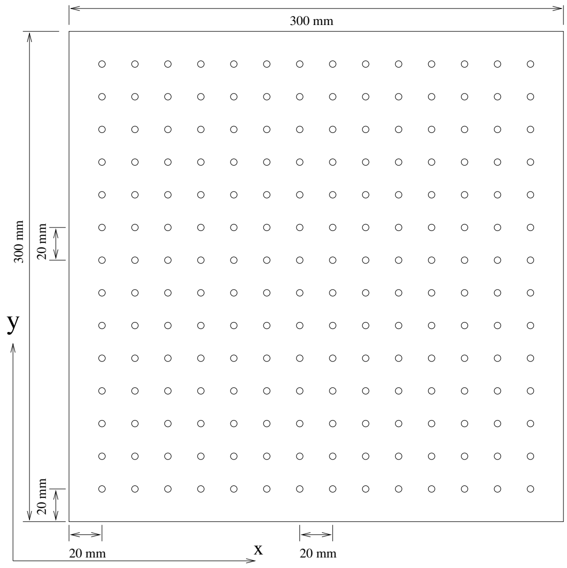


Fig. 3. The selected grid position on the plate

first obtained by making experimental measurements at sensor-actuator locations along a coarse grid on the plate. If the grid points selected are 20 mm apart, it yields 14 lines of 14 points in each. For N = 196 points, $14 \times 14$ points, on the grid as shown in Figure 3, a total of $N^2$ experimental measurements are needed to be made. And, let the grid position be $N_{x,y}$, where $x = 1$ is the bottom line and $y = 1$ is the most left column. The grid point $N_{1,1}$ at the bottom left corner has the coordinates $(20, 20, 0)$.

To obtain an interpolated model between input at a given actuator location $N_{i,j}$ and an arbitrary point $r_s$ on the plate for output, it can be done in the following steps:

(1) Draw a straight line which includes the point $r_s$ and at least four other grid points on the line, the number of points can be more than four. If this is not possible then first interpolations have to be done for other points which enable at least four points with known models to be on a straight line which has $r_s$ on it.
(2) Perform a single-input-multi-output (SIMO) system identification using the Subspace method with input at the actuator location $N_{i,j}$ and output at all the selected four grid locations.
(3) The above identification process will generate matrices **A**, **B**, **C**, **D**. Let the order of the system be $n$, then **C** will be a $4 \times n$ matrix corresponding to 4 outputs and **D** will be a $4 \times 1$ matrix. The interpolated model for the desired output location $r_s$ will be the same **A** and **B** but a different $1 \times n$ **C** matrix and $1 \times 1$ **D** matrix. Let these **C** and **D** be $\mathbf{C}(\mathbf{r}_s)$ and $\mathbf{D}(\mathbf{r}_s)$.
(4) The interpolation is done column-wise for **C** and **D**, i.e., the first column of **C** is

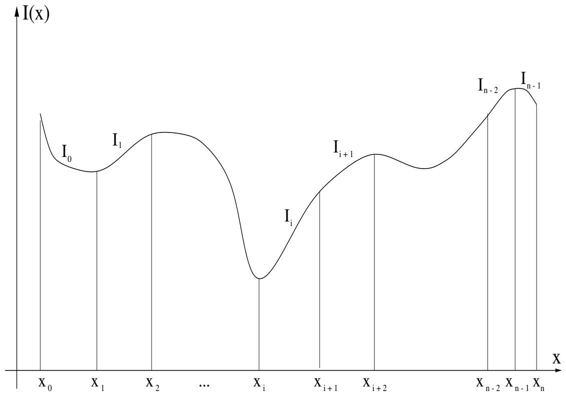

Fig. 4. A cubic spline interpolant

used to obtain the first element of $\mathbf{C}(\mathbf{r}_s)$, the second column the second element, etc. Whereas, only one column for $\mathbf{D}(\mathbf{r}_s)$ is to be interpolated.

To obtain a model between a given sensor location $N_{l,m}$ on the grid and an arbitrary input location $r_a$, the same process as given above can be followed. But this will be a multi-input-single-output (MISO) identification. The expected model for the input location $r_a$ will be the same $\mathbf{A}$ and $\mathbf{C}$, which are derived from the identification process. But a different $n \times 1$ $\mathbf{B}$ and $1 \times 1$ $\mathbf{D}$, where $n$ is the system order. The new obtained $\mathbf{B}$ and $\mathbf{D}$ are defined as $\mathbf{B}(\mathbf{r}_a)$ and $\mathbf{D}(\mathbf{r}_a)$. Elements of the $\mathbf{B}(\mathbf{r}_a)$ and $\mathbf{D}(\mathbf{r}_a)$ have to be then interpolated row-wise to obtain the desired model. $\mathbf{D}(\mathbf{r}_a)$ is derived from the interpolation of only one row.

### *3.3 Cubic spline interpolation*

The cubic spline interpolation is the piecewise cubic polynomial approximation between each successive pair of nodes, positions used for interpolation. It provides sufficient flexibility in the procedure for continuously differentiable with a continuous second derivative on the interval, an example as interpolated curve shown in Figure 4. It presents a cubic spline interpolation $I$ for a function $f$, which defined on $[a, b]$ with a set of numbers $x_i$ (nodes, $a = x_0 < x_1 < \ldots < x_n = b$), satisfied the following conditions:

**1.** $I_i$ is subinterval of $I$ on $[x_i, x_{i+1}]$ for $i = 0, 1, \ldots, n-1$.
**2.** $I(x_i) = f(x_i)$ for $i = 0, 1, \ldots, n$.
**3.** $I_{i+1}(x_{i+1}) = I_i(x_{i+1})$ for $i = 0, 1, \ldots, n-2$.
**4.** $I'_{i+1}(x_{i+1}) = I'_i(x_{i+1})$ for $i = 0, 1, \ldots, n-2$.
**5.** $I''_{i+1}(x_{i+1}) = I''_i(x_{i+1})$ for $i = 0, 1, \ldots, n-2$.

### *3.4 System Identification*

The interpolation method is now demonstrated by obtaining a model for two conditions: (a) for a given input location on the grid and an arbitrary sensor location and (b) for sensor location on the grid and an arbitrary actuator location. The identification is done from measured frequency response data. The model order is assumed 7 in all the models obtained in this paper. It is not practical to use very high order model to design a controller (Pota *et al.*, 2004). The accuracy of the interpolated model is shown by comparing the control performance of the controller using the interpolated model and the controller using the directly identified model from the measured data.

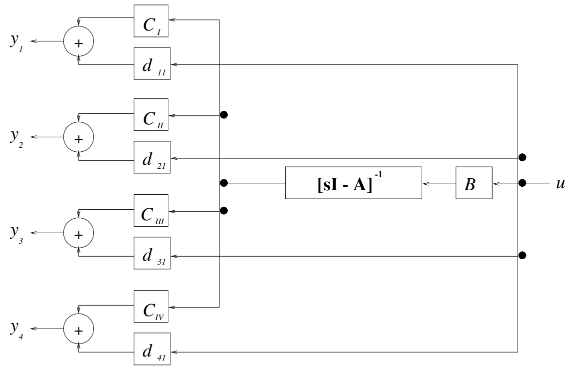

Fig. 5. Block diagram of SS in frequency domain based on a SIMO system with four outputs

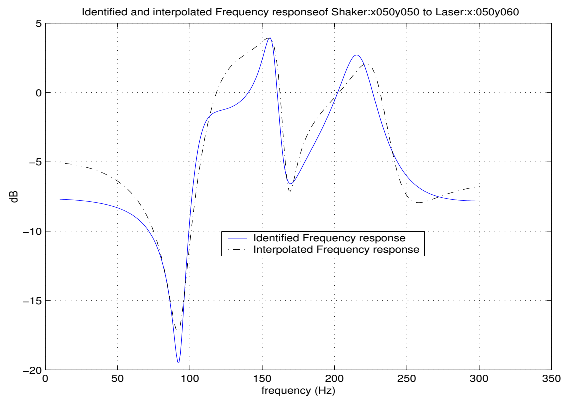


Fig. 6. Identified and interpolated (from outputs) frequency response of input at $(050, 050, 0)$ to output at $(050, 060, 0)$

## 4. EXPERIMENTAL RESULTS

### *4.1 Single-Input-Multi-Output Identification*

A model for an arbitrary sensor location at $r_s = (050, 060, 0)$ and an given input at $r_a = (050, 050, 0)$ is interpolated. The interpolated model is based on a SIMO system identification from four measured frequency responses with input at $r_a$ and outputs at $N_{1,3} = (020, 060, 0)$, $N_{2,3} = (040, 060, 0)$, $N_{3,3} = (060, 060, 0)$, and $N_{4,3} = (080, 060, 0)$. The SS representation of the SIMO system can be described in time domain as

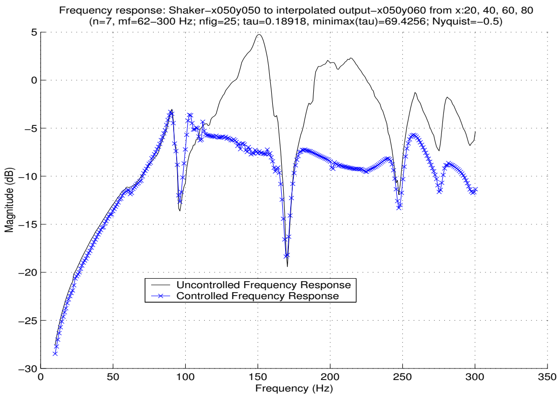


Fig. 7. Control performance using interpolated frequency response of the input at $(050, 050, 0)$ to the interpolated output at $(050, 060, 0)$

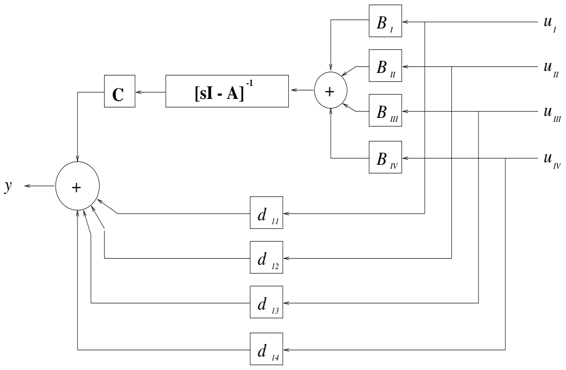


Fig. 8. Block diagram of SS in frequency domain based on a MISO system with four inputs

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\
y_1(t) &= C_I\mathbf{x}(t) \quad + d_{11}\mathbf{u}(t) \\
y_2(t) &= C_{II}\mathbf{x}(t) \;\, + d_{21}\mathbf{u}(t) \\
y_3(t) &= C_{III}\mathbf{x}(t) + d_{31}\mathbf{u}(t) \\
y_4(t) &= C_{IV}\mathbf{x}(t) \;\, + d_{41}\mathbf{u}(t)
\end{aligned}
$$

where, $C_I, C_{II}, \ldots C_{IV}$ are rows of $\mathbf{C}$ and $d_{11}$, $d_{21}$, ..., $d_{41}$ are elements of $\mathbf{D}$ corresponding to positions $N_{1,3}$, $N_{2,3}$, $N_{3,3}$, and $N_{4,3}$. Or, it can be presented in frequency domain as a block diagram shown in Figure 5. The SS representation and the block diagram show that the interpolated model can be obtained from interpolating coefficient matrices $\mathbf{C}$ and $\mathbf{D}$ of the SIMO SS representation.

Figure 6 presents the identified and the interpolated model for actuator location $r_a$ and arbitrary sensor location at $r_s$. Figure 7 shows the closed-loop response of the controller using minimax LQG control design method based on this interpolated model and the measured frequency response of the input $r_a$ and the output $r_s$ in open-loop.

### *4.2 Multi-Input-Single-Output Identification*

Since the model is generated from low frequency response, it can be considered as Linear Time

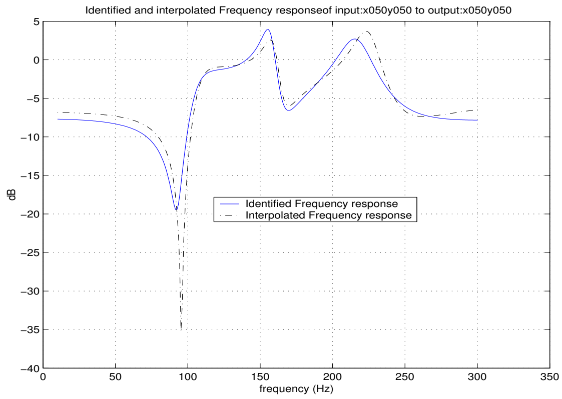


Fig. 9. Identified and interpolated (from inputs) frequency response of input at $(050, 050, 0)$ to output at $(050, 060, 0)$

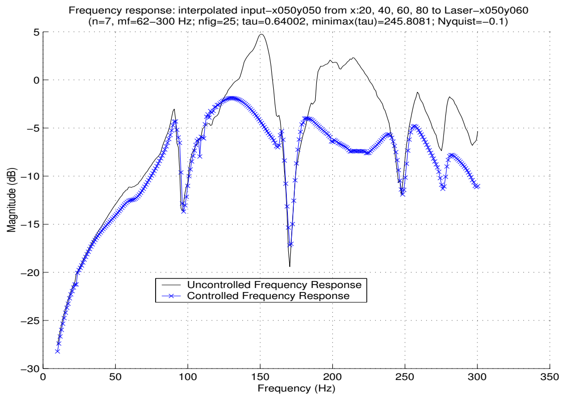


Fig. 10. Control performance using interpolated frequency response of the interpolated input at $(050, 050, 0)$ to the output at $(050, 060, 0)$

Invariant system which leads the results of the MISO identification very similar to the results of the SIMO identification. The similar experimental conditions for the same input $r_a = (050, 050, 0)$, which is purposed to be an arbitrary position and the same output at $r_s = (050, 060, 0)$, which is expected to be an given position, have been done again. But, the interpolated model is based on a MISO system identification obtained from four measured frequency responses with output at $r_s$ and inputs at $N_{1,3} = (020, 050, 0)$, $N_{2,3} = (040, 050, 0)$, $N_{3,3} = (060, 050, 0)$, and $N_{4,3} = (080, 050, 0)$. The SS representation of the SIMO system in time domain is described as

$$
\begin{aligned}
\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) &+ B_I u_1(t) \quad + B_{II} u_2(t) \\
&+ B_{III} u_3(t) + B_{IV} u_4(t) \\
\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) &+ d_{11} u_1(t) \quad + d_{12} u_2(t) \\
&+ d_{13} u_3(t) \quad + d_{14} u_4(t)
\end{aligned}
$$

where, $B_I, B_{II}, \ldots B_{IV}$ are column of $\mathbf{B}$ and $d_{11}, d_{12}, \ldots d_{14}$ are element of $\mathbf{D}$ at positions $N_{1,3}$, $N_{2,3}$, $N_{3,3}$, and $N_{4,3}$ respectively. It is also described in frequency domain as a block diagram depicted in Figure 8. The interpolated and the identified model is presented in Figure 9.The obtained control performance based on the interpolated model in comparison with the measured frequency response of the input-output in open loop is presented in Figure 10.

Furthermore, different number of nodes and different distance between nodes have been investigated. Interpolation derived from four nodes and the distance of 20 $mm$ between nodes provides the best model for the minimax LQG controller. Increasing the nodes number or the distance reduces the stability of the controller clearly, and also decreases the ability of vibration damping gradually. On the other hand, decreasing of the nodes number does not increase the stability and not improve the damping ability neither, whereas it would rather be reduced.

The experimental results can be explained that with proper nodes number the interpolated model can be used for an high performance controller.

## 5. CONCLUSIONS

System identification at specific locations of an already build structure without any measured data can be approximated satisfyingly by cubic spline interpolation method using the neighbouring data for designing a high performance controller. Because of less measurement, the interpolation method can save more than an half of measurement work. It is useful for applications that need a large number of measured frequency responses of arbitrary positions. In this experiment, proper number of nodes used for interpolation is four. And, proper distance between nodes is 20 $mm$.

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