DISCRETE-TIME ADAPTIVE MODEL PREDICTIVE CONTROL BASED ON COMPARISON MODEL

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Abstract: This paper proposes a discrete-time adaptive model predictive control (MPC) algorithm for a class of constrained linear time-invariant systems, which updates the estimation of system parameters on-line and produces the control input subject to the given input/state constraints. This method is based on a robust MPC algorithm using comparison model which enable us to estimate the prediction error bound of uncertain systems and an adaptive mechanism. First, a new parameter update method based on the moving horizon estimation is proposed, which allows us to predict the worst-case estimation error bound over prediction horizon. Second, we propose an adaptive MPC algorithm developed by combining the on-line parameter estimation with MPC method based on the modified comparison model. This method guarantees the feasibility and stability of closed-loop systems.

Keywords: Predictive control, Constraint satisfaction problems, Parameter estimation, Uncertain linear system, Robust stability.

1. INTRODUCTION

Model predictive control (MPC) is one of the most promising ways to handle control problems for the systems having input/state constraints (see e.g. Mayne, *et al*., 2000). It determines the input sequence by solving on-line, at each time step, a finite horizon optimization problem based on the system model and subsequently only the first control in this sequence is implemented. Therefore the model quality plays a vital role in MPC, but in reality there always exist model uncertainties and these may cause a large effect on the system performance.

One way to cope with such problems is to develop robust MPC methods, which guarantee certain control performances against modeling uncertainties. Several different robust MPC schemes have been proposed for many years (Badgwell, 1997; Kothare, *et al*., 1996; Lee and Kouvaritakis, 2000). In this line of research, the system model is fixed though its uncertainties are taken into account explicitly. Therefore, its control performance is limited by the quality of the fixed (initial) model.

Another attractive way to handle model uncertainties in MPC is to update the system model on-line based on measurement data. Although the development of adaptive type MPC scheme is one of the research issues for the control of constrained systems, there have been few reports on this topic so far (Mayne, *et al*., 2000). One of the main reasons for this is the difficulty to guarantee the fulfillment of system constraints in the presence of adaptive mechanism through the receding horizon strategy. In order to overcome that problem, we have to estimate the future behavior of real system while updating the system parameters on-line. In addition, it seems to be extremely difficult to guarantee both feasibility and stability theoretically when we adopt any adaptive approaches in MPC.

The purpose of this paper is to develop a discrete-time adaptive MPC algorithm for a class of constrained linear time-invariant systems, which updates the estimation of system parameters on-line and produces the control input satisfying given state and input constraints for possible parameter estimation errors. The key idea of this paper is based on the adaptive MPC algorithm for a class of continuous-time constrained linear systems proposed by Kim, *et al*. (2004), that is, the combination of the robust MPC method (Fukushima and Bitmead, 2004) and a new parameter estimation method suitable for discrete-time MPC.

First, a new parameter estimation method based on the moving horizon estimation is proposed. This method allows us to predict the worst-case estimation error bound over prediction horizon, with which the future model improvement can be taken account of. Then it is shown that the proposed estimation method can be incorporated into robust MPC method (Fukushima and Bitmead, 2004), which can handle state-dependent disturbances, by modifying the comparison model to handle the time-varying parameter estimation errors. Using such comparison model, the original MPC problem based an uncertain model can be transformed into a nominal one without uncertain parameters. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method. In this paper, only noise-free case is considered.

Notation: The symbol x_i denotes the ith element of a vector x. Let $\| x \|$ and $\| x \|_p$ denote the Euclidean and p-norms of a vector x. Let $\overline{\sigma}(M)$ denotes the largest singular value of matrix M. The maximum and minimum eigenvalues of matrix M are denoted by $\overline{\lambda}(M)$ and $\underline{\lambda}(M)$, respectively. The notation $M \succ 0$ means that M is a symmetric positive definite, and $M^{1/2}$ denotes the unique positive definite square root of $M \succ 0$.

2. PROBLEM FORMULATION

Consider the following constrained linear time-invariant system in controllable canonical form:

$$x(t+1) = A(\theta^*)x(t) + Bu(t), \quad x(0) = x \tag{1}$$

where

$$A(\theta^*) = \begin{bmatrix} a_1 & \cdots & a_n \\ \hline I_{n-1} & \vdots & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$$

and $\theta^* = (a_1, \cdots, a_n)^T$ denotes uncertain parameter vector. The constraints for measurable state $x(t) \in R^n$ and control input $u(t) \in R$, which are fulfilled at all time instants $t \geq 0$, are given as

$$\begin{aligned} &x(t) \in X, \quad X := \{x(t) \in R^n : |x_i(t)| \leq \psi_i, \forall i\}, \\ &u(t) \in U, \quad U := \{u(t) \in R : |u(t)| \leq \eta\} \end{aligned} \tag{2}$$

We assume that the following $z(t)$ is measurable

$$z(t) = \theta^{*T} x(t) \tag{3}$$

and the initial estimate θ_0 of θ^* and its estimation error bound v_0 satisfying

$$\| \theta_0 - \theta^* \| \leq v_0 \tag{4}$$

are given. We also assume that given $K_0^T \in R^n$ and $P \succ 0$ satisfy the similar assumptions to those used in Kim, *et al*. (2004) as follows:

Assumption 1: $1 \leq \sqrt{\underline{\lambda}(P)} \min_i \psi_i, \| K_0^T \| \leq \sqrt{\underline{\lambda}(P)}\, \eta$

Assumption 2:
$$0 < \sqrt{1 - \frac{\underline{\lambda}(Q)}{\overline{\lambda}(P)}} + \frac{\overline{\sigma}(P^{\frac{1}{2}}B)v_0}{\sqrt{\underline{\lambda}(P)}} < 1,$$
$$Q := P - F^T P F, \; F := A(\theta_0) + BK_0$$

Note that Assumption 1 implies that the given feedback control $u(t) = K_0 x(t)$ always satisfies the given constraints in the following set.

$$X_f = \{x(t) \in R^n : V(x) \leq 1\}, \; V(x) := \sqrt{x^T P x} \tag{5}$$

Assumption 2, on the other hand, implies that X_f is a robustly invariant set by $u(t) = K_0 x(t)$ (Blachini, 1999; Fukushima and Bitmead, 2004; Kim, *et al*., 2004)).

Under these assumptions, our goal is to construct a discrete-time adaptive MPC algorithm, which obtains the estimate $\theta(t)$ of θ^* on-line and steers the state $x(t)$ to the origin without violating the constraints.

3. NEW ADAPTIVE PARAMETER ESTIMATION ALGORITH FOR DISCRETE-TIME MPC

In this section, we propose the following recursive parameter update algorithm, which is more compatible with robust MPC than the conventional methods and a less conservative approach for adaptive MPC. We define $f(\tau) := 0$ for $\tau < 0$. The estimation horizon N_e is a design parameter. The parameter update algorithm at the current time t is described as follows. $\alpha \geq 0$ and κ are design parameters.

Step 0: At the update starting time t_0, initialize $\overline{\gamma}$, $c_\phi(t_0)$ and $c_z(t_0)$ as follows and go to Step 3.

$$\begin{aligned} &\overline{\gamma} = \sqrt{\overline{\lambda}(\Pi_0^T(t)\Pi_0(t))} \\ &\Pi(t_0) := I_n - \kappa \sum_{s=t_0}^{t_0-N_e+1} x(s)x^T(s) \Bigg/ \left[\alpha + \sum_{s=t_0}^{t_0-N_e+1} x^T(s)x(s)\right] \\ &c_\phi(t_0) = \sum_{s=t_0}^{t_0-N_e+1} x(s)x^T(s) \Bigg/ \left[\alpha + \sum_{s=t_0}^{t_0-N_e+1} x^T(s)x(s)\right] \\ &c_z(t_0) = \sum_{s=t_0}^{t_0-N_e+1} x(s)z^T(s) \Bigg/ \left[\alpha + \sum_{s=t_0}^{t_0-N_e+1} x^T(s)x(s)\right] \end{aligned} \tag{6}$$

Step 1: If $\overline{\gamma} \leq \sqrt{\overline{\lambda}(\Pi^T(t)\Pi(t))}$, update $c_\phi(t)$ and $c_z(t)$ as follows and go to Step 3. Otherwise, go to Step 2.

$$c_\phi(t) = c_\phi(t-1), \quad c_z(t) = c_z(t-1) \tag{7}$$

Step 2: Update $\bar{\gamma}$, $c_\phi(t)$ and $c_z(t)$ using (6) and then go to Step 3.

Step 3: Apply the following parameter update law.

$$\theta(t) = \theta(t-1) + \kappa(c_z(t) - c_\phi(t)\theta(t-1)) \tag{8}$$

At the next time instant, go to Step 1.

It is important to note that one of the differences from conventional update laws is to use the summation c_z and c_ϕ over the estimation horizon N_e . Another difference is that Step 1 above aims at choosing the "best" data set for parameter estimation in terms of the excitation of $x(\tau)$ over the horizon.

The value of $\bar{\gamma}$ determined in the above algorithm at time t satisfies

$$\bar{\gamma} := \min_{t_0 \le t_i \le t} \sqrt{\bar{\lambda}(\Pi^T(t_i)\Pi(t_i))} \tag{9}$$

where t_i denotes the time step of MPC. We define the parameter estimation error as $\tilde{\theta}(t) := \theta(t) - \theta^*$. Although $\|\tilde{\theta}(t+k)\|$ over the prediction horizon of MPC is unknown, its upper bound $\nu_{t+k|t}$ can be predicted as follows:

$$\nu_{t+k+1|t} = \bar{\gamma}\,\nu_{t+k|t}\ ,\ \nu_{t|t} = \nu(t),\ k = 0,1,\cdots,N-1 \tag{10}$$

where $\nu_{t+k|t}$ denotes the predicted value of upper bound of parameter estimation error at the current time t and N denotes the length of prediction horizon of MPC. The predicted $\nu_{t+k|t}$ satisfies the following lemma.

Lemma 3 *The parameter estimation error bound satisfies*

$$\|\tilde{\theta}(t+k)\| \le \nu_{t+k|t}\ , \quad k = 0,1,\cdots,N \tag{11}$$

This result shows that we can take account of the future improvement of $\theta(t)$ by using $\nu_{t+k|t}$ over the prediction horizon. Moreover, (9) and (10) show that the proposed algorithm tries to choose the "best" estimate $\theta(t)$ in the sense that the predicted error bound $\nu_{t+k|t}$ is minimized more rapidly than that of the previous time instant.

Remark 1: The proposed estimation algorithm requires the additional computational burden such as the computation of the smallest eigenvalue in Steps 1 and 2 as compared with the conventional estimation algorithm. However, since its computational burden may be much less than that of MPC, the proposed method could be implementable in many applications.

Similarly to the existing robust MPC methods (Bemporad, 1998; Fukushima and Bitmead, 2004; Kim, *et al.*, 2004), we adopt the following feedback control

$$u(t) = K(\theta(t))x(t) + \tilde{u}(t) \tag{12}$$

which consists of feedback gain $K(\theta(t))$ and open-loop trajectory $\tilde{u}(t)$. The key difference from other approaches is that $K(\theta(t))$ depending on $\theta(t)$ is updated at each time step of MPC as follows:

$$K(\theta(t)) := -\theta^T(t) + \theta_0^T + K_0 \tag{13}$$

Substituting (12) into (1) results in the following equation.

$$x(t+1) = Fx(t) + B\tilde{u}(t) - Bd(t),\ d(t) := \tilde{\theta}^T(t)x(t) \tag{14}$$

Since we know the upper bound $\nu_{t+k|t}$ of $\|\tilde{\theta}(t+k)\|$ as in (11), the disturbance, the effect of parameter uncertainty, $d_{t+k|t}$ is bounded as follows:

$$\begin{aligned} &d(t+k) \in D, \quad k = 0,1,\cdots,N-1, \\ &D := \{d \in R : \|d(t+k)\| \le \nu_{t+k|t}\|x(t+k)\|\} \end{aligned} \tag{15}$$

Therefore, the robust MPC method (Fukushima and Bitmead, 2004) is applicable to the system in (14).

The proposed adaptive MPC algorithm is composed of two methods, one which estimate the uncertain parameters using the proposed algorithm in this section and another one, the modified robust MPC method, described in the next section.

Remark 2: The conventional parameter estimation method (Åström & Wittenmark, 1995; Krstic, *et al.*, 1995), which is described as

$$\theta(t) = \theta(t-1) + \frac{\kappa x(t)}{\alpha + x^T(t)x(t)}(z^T(t) - x^T(t)\theta(t-1))$$

could be incorporated into a robust MPC method. However, this method makes the MPC problem too conservative in the sense that the possible model improvement in the future cannot be considered. In other words, the error bound $\nu_{t+k|t}$ is fixed over the prediction horizon of MPC, although the estimation error bound $\|\tilde{\theta}(t+k)\|$ could be decreased by parameter estimation.

4. MODIFIED ROBUST MPC ALGORITHM

We predict the state $x(t+k)$ of (1) and (14) based on the nominal model such as

$$\begin{aligned} &\hat{x}_{t+k+1|t} = A(\theta_t(t+k))\hat{x}_{t+k|t} + B\hat{u}_{t+k|t}, \\ &\hat{x}_{t|t} = x(t), \quad k = 0,1,\cdots,N-1. \end{aligned} \tag{16}$$

where $\hat{x}_{t+k|t}$ and $\theta_t(t+k)$ denote the predicted value of $x(t+k)$ and the estimated parameters based on (8) at the current time step t , respectively. The predicted control $\hat{u}_{t+k|t}$ in (16) has the following form as mentioned in Section 3.

$$\hat{u}_{t+k|t} = K(\theta_t(t+k))\hat{x}_{t+k|t} + \tilde{u}_{t+k|t} \tag{17}$$

Substituting the control law (17) into the system (16) results in the following equation.

$$\hat{x}_{t+k+1|t} = F\hat{x}_{t+k|t} + B\tilde{u}_{t+k|t} \tag{18}$$

It is necessary for the robustness analysis of closed-loop to evaluate the prediction error bound of $\hat{x}_{t+k|t}$ due to disturbance $d(t+k)$ caused by the parameter estimation error. Moreover it is also required to derive constraints for $\hat{x}_{t+k|t}$ and $\hat{u}_{t+k|t}$, which guarantee that the constraints in (2) for real system are also satisfied. However, since the estimation error $d(t+k)\in D$ depends on the state $x(t+k)$ of real system as in (15), it is difficult to evaluate the prediction error from only the nominal model in (18).

In order to overcome this difficulty, we now introduce the following additional scalar system, comparison model, into the optimization problem of MPC. This system is constructed based on *a priori* information on the estimation error bound in (11) as a similar method described in Kim, *et al*. (2004) and Fukushima and Bitmead (2004) as follows:

$$\begin{aligned} &w_{t+k+1|t} = a_{t+k|t}w_{t+k|t} + b|\tilde{u}_{t+k|t}|,\ \ w_{t|t} = V(x(t)),\\ &a_{t+k|t} := \sqrt{1-\frac{\underline{\lambda}(Q)}{\overline{\overline{\lambda}}(P)}} + \frac{\overline{\sigma}(P^{\frac{1}{2}}B)v_{t+k|t}}{\sqrt{\underline{\lambda}(P)}},\ b := \| P^{\frac{1}{2}}B \| \end{aligned} \tag{19}$$

This comparison model enables us to obtain an upper bound of the future value of $V(x)$, as shown in the following lemma.

Lemma 4 *For any $\tilde{u}_{t+k|t}$, the states of comparison model in (19) and real system in (14) satisfy*

$$V(x(t+k)) \le w_{t+k|t},\quad k = 0,1,\cdots,N \tag{20}$$

Once we found the model for predicting an upper bound $w_{t+k|t}$ of $V(x)$, the constraint sets $\hat{X}$ and $\hat{U}$ for $\hat{x}_{t+k|t}$ and $\tilde{u}_{t+k|t}$ depending on $v_{t+k|t}$ and $w_{t+k|t}$ can be derived as follow:

$$\begin{aligned} &\hat{X}(t+k+1, v_{t+k+1|t}, w_{t+k+1|t})\\ &:= \{\hat{x}\in R^n : |\hat{x}_i| \le \psi_i - \hat{\psi}_i(t+k+1, v_{t+k+1|t}, w_{t+k+1|t}), \forall i\}\\ &\hat{U}(t+k, v_{t+k|t}, w_{t+k|t})\\ &:= \{K_0\hat{x}+\tilde{u}\in R : |K_0\hat{x}+\tilde{u}| \le \eta - \hat{\eta}(t+k, v_{t+k|t}, w_{t+k|t})\} \end{aligned}$$

where

$$\begin{aligned} &\hat{\psi}_i(t+k+1, v_{\cdot|t}, w_{\cdot|t}) := \sum_{s=0}^{k} |\zeta_i(s)| \frac{v_{t+k+1-s|t}\, w_{t+k+1-s|t}}{\sqrt{\underline{\lambda}(P)}}\\ &\hat{\eta}(t+k, v_{\cdot|t}, w_{\cdot|t}) := (v_{t+k|t}+v_0)(2\hat{\eta}_1+\hat{\eta}_3)+\hat{\eta}_2\\ &\hat{\eta}_1(t+k, v_{\cdot|t}, w_{\cdot|t}) := \sum_{s=0}^{k} \|\xi(s)\| \frac{v_{t+k-s|t}\, w_{t+k-s|t})}{\sqrt{\underline{\lambda}(P)}}\\ &\hat{\eta}_2(t+k, v_{\cdot|t}, w_{\cdot|t}) := \sum_{s=0}^{k} |K_0\xi(s)| \frac{v_{t+k-s|t}\, w_{t+k-s|t}}{\sqrt{\underline{\lambda}(P)}}\\ &\hat{\eta}_3(t+k, v_{\cdot|t}, w_{\cdot|t}) := w_{t+k|t} \Big/ \sqrt{\underline{\lambda}(P)}\\ &\zeta(s) := F^sB,\quad \xi(s) := \begin{cases} F^{s-1}B, & for\ s\neq 0\\ 0\quad, & for\ s=0 \end{cases} \end{aligned} \tag{21}$$

and $\zeta_i(s)$ denotes the *i*th row of $\zeta(s)$.

The constraint sets modified by $\hat{X}$ and $\hat{U}$ above and two scalar systems (10) and (19) satisfy the following properties.

Theorem 5 *For a given $x(t)\in R^n$, any open-loop trajectory $\tilde{u}_{t+k|t}$, $k = 0,\cdots,N-1$, which satisfies*

$$\begin{aligned} &\hat{x}_{t+k+1|t} = F\hat{x}_{t+k|t} + B\tilde{u}_{t+k|t},\ \hat{x}_{t|t} = x(t)\\ &v_{t+k+1|t} = \overline{\gamma}\, v_{t+k|t},\ v_{t|t} = v(t)\\ &w_{t+k+1|t} = a_{t+k|t}w_{t+k|t} + b|\tilde{u}_{t+k|t}|,\ w_{t|t} = V(x(t))\\ &\hat{x}_{t+k+1|t} \in \hat{X}(t+k+1, v_{t+k+1|t}, w_{t+k+1|t})\\ &K_0\hat{x}_{t+k|t} + \tilde{u}_{t+k|t} \in \hat{U}(t+k, v_{t+k|t}, w_{t+k|t}) \end{aligned} \tag{22}$$

also satisfies the constraints for real system in (1)

$$\begin{aligned} &x(t+k+1)\in X,\\ &K(\theta_t(t+k))x(t+k) + \tilde{u}(t+k\,|\,t)\in U \end{aligned} \tag{23}$$

for all possible $d(t+k)\in D$.

Based on Theorem 5, the optimization problem for the proposed adaptive MPC method is described for a given diagonal matrix $R\succ 0$ as follows:

Optimization problem for MPC:

$$\begin{aligned} &\min_{\tilde{u}} J(x(t), \tilde{u}_{t+|t}) := \sum_{k=0}^{N-1} \tilde{u}_{t+k|t}^T R\, \tilde{u}_{t+k|t}\\ &\text{subject to (22) and}\\ &w_{t+k|t} \le \omega,\ w_{t+N|t} \le 1,\ k = 0,\ 1,\cdots,N-1. \end{aligned} \tag{24}$$

In (24), the finite-horizon optimal control problem without the disturbance term due to parameter estimation error is solved from the measured state $x(t)$ at the current time instant *t*. It is easily verified from Theorem 5 that, if the problem in (24) is feasible at each update time, the given constraints are always satisfied.

In (24), the additional constraints for $w_{t+k|t}$ were introduced to guarantee the feasibility at each time instance. The constant ω is a number satisfying $\omega \ge \max\{V(x_0), 1\}$ and the terminal condition $w_{t+N|t} \le 1$ guarantees $x(k+N)\in X_f$ for real system. Although ω is desired to be as large as possible for the feasibility at the current time instant, the value of ω at the next time instant should be bounded to guarantee the feasibility, which is verified in Section 5.

Remark 3: The third constraint in (22) is a nonlinear equation of $\tilde{u}_{t+k|t}$. By introducing a new variable $\chi_{t+k|t} \in R$, we can modify this constraint to

$$w_{t+k+1|t} = a_{t+k|t} w_{t+k|t} + b\chi_{t+k|t}$$
$$|\tilde{u}_{t+k|t}| \le \chi_{t+k|t}, \quad w_{t|t} = V(x(t)) \tag{25}$$

and the cost function $J(x, \tilde{u}_{t+k|t})$ to $J(x, \chi_{t+k|t})$. Therefore, the modified optimization problem has only linear constraints and can be solved by standard quadratic programming (QP) method with free variables $\tilde{u}_{t+k|t}$ and $\chi_{t+k|t}$.

The algorithm iterating the parameter update in (8) and the modified robust MPC method in this section is an adaptive MPC method proposed in this paper.

5. FEASIBILITY AND STABILITY

In order to ensure that the optimization problem in (24) is feasible at each time step, we need the following assumption defining the upper bound of ω, as mentioned in Section 4.

Assumption 1' *The given* $\omega (\ge \max\{V(x_0), 1\})$ *in (24) satisfies*

(i) $\omega \nu(t) c_\zeta \le \sqrt{\underline{\lambda}(P)} \min_i \psi_i - 1,$

(ii) $\omega[(\nu(t)+\nu_0)(2\nu(t)c_{\xi 1}+1) + \nu(t)c_{\xi 2}] \le \eta\sqrt{\underline{\lambda}(P)} - \| K_0^T \|$

where

$$c_\zeta := \sum_{s=0}^{N-1} \| \zeta(s) \|_\infty, \; c_{\xi 1} := \sum_{s=0}^{N-1} \| \xi(s) \|, \; c_{\xi 2} := \sum_{s=0}^{N-1} | K_0 \xi(s) |$$

Note that Assumption 1' is a sufficient condition for Assumption 1. If Assumption 1' cannot be satisfied for any $\omega \ge \max\{V(x_0), 1\}$, we need to consider a smaller terminal set X_f or modify the term K_0 of feedback gain in (13). It is also important to notice that once the state is steered into the robustly invariant constraint set X_f, the control law in (17) is completely switched to the feedback law $\hat{u} = K(\theta_t)\hat{x}$, since it is the optimal control in X_f in terms of the cost function in (24). That is, the control law of the proposed method converges to the given feedback law $\hat{u} = K(\theta_t)\hat{x}$.

The following Theorem 6 and Lemma 7 describe the properties of feasibility and stability of the proposed adaptive MPC method.

Theorem 6 (Stability) *Assume the optimization in (24) is feasible at* $t = 0$ *for* ω *which satisfies Assumption 1'. Then, the proposed MPC method has the following properties.*

(i) The optimization in (24) is feasible at time $t > 0$

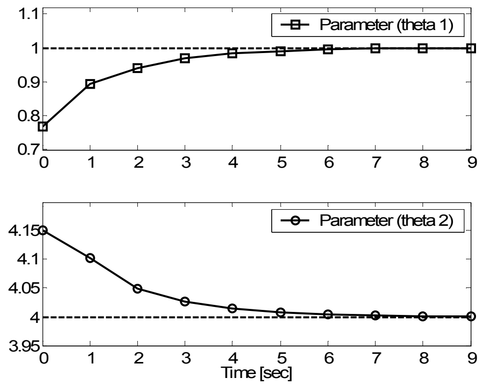

Fig. 1. Estimated uncertain parameter $\theta(t)$ using (8)

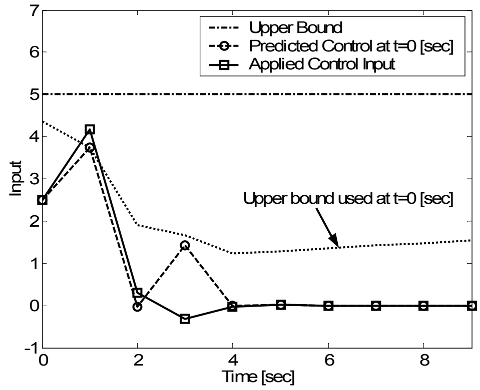

Fig. 2. Control input $u(t)$ (solid line) and predicted trajectory $K_0\hat{x}_{\cdot|t} + \tilde{u}_{\cdot|t}$ at $t = 0$ (dashed line)

(ii) $x(t)$ *converges to the origin as* $t \to \infty$

The following Lemma 7 is a key result to prove Theorem 6 and means that if the proposed MPC problem is feasible at $t = 0$, then the problem is feasible at each time step.

Lemma 7 (Feasibility) *Assume the optimization problem in (24) is feasible at the current time t for* ω *which satisfies Assumption 1'. Then, at the next time step* $t+1$,

$$\tilde{u}_{t+k|t+1} = \begin{cases} \tilde{u}^*_{t+k|t}, & k = 1, 2, \cdots, N-1 \\ 0 \quad , & k = N \end{cases}$$

could be a feasible solution of (24), where $\tilde{u}^*_{t+k|t}$ *denotes the optimal solution at* t.

6. NUMERICAL EXAMPLE

Consider the following discrete-time linear time-invariant system in controllable canonical form:

$$x(t+1) = \begin{bmatrix} 1 & 4 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t), \theta^* = [1, 4]^T, x_0 = \begin{bmatrix} -1 \\ -0.4 \end{bmatrix}$$

We assume the initial estimate θ_0 of θ^* and its estimation error bound ν_0 as follows:

$\theta_0 = [0.77,\ 4.15]^T, \quad \nu_0 = 0.275.$

The state and input constraints are given as

$|x_i(t)| \le 1.0, \quad |u(t)| \le 5.0, \quad i = 1, 2.$

A feedback gain K_0 and matrix P are chosen as

$$K_0 = [-0.7745 \quad -4.1370],\ P = \begin{bmatrix} 15.0054 & -0.0689 \\ -0.0689 & 30.0054 \end{bmatrix}$$

We choose the length of prediction horizon $N = 100$ and the terminal set as follows:

$$X_f = \{x(t) \in R^n : V(x(t)) \le 0.7\}$$

For an adaptive parameter estimation algorithm, we choose the estimation horizon $N_e = 10$ and other design parameters $\alpha = 0.1$ and $\kappa = 1.8$.

The convergence of estimated parameters by the proposed method in Section 3 to their true values is shown in Fig. 1. In Fig. 2, the solid line shows the applied control trajectory $u(t)$ and the dashed line shows the predicted trajectory $K_0\hat{x}_{t+k|t} + \tilde{u}_{t+k|t}$ at $t = 0$, which is obtained by the proposed adaptive MPC method. This shows that the control input obtained by the proposed adaptive MPC satisfies the given constraints. The doted lines show the upper bound calculated based on the constraint sets $\hat{X}$ and $\hat{U}$ in (22) at time step $t = 0.0$[sec]. These show that, since the adaptation algorithm is applied in MPC as shown in Section 4, the parameter estimation error bound $\nu_{t+k|t}$ and thus $\hat{\eta}(t+k, \nu_{\cdot|t}, w_{\cdot|t})$ in (21) become smaller over the prediction horizon. In other words, the parameter uncertainty is considered explicitly by the proposed adaptive parameter estimation algorithm and therefore MPC problem becomes less conservative as steps passes on. The trajectory of the state is shown in Fig. 3. This shows that the trajectory of the state goes into the terminal set and the state $x(t)$ converges to the origin as $t \to \infty$.

7. CONCLUSION

In this paper, we have proposed a discrete-time adaptive MPC algorithm for a constrained linear systems with uncertain parameters, which updates the estimation of system parameters on-line and produces the control input subject to the given input/state constraints. In order to construct such an adaptive MPC method, we first have proposed a new parameter update algorithm based on the moving horizon estimation method. It allows us to predict the worst-case estimation error bound over the given prediction horizon. We then have incorporated the above algorithm with the robust MPC method based on the modified comparison model extended to be applicable to the time varying-case. Furthermore, we have shown that the proposed algorithm guarantees the feasibility and stability of the closed-loop systems in the presence of constraints. Future work will address the extension of applicable class of plants, the development of less conservative error estimation methods and so on.

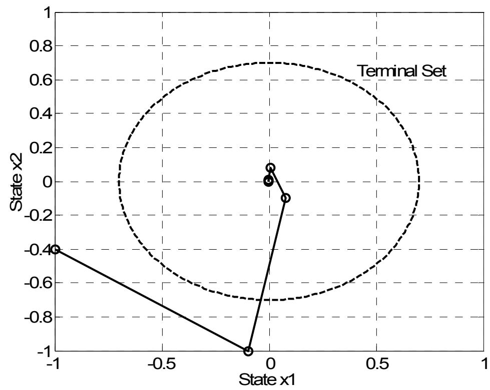

Fig. 3. Trajectory of the states $x_1(t)$ and $x_2(t)$

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